チャーム・ボトムの
エキゾチックハドロン物理の最近の発展

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内容

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3. Heavy exotic hadrons -X, Y, Z hadrons-

Charm/bottom exotic hadrons

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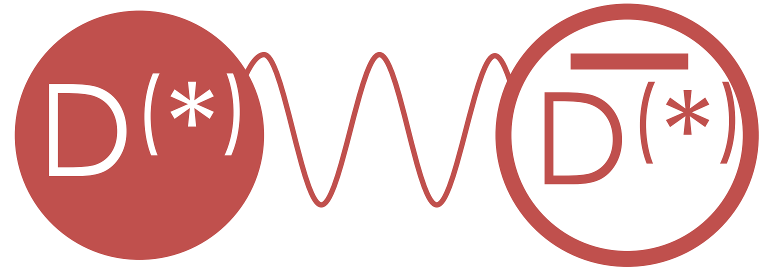
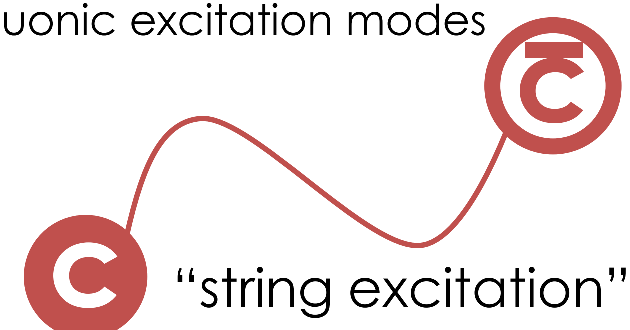
Exotic Hadrons

Compact multiquark



Hadronic molecule

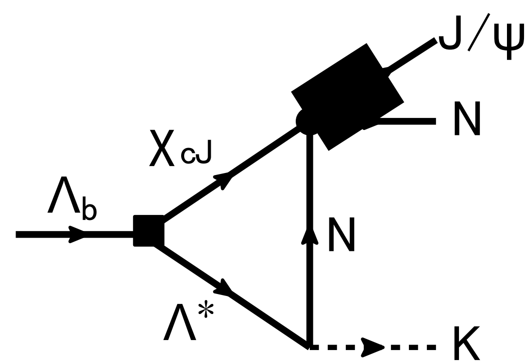
gluonic excitation modes



“string excitation”

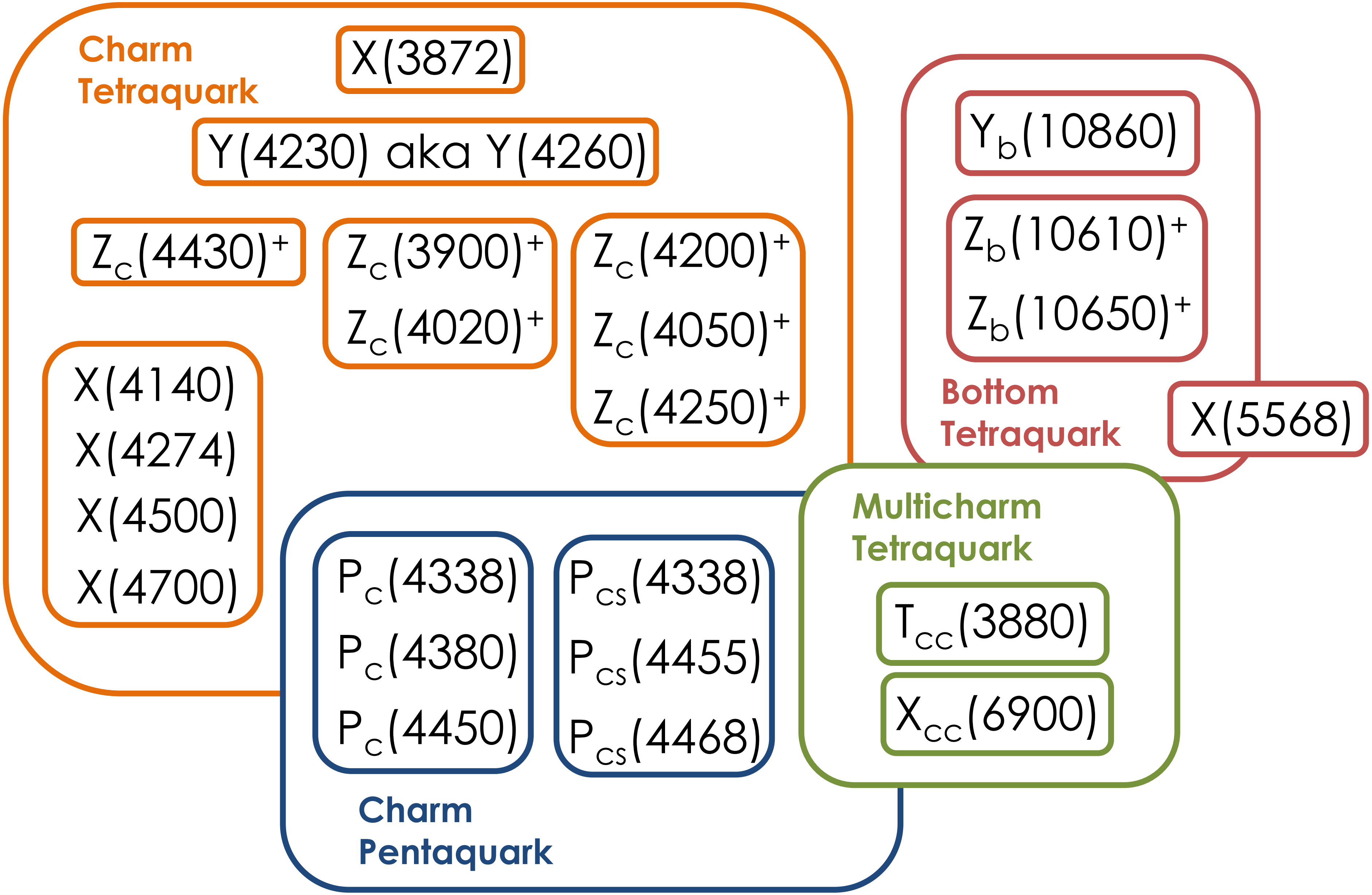
Σ, Π, Δ, Φ, ...

(S, P, D, F, ...)



Kinematic effect

3. Heavy exotic hadrons -X, Y, Z hadrons-



3. Heavy exotic hadrons -X, Y, Z hadrons-

Charm

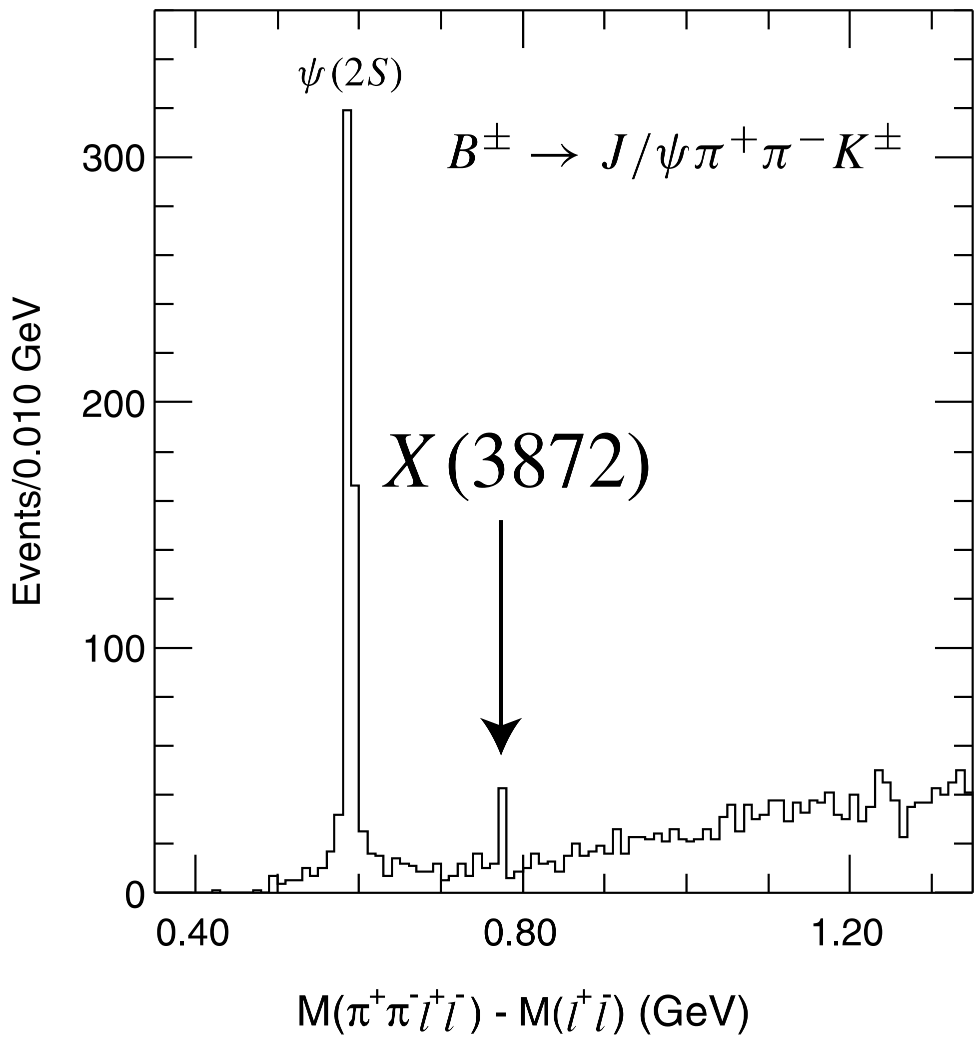
Tetraquark

3. Heavy exotic hadrons -X, Y, Z hadrons-

X(3872)

3. Heavy exotic hadrons -X, Y, Z hadrons-
X(3872)

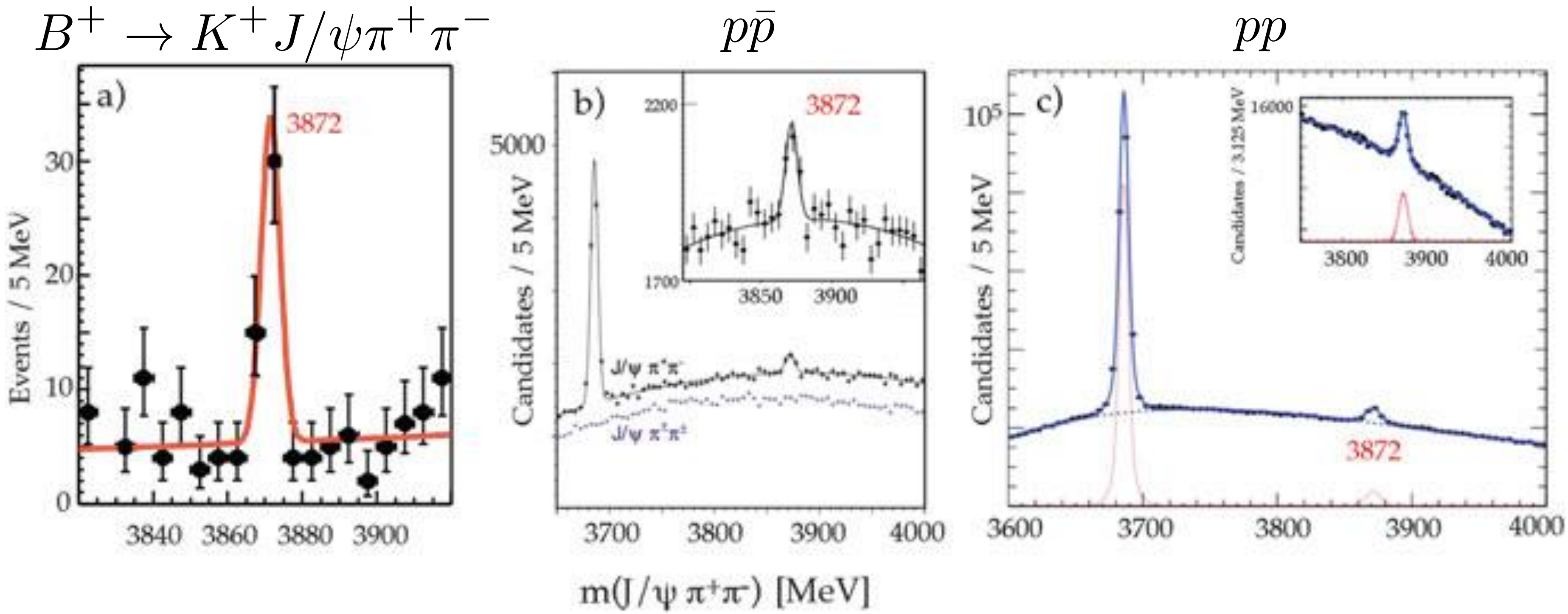
S. K. Choi et al. [Belle Collaboration], Phys. Rev. Lett. 91, 262001 (2003)



> 2600 citations

3. Heavy exotic hadrons -X, Y, Z hadrons- X(3872)

X(3872)→J/ψπ⁺π⁻ seen in different reactions



a) Choi, S.-K., et al.: Phys. Rev. Lett. 91, 262001 (2003)
b) Acosta, D., et al.: Phys. Rev. Lett. 93, 072001 (2004)
c) Chatrchyan, S., et al.: JHEP 04, 154 (2013)
Cf. Amsler, "The Quark Structures of Hadrons", Springer (2018)

3. Heavy exotic hadrons -X, Y, Z hadrons- X(3872)

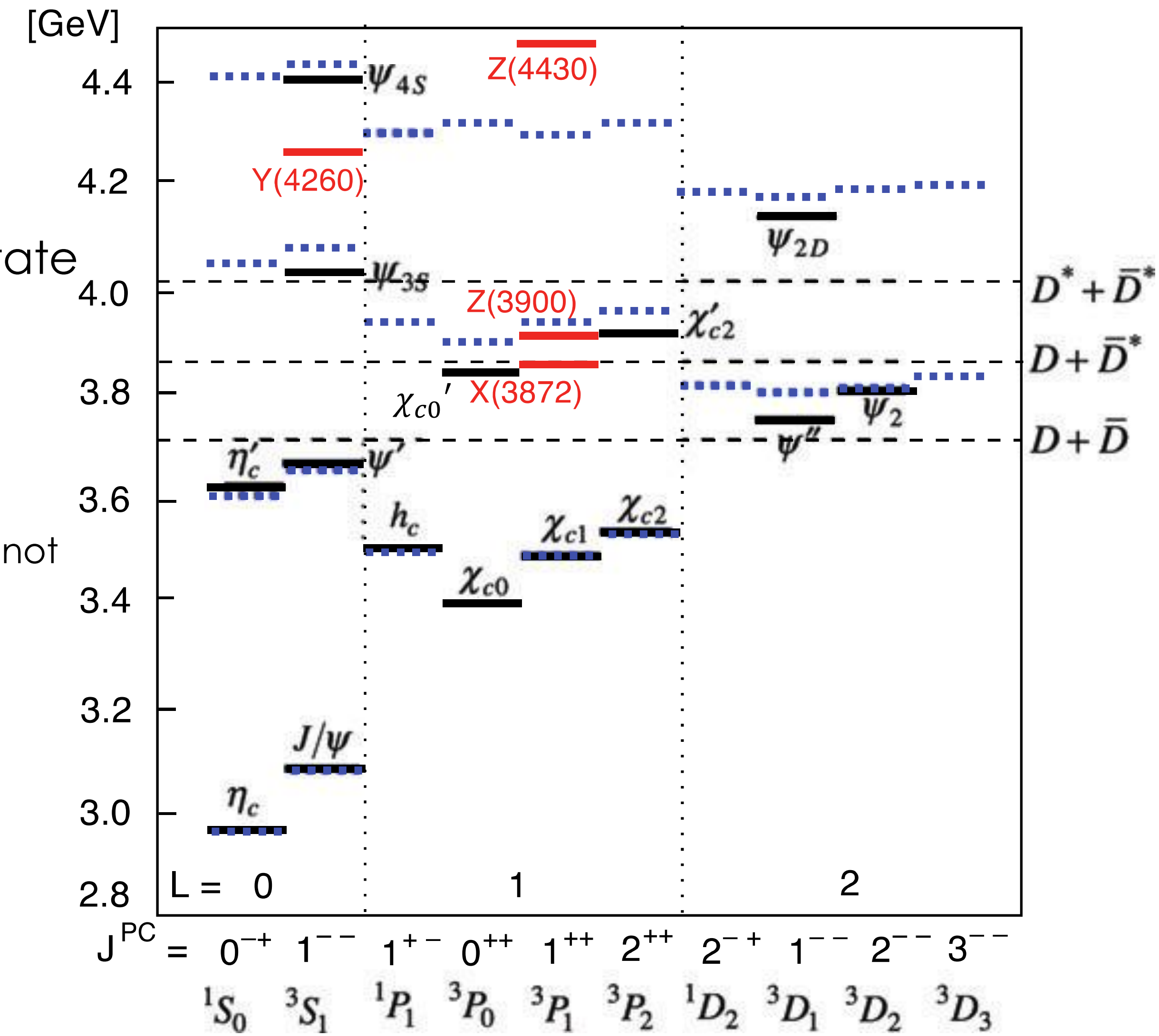
Mass of X(3872) is very close to DD*bar threshold.

Observed 2P state

$\chi_{c0}'(3860)$

$\chi_{c2}'(3930)$

Note:
$\chi_{c1}'(2P)$ and $h_c'(2P)$ not observed yet



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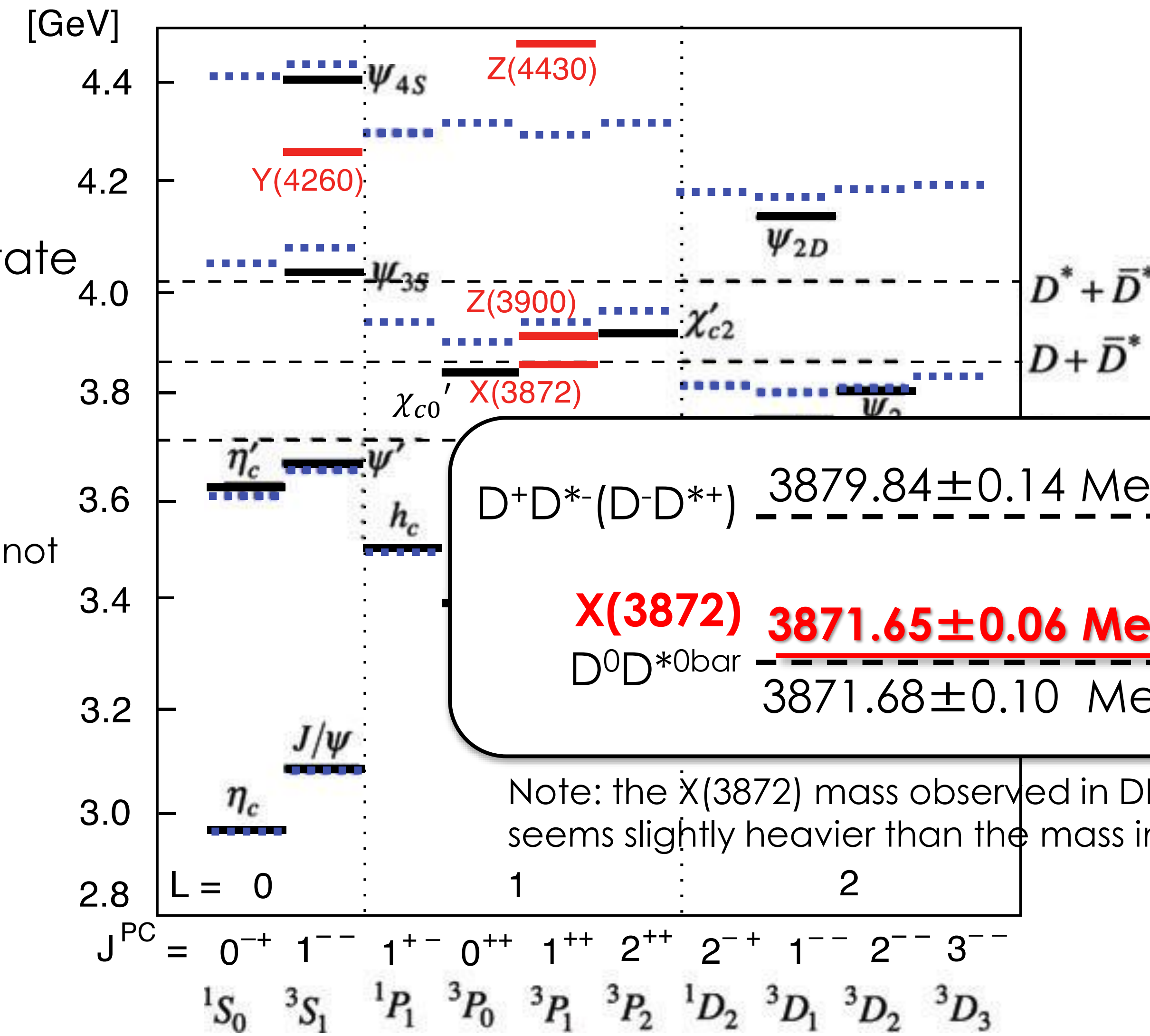


$D^+D^{*-}(D^-D^{*+})$ 3879.84±0.14 MeV

isospin symmetry breaking

X(3872) **3871.65±0.06 MeV**

D^0D^{*0bar} 3871.68±0.10 MeV

Note: the X(3872) mass observed in DD* channel seems slightly heavier than the mass in J/ψππ channel.

3. Heavy exotic hadrons -X, Y, Z hadrons- X(3872)

What is quantum number (J^{PC}) ?

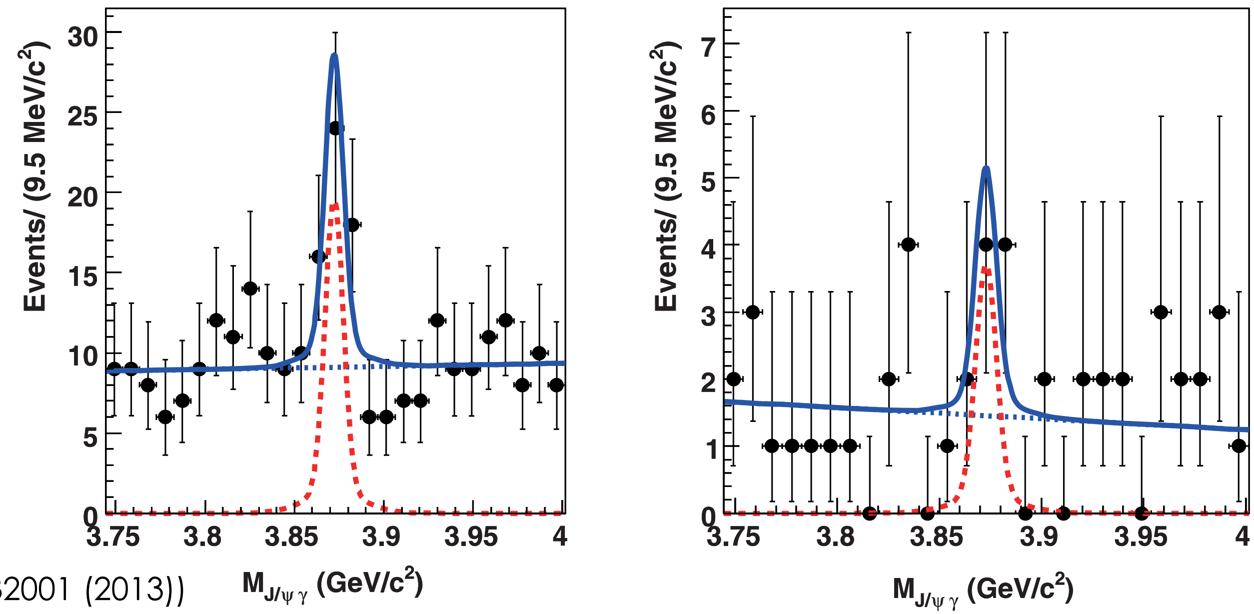
V. Bhardwaj et al. [Belle], Phys. Rev. Lett. 107, 091803 (2011)
(Cf. B. Aubert et al. [BaBar], Phys. Rev. Lett. 102, 132001 (2009))

① photon decay

$$X(3872) \rightarrow J/\psi\,\gamma$$

C=-1 C=-1

X(3872) C=+1



No signature for C=-1: $J/\psi\eta$(PTEP2014, 043C01 (2014)) and χ_{c1}(2P) (PRL111, 032001 (2013))

② angular distribution of decay products

$$X(3872) \longrightarrow J/\psi\pi^+\pi^-$$

A. Abulencia et al. [CDF], Phys. Rev. Lett. 98, 132002 (2007)
B. S.-K. Choi et al. [Belle], Phys. Rev. D 84, 052004 (2011)
R. Aaij et al. [LHCb], Phys. Rev. Lett. 110, 222001 (2013)

$$X(3872) \longrightarrow J/\psi\pi^+\pi^-\pi^0$$

P. del Amo Sanchez et al. [BaBar], Phys. Rev. D 82, 011101 (2010)

X(3872) $J^{PC}=1^{++}$ or 2^{-+} ➡ $J^{PC}=$**1^{++}**

3. Heavy exotic hadrons -X, Y, Z hadrons-
X(3872)
Tetrquark interpretation

L. Maiani, F. Piccinini, A. D. Polosa, and V. Riquer, Phys. Rev. D 71, 014028 (2005)

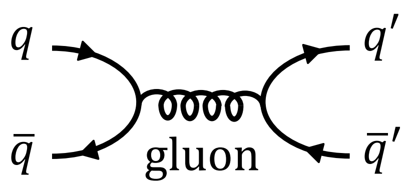
① Isospin symmetry breaking → particle eigenstates (isospin sym. discarded)

$$X_u = [cu][\bar{c}\,\bar{u}] \qquad X_d = [cd][\bar{c}\,\bar{d}]$$

If isospin eigenstate exists… $\underset{\mathrm{I=0}}{f_{c\bar{c}}} = (X_u + X_d)/\sqrt{2}$ $\underset{\mathrm{I=1}}{a_{c\bar{c}}} = (X_u - X_d)/\sqrt{2}$

② Mixing of uu$^{\mathrm{bar}}$ and dd$^{\mathrm{bar}}$ via gluon exchange (interaction strength δ)

Hamiltonian: $$\begin{array}{c|cc} & X_u & X_d \\ \hline X_u & \multicolumn{2}{c}{\multirow{2}{*}{$\begin{pmatrix} 2m_u + \delta & \delta \\ \delta & 2m_d + \delta \end{pmatrix}$}} \\ X_d & & \end{array}$$



δ: flavor-blind coupling (same for $u\bar{u}$ and $d\bar{d}$)

③ Eigenstates of Hamiltonian: *two states* appear

$$X_{\mathrm{low}} = \cos\theta X_u + \sin\theta X_d \qquad X_{\mathrm{high}} = -\sin\theta X_u + \cos\theta X_d$$

mass difference: $$M(X_h) - M(X_l) = 2(m_d - m_u)/\cos(2\theta) = \\ = (7 \pm 2)/\cos(2\theta)\ \mathrm{MeV}$$

3. Heavy exotic hadrons -X, Y, Z hadrons-
X(3872)
Tetrquark interpretation

Searching *two states* in B decays (by experiments)

$B^{\pm} \to X K^{\pm}$ $\quad B^0 \to X K^0$ (Replacing u and d)

$X(3872) \longrightarrow J/\psi \pi^+ \pi^-$

mass difference

B. Aubert et al. [BaBar], Phys. Rev. D 77, 111101 (2008)

$$(2.7 \pm 1.6\,(\mathrm{stat}) \pm 0.4\,(\mathrm{syst}))\ \mathrm{MeV}/c^2$$

S.-K. Choi et al. [Belle], Phys. Rev. D 84, 052004 (2011)

$$(-0.71 \pm 0.96\,(\mathrm{stat}) \pm 0.19\,(\mathrm{syst}))\ \mathrm{MeV}/c^2$$

No mass difference → Inconsistent with tetraquark...

Charged state？

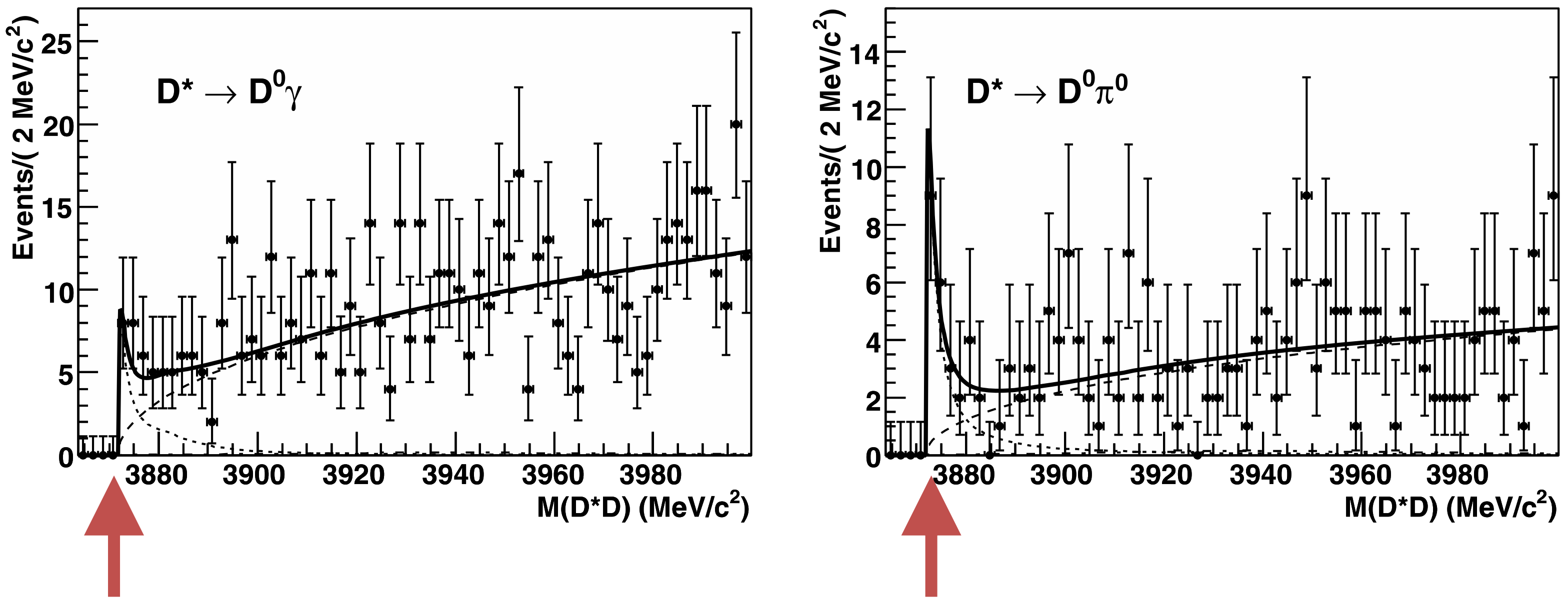
$X(3872) \longrightarrow J/\psi \pi^{\pm} \pi^0$(？) No signature in experiments.

S.-K. Choi et al. [Belle], Phys. Rev. D 84, 052004 (2011)

3. Heavy exotic hadrons -X, Y, Z hadrons- X(3872)
Dominance of D^0D^{*0bar} component？

$$B \to \underline{D^0 \bar{D}^{*0}} K$$

X(3872)？



Excess at D^0D^{*0bar} threshold was found.

$$\mathcal{B}\left(X(3872) \to D^0\overline{D}^{*0}\right) \approx \underline{10\times}\mathcal{B}\left(X(3872) \to J/\psi\pi^+\pi^-\right)$$

Branching fraction B(channel):
the ration of the decay width by the specific channel against the total decay width

T. Aushev et al. [Belle], Phys. Rev. D 81, 031103 (2010)
(cf. B. Aubert et al. [BaBar], Phys. Rev. D 77, 011102 (2008))

3. Heavy exotic hadrons -X, Y, Z hadrons-
X(3872)
$D^0 D^{*0bar}$ molecule interpretation

$D^+D^{*-}(D^-D^{*+})$ 3879.84±0.14 MeV

X(3872) **3871.65±0.06 MeV**

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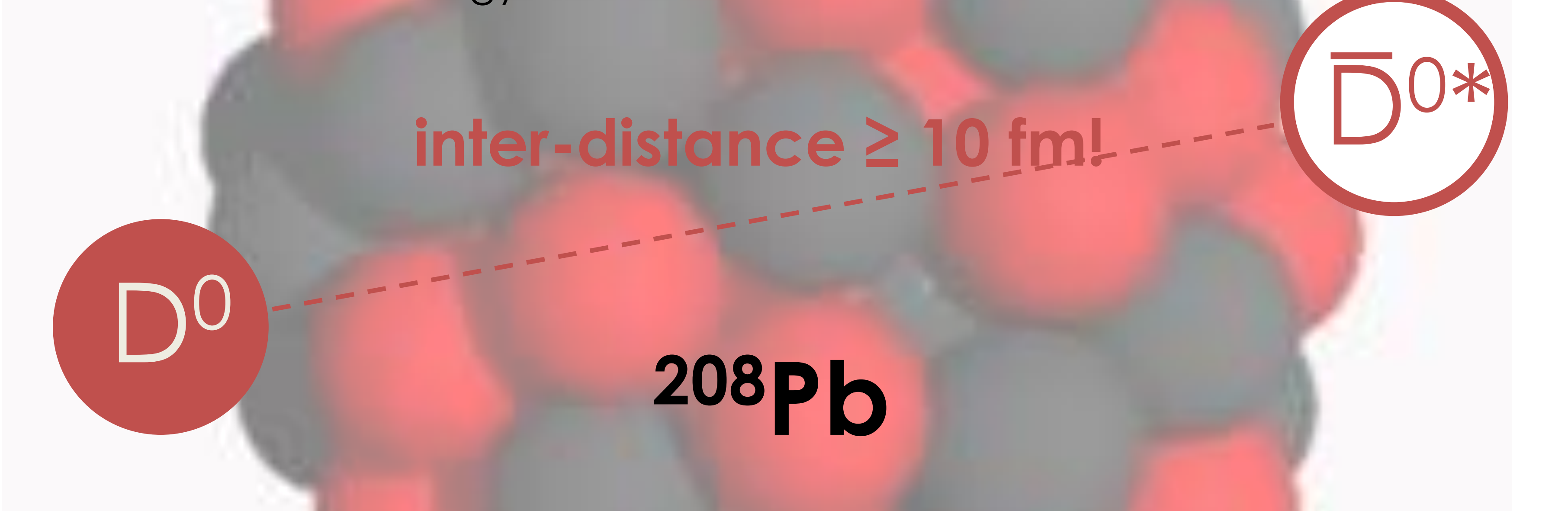
isospin symmetry breaking

The energy from threshold is less than 0.1 MeV...



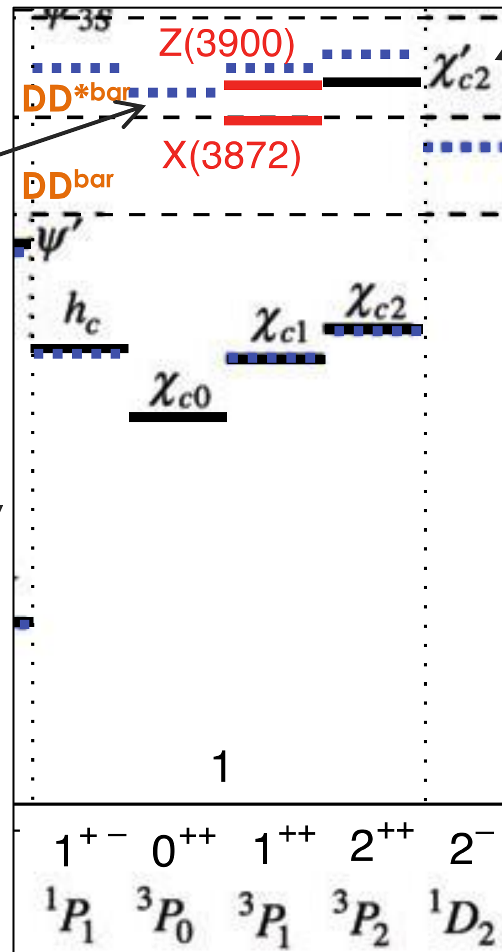
Such fragile particle must be difficult to be produced in pp^{bar} collisions...

3. Heavy exotic hadrons -X, Y, Z hadrons-
X(3872)
Likely NOT a simple $D^0 D^{*0bar}$ molecule... What is this?

R. Aaij et al. [LHCb], Phys. Rev. Lett. 110, 222001 (2013)

Hint: absence of $\chi_{c1}(2P)$

Observed by Belle, Phys. Rev. Lett. 96, 082003 (2006)



?

$\chi_{c0}(2P) = \chi_{c0}(3915)$

Observed by Belle, Phys. Rev. Lett. 104, 092001 (2010)

Under the debates:

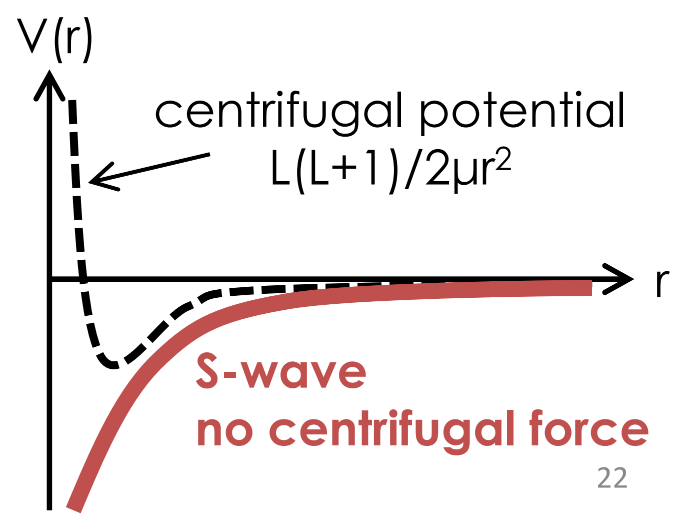
- Suppression in decay to DD^{bar} in experiments
- $cc^{bar}ss^{bar}$ tetraquark?

No observation of $\chi_{c1}(2P)$

↓

Is X(3872) mixed with $\chi_{c1}(2P)$?

DD^{*bar} (S-wave) can easily couple to $\chi_{c1}(1^{++})$.



3. Heavy exotic hadrons -X, Y, Z hadrons-
X(3872)
Admixture of $D^0\bar{D}^{*0}$ molecule and $\chi_{c1}(2P)$

Model setting (simple!) E. J. Eichiten et al., PRD73,014014(2006); A. M. Badalian et al., PRD85,031103(2012)
M. Takizawa and S. Takeuchi, Prog. Theor. Exp. Phys. 2013, 093D01 (2013)

① Wave function as a superposition

$$|X\rangle = c_1|\bar{c}c\rangle + c_2\left|D^0\bar{D}^{*0}\right\rangle + c_3\left|D^{\pm}\bar{D}^{*\mp}\right\rangle$$

② Coupling between $D^0\bar{D}^{*0}$ (D^+D^{*-}) and $\chi_{c1}(2P)$: $D\bar{D}^{*} \rightleftarrows \chi_{c1}(2P)$

$$\langle D^0\bar{D}^{*0}(\boldsymbol{q})|V|c\bar{c}\rangle = \langle D^+D^{*-}(\boldsymbol{q})|V|c\bar{c}\rangle = \frac{g}{\sqrt{\Lambda}}\left(\frac{\Lambda^2}{q^2+\Lambda^2}\right)$$

③ Hamiltonian (3 × 3 matrix)

g: coupling constant
Λ: momentum cutoff

$$H = \begin{pmatrix} m_{\bar{c}c} & \boxed{V} & \boxed{V} \\ \boxed{V} & m_{D^0\bar{D}^{*0}} + K & 0 \\ \boxed{V} & 0 & m_{D^{\pm}\bar{D}^{*\mp}} + K \end{pmatrix}$$

Parameter set: 0.3 GeV ≤ Λ ≤ 1.0 GeV (g is fixed to reproduce 3872 MeV mass.)

Λ [GeV]	0.3	0.5	1.0
g	0.054 35	0.051 10	0.048 35

3. Heavy exotic hadrons -X, Y, Z hadrons- X(3872)
Admixture of $D^0 D^{*0bar}$ molecule and $\chi_{c1}(2P)$

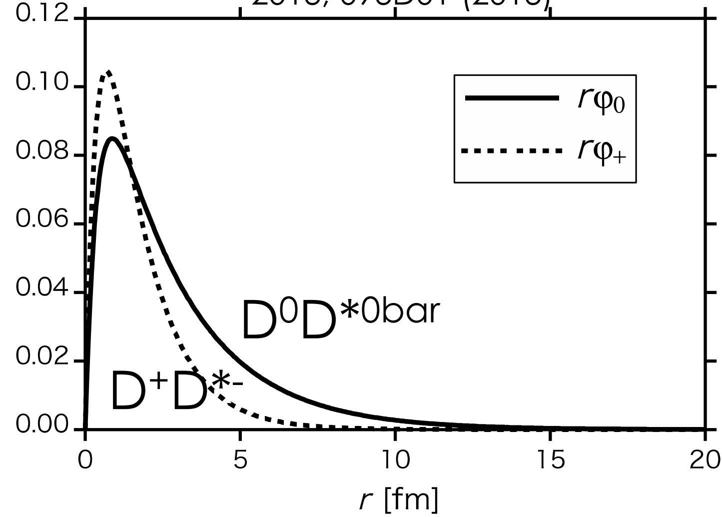
Result

M. Takizawa and S. Takeuchi, Prog. Theor. Exp. Phys. 2013, 093D01 (2013)

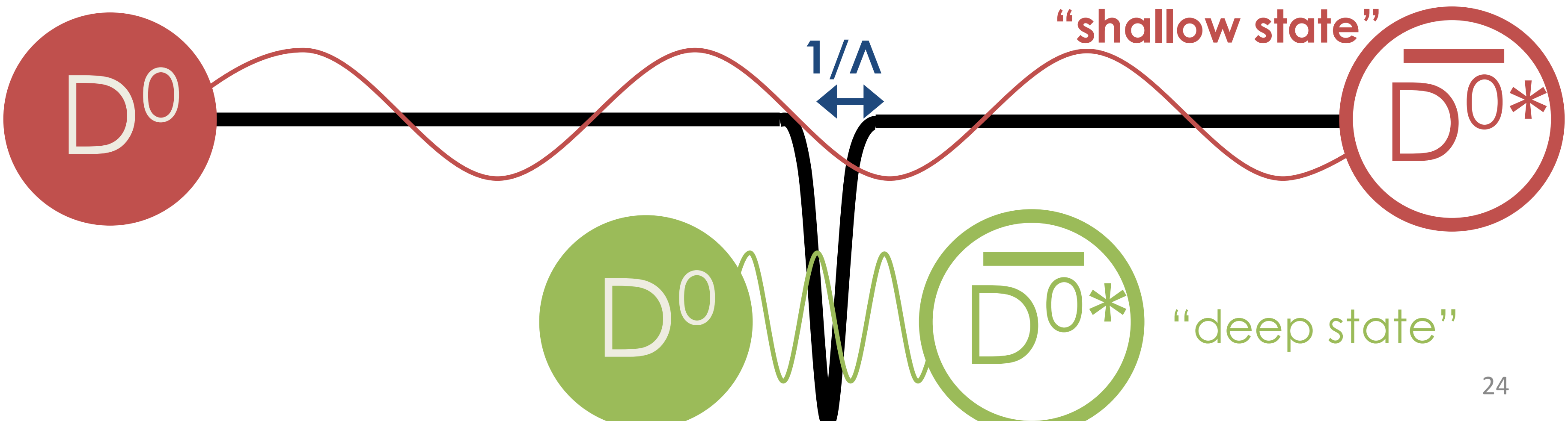
① Components in wave function

$$|X\rangle = c_1|\bar{c}c\rangle + c_2\left|D^0\bar{D}^{*0}\right\rangle + c_3\left|D^{\pm}\bar{D}^{*\mp}\right\rangle$$

Λ [GeV]	c_1	c_2	c_3
0.3	0.227	−0.947	−0.228
0.5	0.293	−0.920	−0.259
1.0	0.404	−0.871	−0.280



$c_{1,2,3}$ do not depend strongly in cutoff Λ.



3. Heavy exotic hadrons -X, Y, Z hadrons-
X(3872)
Admixture of $D^0 D^{*0bar}$ molecule and $\chi_{c1}(2P)$

Result

② Spectrum

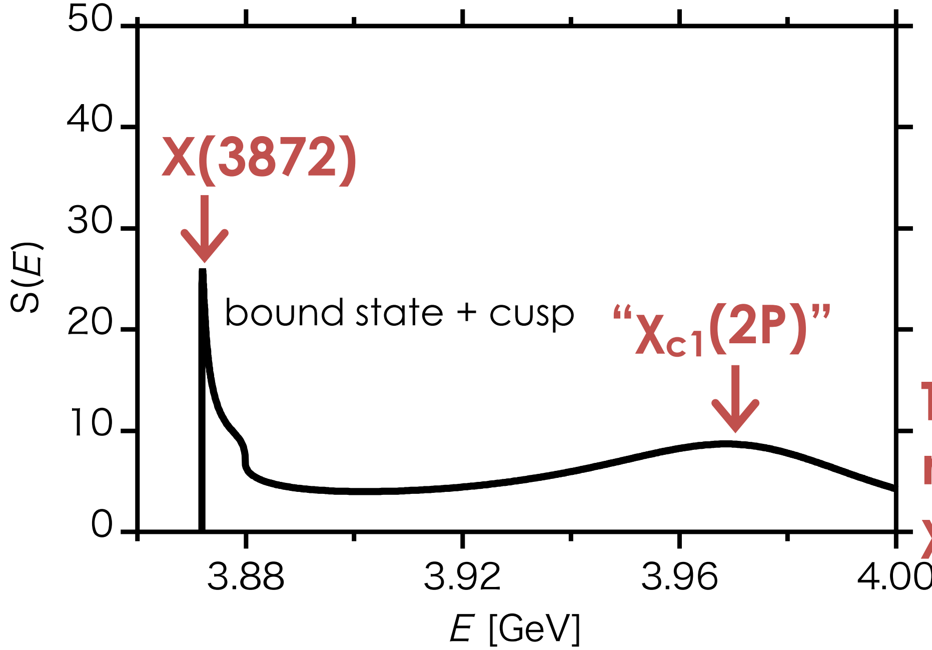
Where is this $\chi_{c1}(2P)$?

M. Takizawa and S. Takeuchi, Prog. Theor. Exp. Phys. 2013, 093D01 (2013)

$$|X\rangle = c_1 \boxed{|\bar{c}c\rangle} + c_2 \left|D^0 \bar{D}^{*0}\right\rangle + c_3 \left|D^{\pm} \bar{D}^{*\mp}\right\rangle$$

Spectral function of X(3872)

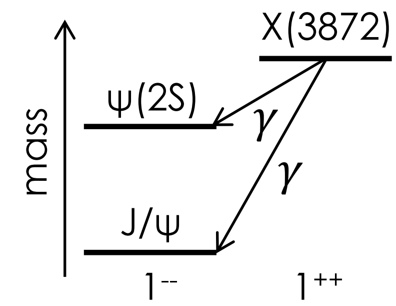
$$S(E) = \frac{-1}{\pi} \mathrm{Im}\langle c\bar{c}|G(E)|c\bar{c}\rangle \quad G(E) = \frac{1}{E - \hat{H} + i\varepsilon}$$



"χ_{c1}(2P)" exists as a broad peak.

This explains naturally no confirmation of χ_{c1}(2P) in experiments.

3. Heavy exotic hadrons -X, Y, Z hadrons-
X(3872)
Radiative decay



Branching ratios (R_γ): $X(3872)\to\psi(2S)\gamma$, $J/\psi\gamma$

$$\frac{\mathcal{B}(X(3872)\to\psi(2S)\gamma)}{\mathcal{B}(X(3872)\to J/\psi\gamma)} = 3.4 \pm 1.4$$

B. Aubert et al. [BaBar], Phys. Rev. Lett. 102, 132001 (2009)

$$\frac{\mathcal{B}(X(3872)\to\psi(2S)\gamma)}{\mathcal{B}(X(3872)\to J/\psi\gamma)} < 2.1 \text{ at } 90\% \text{ C.L.}$$

V. Bhardwaj et al. [Belle], Phys. Rev. Lett. 107, 091803 (2011)

$$\frac{\mathcal{B}(X(3872)\to\psi(2S)\gamma)}{\mathcal{B}(X(3872)\to J/\psi\gamma)} = 2.46 \pm 0.64\,(\text{stat}) \pm 0.29\,(\text{syst})$$

R. Aaij et al. [LHCb], Nucl. Phys. B886, 665 (2014)

Those suggest

$$\boxed{B(X(3872)\to\psi(2S)\gamma) \approx 3 \times B(X(3872)\to J/\psi\gamma)}$$

$X(3872)=\chi_{c1}(1P)+\chi_{c1}(2P)+D^0D^{*0\text{bar}}+D^{+/-}D^{*-/+\text{bar}}$

S. Takeuchi, M. Takizawa, K. Shimizu, arXiv:1602.04297 [hep-ph]

$$\Gamma\left(X(3872)\to\psi+\gamma\right) = \frac{4}{9}|Q_c|^2\alpha\,\frac{\omega_\gamma^3 E_\psi}{M_X}\left| Z_{c\bar{c}(1P)}\langle\psi|\,r\,|c\bar{c}(1P)\rangle + Z_{c\bar{c}(2P)}\langle\psi|\,r\,|c\bar{c}(2P)\rangle\right|^2$$

model	$g_{c\bar{c}(1P)-D\bar{D}}$	$Z^2_{c\bar{c}(1P)}$	$Z^2_{c\bar{c}(2P)}$	$\Gamma(X\to J/\psi)$	$\Gamma(X\to\psi(2S))$	R_γ	R_γ (spectrum)
A00	0	0	0.036	0.6	2.1	3.6	3.4
A01	$\frac{1}{10}g_{c\bar{c}(2P)-D\bar{D}}$	0.001	0.036	1.1	2.0	1.8	1.9
A10	$-g_{c\bar{c}(2P)-D\bar{D}}$	0.011	0.060	6.1	6.2	1.0	1.1

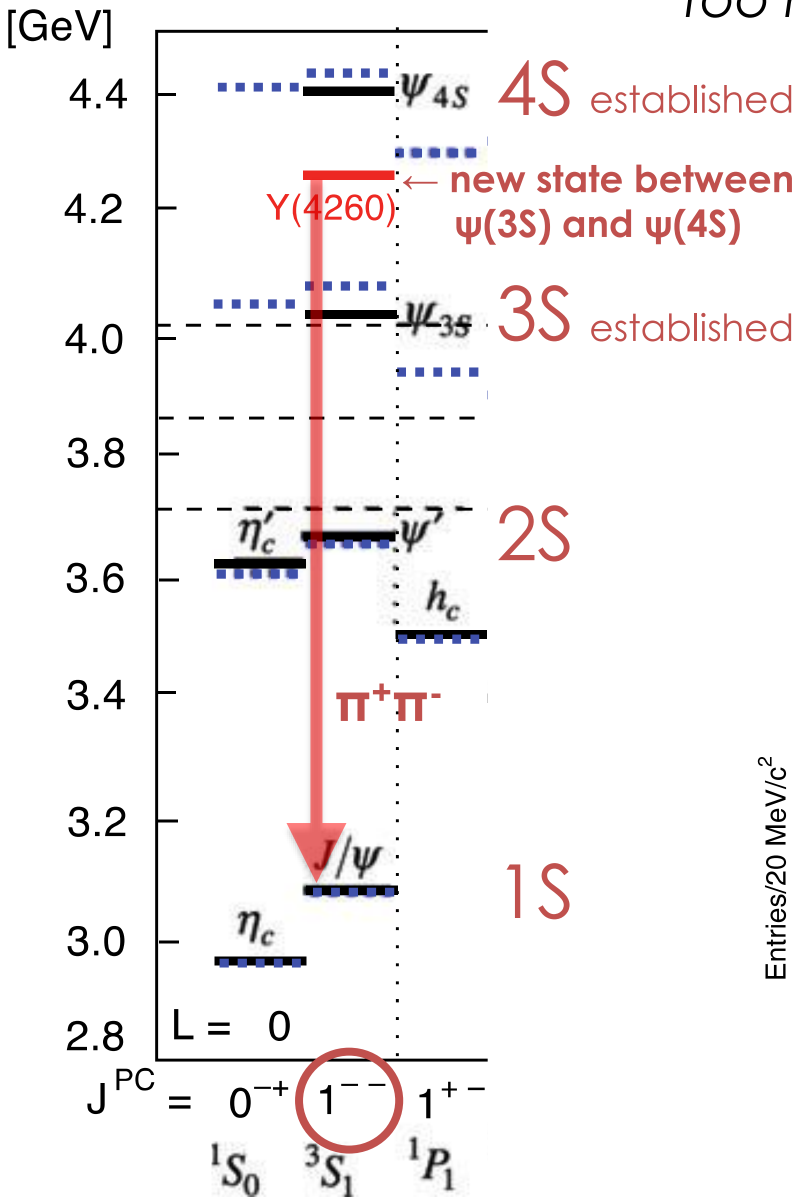
$\psi(4230)$
aka $Y(4260)$

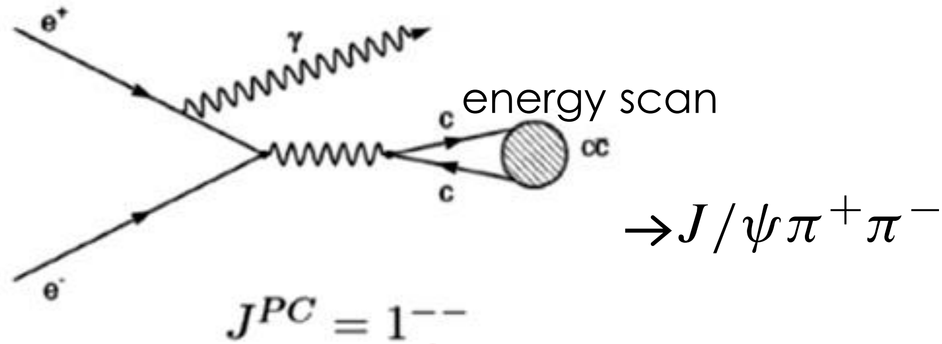
3. Heavy exotic hadrons -X, Y, Z hadrons-

ψ(4230) aka Y(4260)

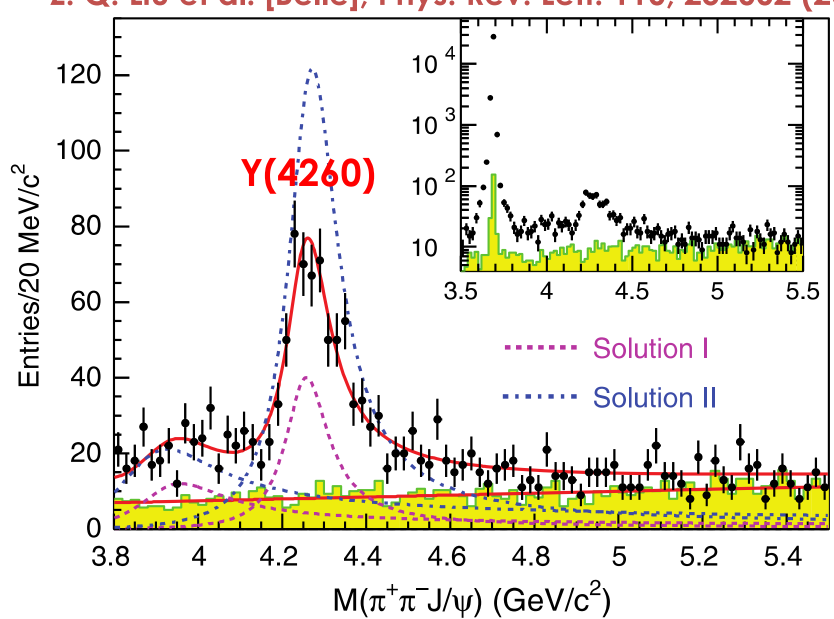
too many ψ'?

BaBar was searching X(3872) in 1^{--}. They did not find X(3872), but found Y(4260).





B. Aubert et al. [BaBar], Phys. Rev. Lett. 95, 142001 (2005)
C. Z. Yuan et al. [Belle], Phys. Rev. Lett. 99, 182004 (2007)
Z. Q. Liu et al. [Belle], Phys. Rev. Lett. 110, 252002 (2013)



3. Heavy exotic hadrons -X, Y, Z hadrons-

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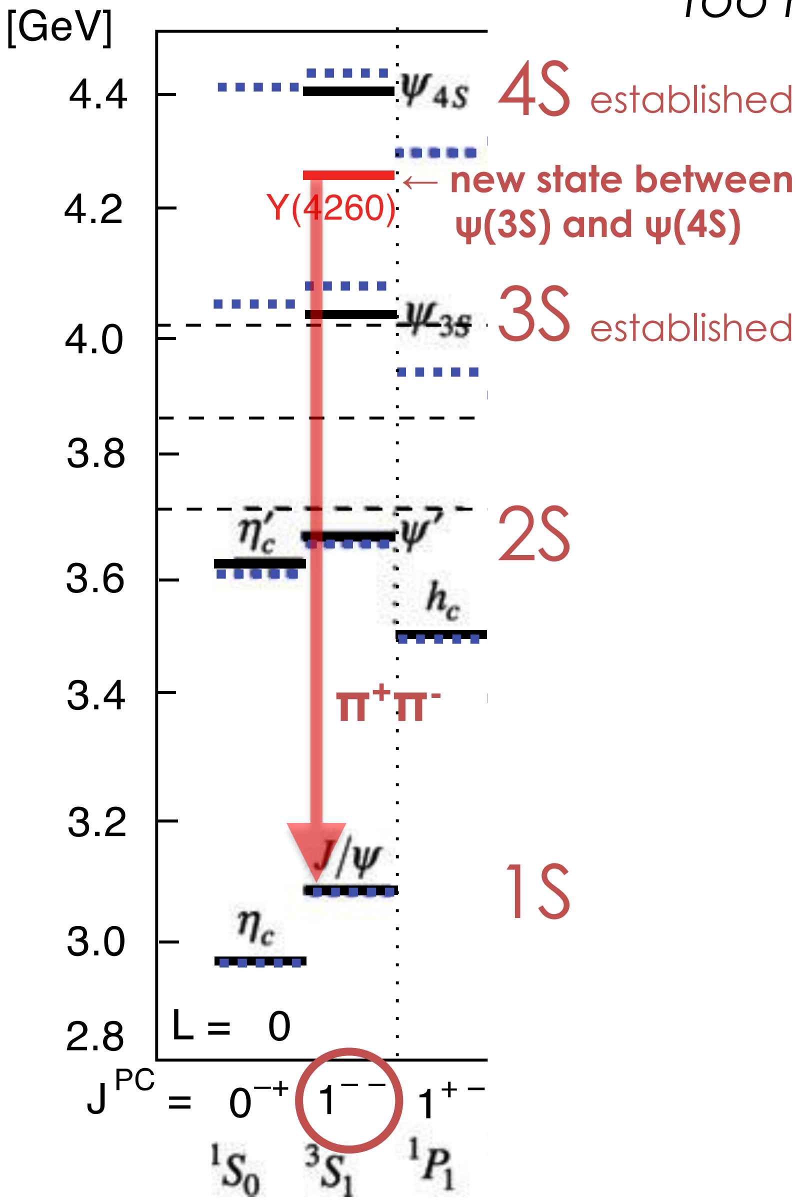
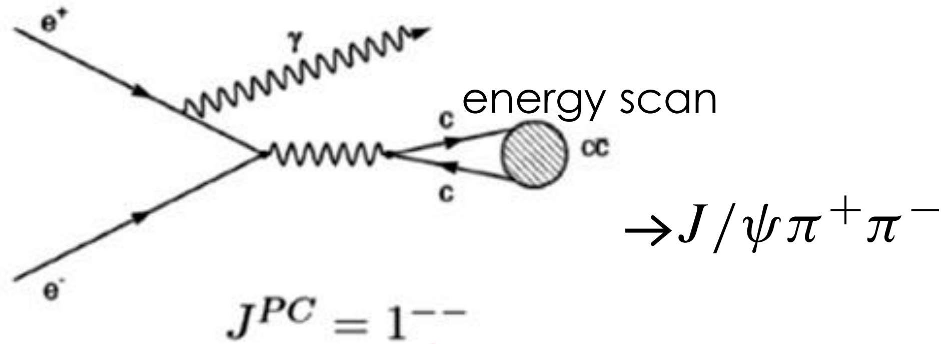




B. Aubert et al. [BaBar], Phys. Rev. Lett. 95, 142001 (2005)
C. Z. Yuan et al. [Belle], Phys. Rev. Lett. 99, 182004 (2007)
Z. Q. Liu et al. [Belle], Phys. Rev. Lett. 110, 252002 (2013)
M. Abikim et al. (BESSIII), Phys. Rev. Lett. 118, 092001 (2017)

Higher statistics

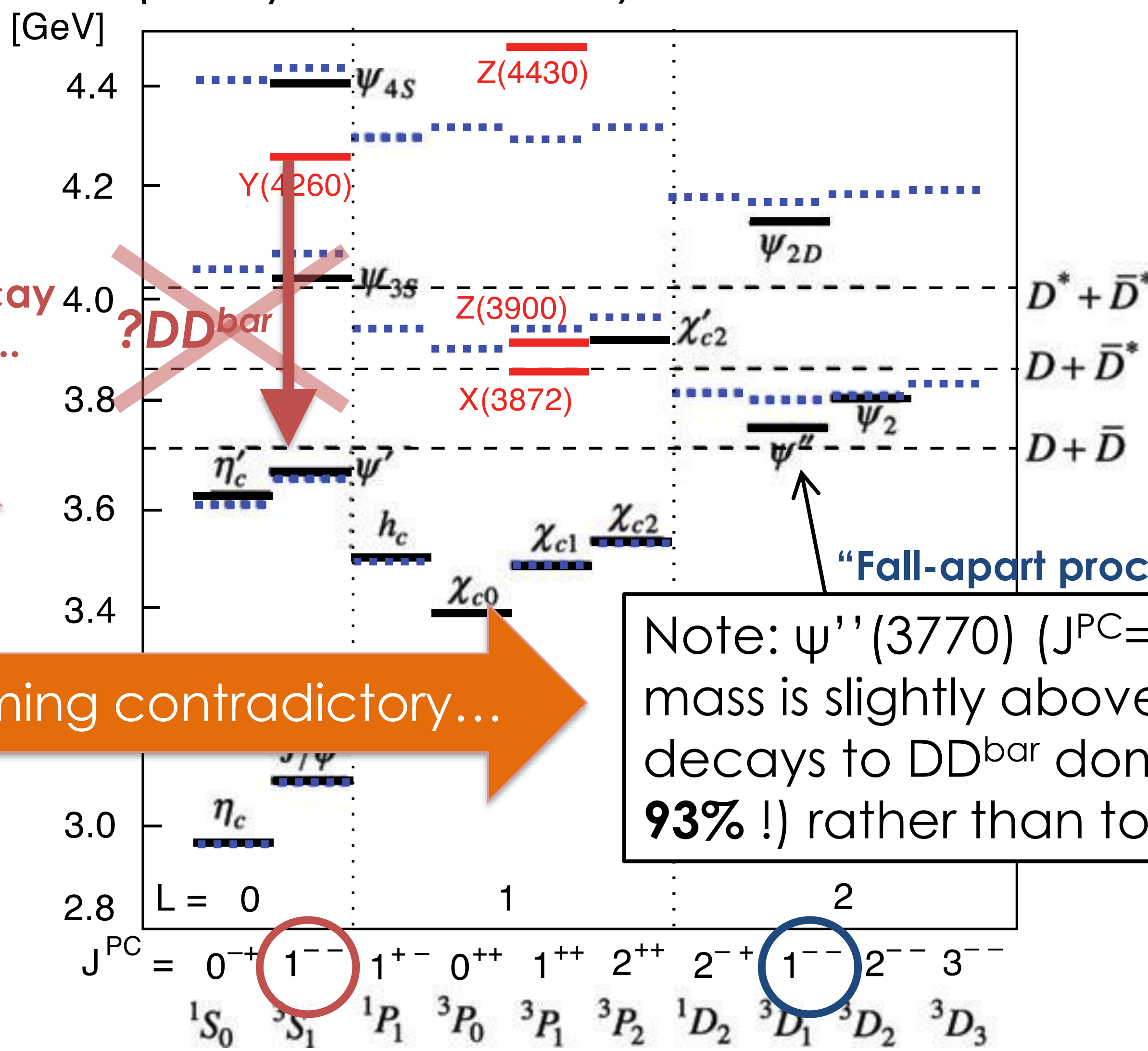
(1) Asymmetry in the cross section
(2) Shift-position of the peak mass to the lower: **ψ(4230)**

3. Heavy exotic hadrons -X, Y, Z hadrons-

ψ(4230) aka Y(4260)

mystery of decay patterns

*Y(4260) **should** decay to DD^bar, but…*



However, this decay is not observed…

Seeming contradictory…

"Fall-apart process"

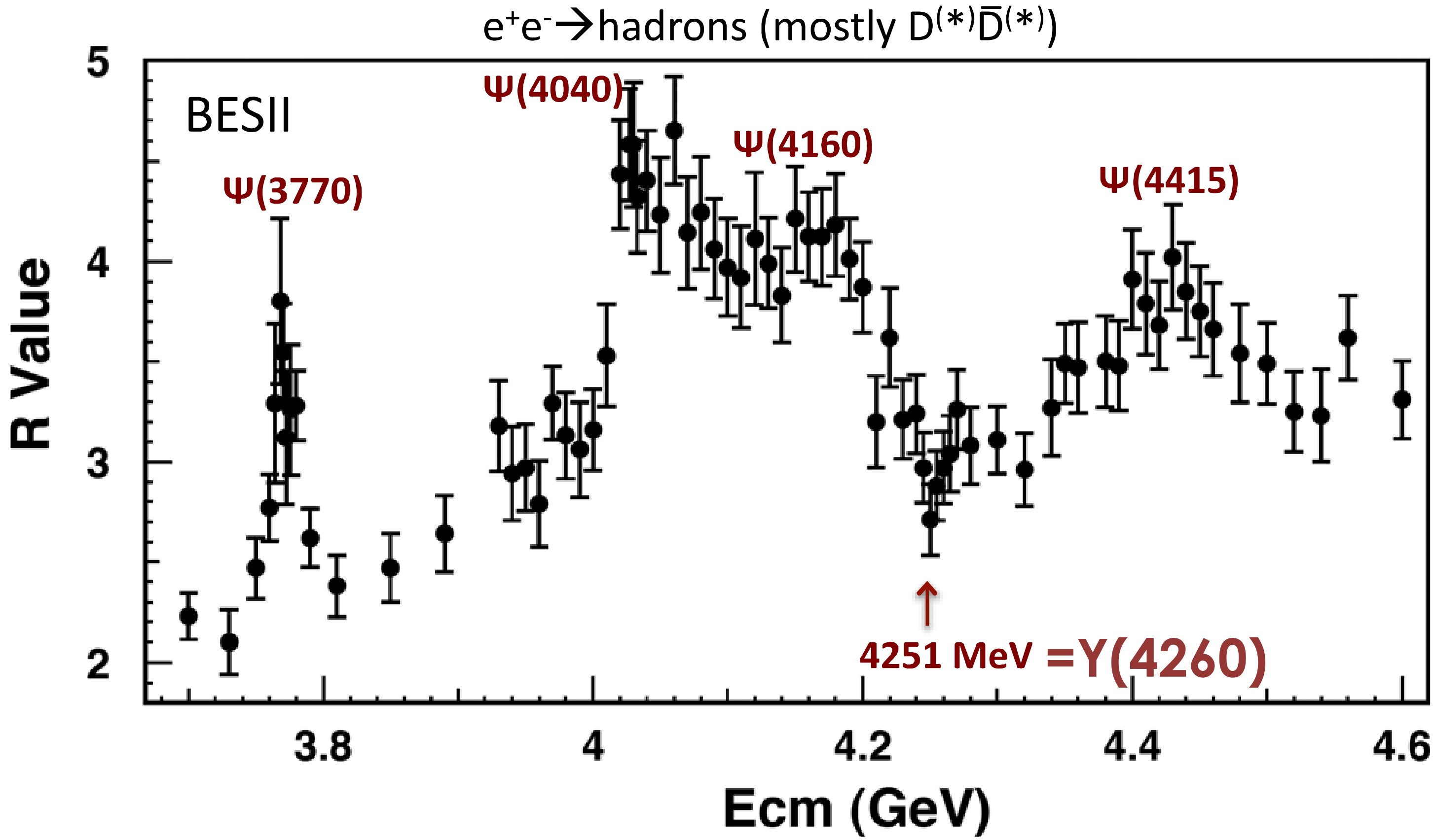
Note: ψ''(3770) ($J^{PC}=1^{--}$), whose mass is slightly above DD^bar, decays to DD^bar dominantly (**B.R. 93%** !) rather than to $J/\psi\pi^+\pi^-$.

3. Heavy exotic hadrons -X, Y, Z hadrons-
ψ(4230) aka Y(4260)

mystery of decay patterns

Y(4260) ***can*** *decay to DD^{bar}, but…*

There is no decay from Y(4260) to DD^{bar}.



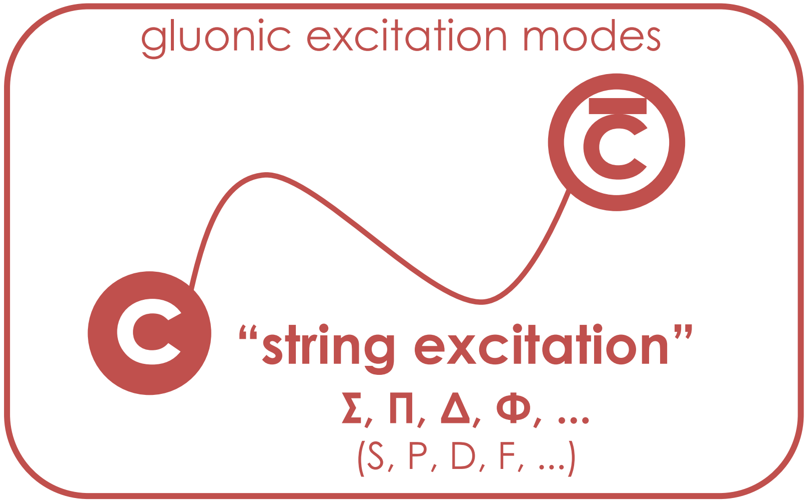
From Olsen et al. Rev. Mod. Phys. 90, 015003 (2018)
Original figure: BESS, Phys. Rev. Lett. 88, 101802 (2002)

3. Heavy exotic hadrons -X, Y, Z hadrons-

ψ(4230) aka Y(4260)

Hybrid state???

F. E. Close and P. R. Page, 2005, Phys. Lett. B 628, 215 (2005)
E. Kou and O. Pene, 2005, Phys. Lett. B 631, 164 (2005)
S.-L. Zhu, 2005, Phys. Lett. B 625, 212 (2005)



$c\bar{c}g$ hybrid

Gluons appear as dynamical d.o.f.

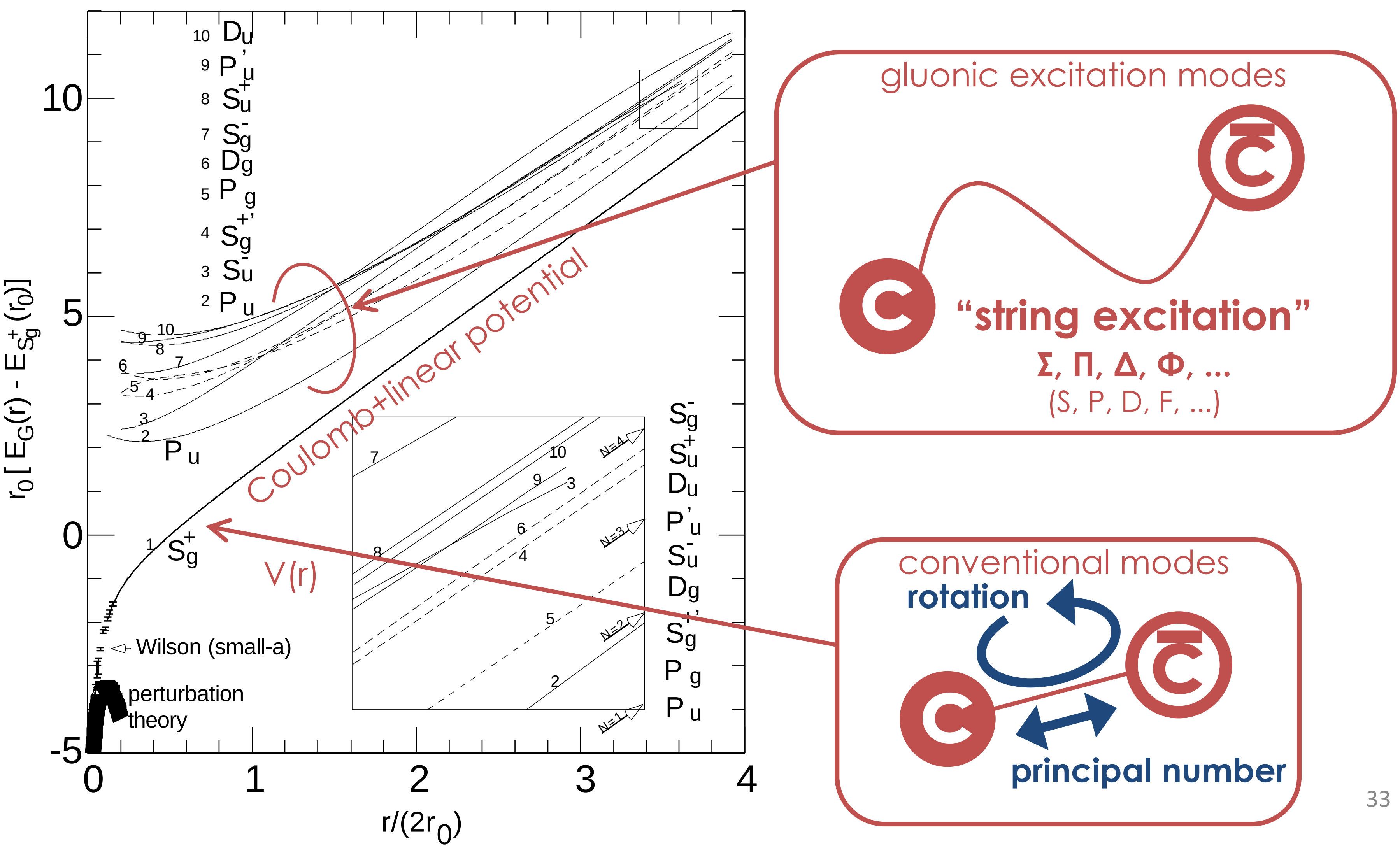
Candidate observed in lattice calculations:
Liu, L., G. Moir, M. Peardon, S. M. Ryan, C. E. Thomas, P. Vilaseca, J. J. Dudek, R. G. Edwards, B. Joo, and D. G. Richards (Hadron Spectrum), 2012, J. High Energy Phys. 07 126

3. Heavy exotic hadrons -X, Y, Z hadrons-

ψ(4230) aka Y(4260)

Hybrid state???

QQ^{bar} potential   J. Kuti, Nucl. Phys. B Proc. Suppl. 73, 72 (1999)



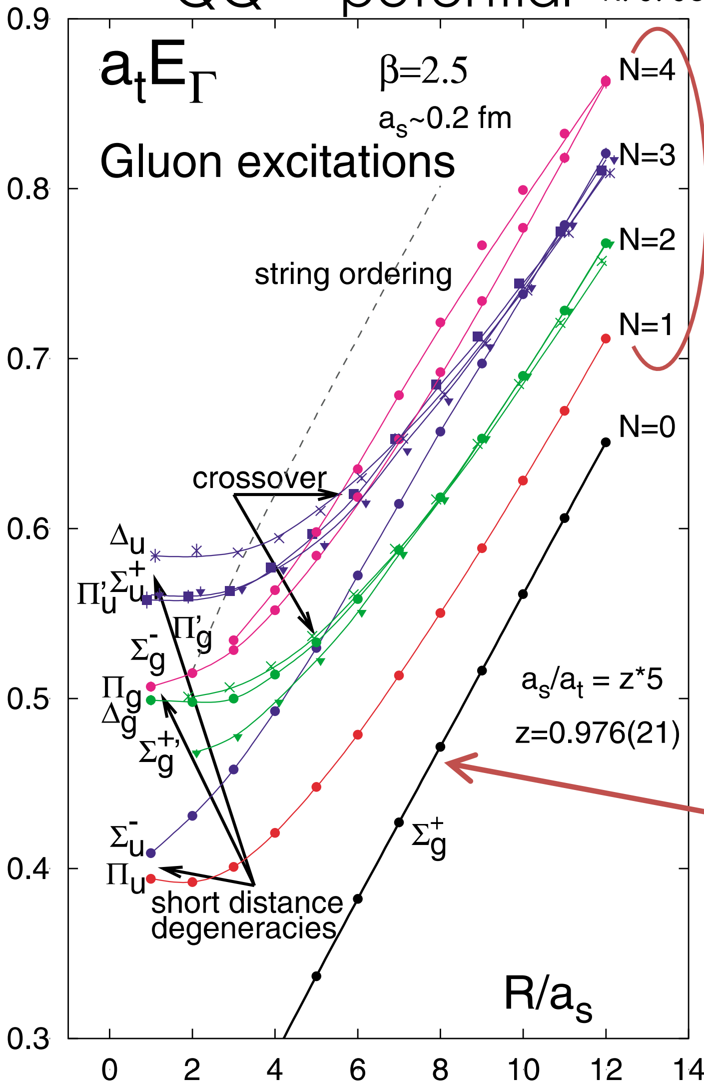
3. Heavy exotic hadrons -X, Y, Z hadrons-

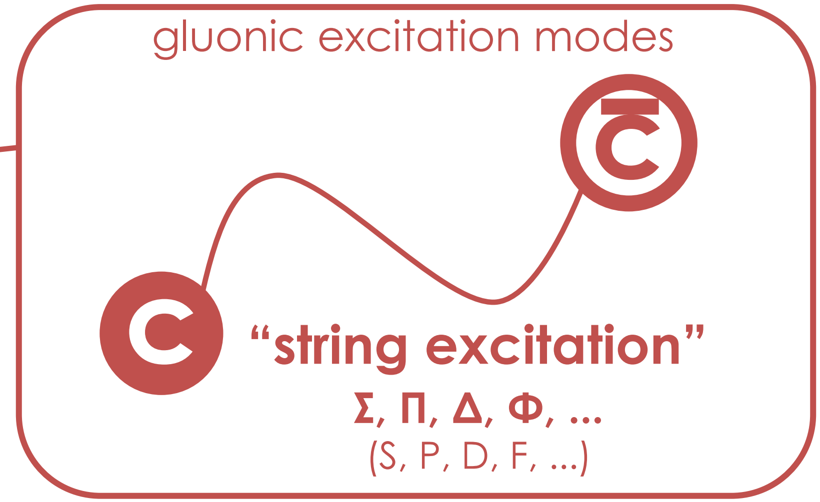
ψ(4230) aka Y(4260)

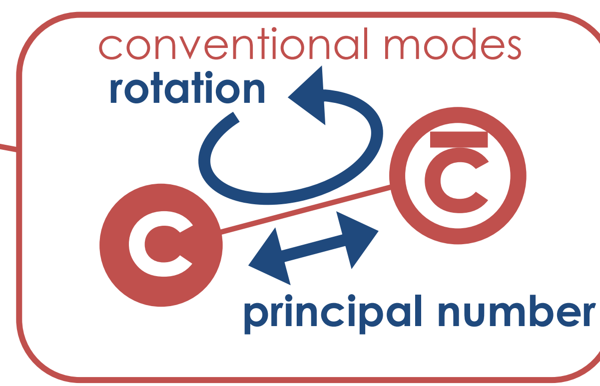
Hybrid state???

QQ^{bar} potential

K. J. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. 16, 161601 (2003)
(Cf. G. S. Bali, A. Pineda, Phys. Rev. D 69, 094001 (2004))



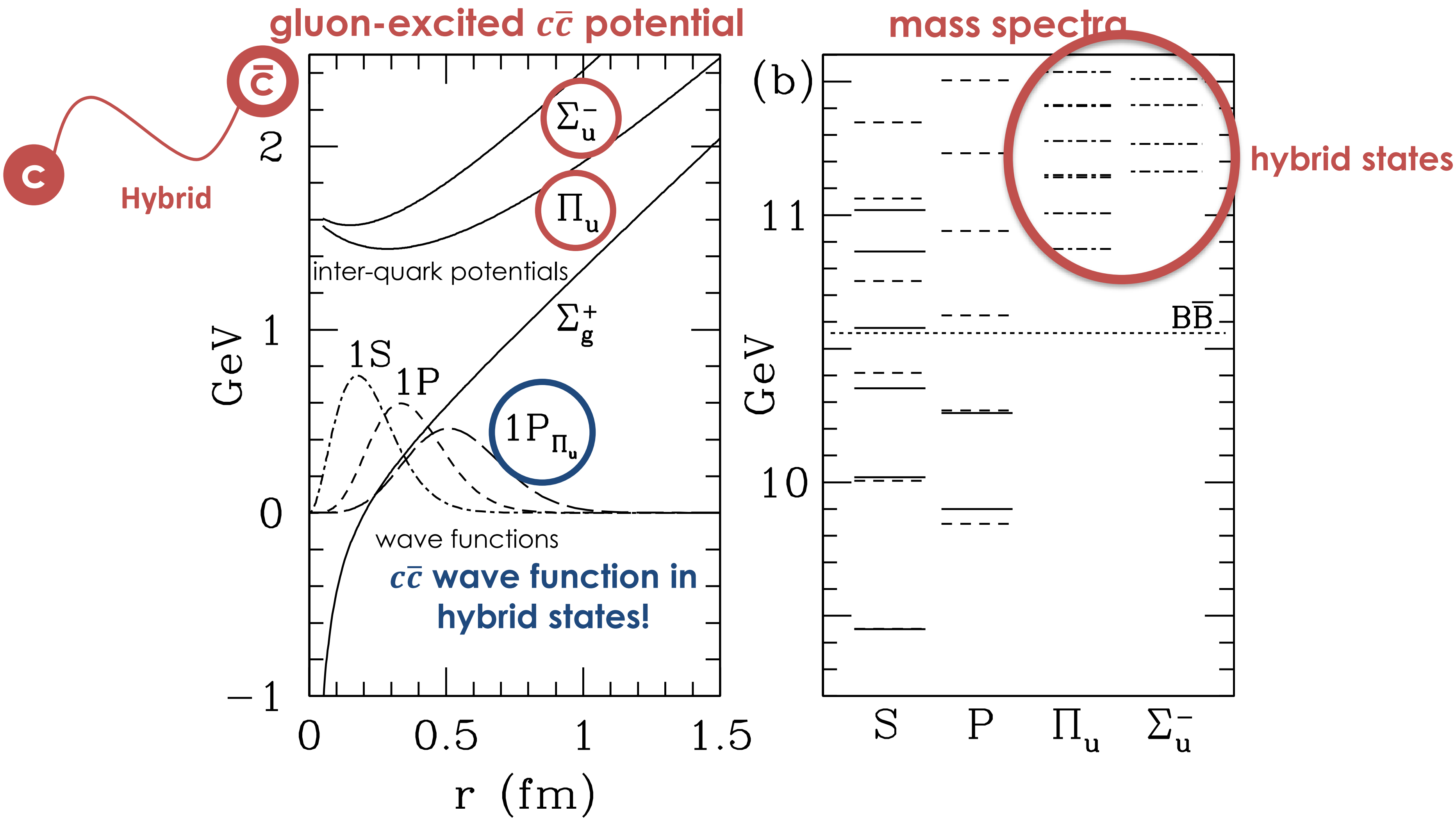




3. Heavy exotic hadrons -X, Y, Z hadrons-
ψ(4230) aka Y(4260)

Hybrid state???

K.J. Juge, J. Kuti, C.J. Morningstar,
Phys. Rev. Lett. 82, 4400 (1999)

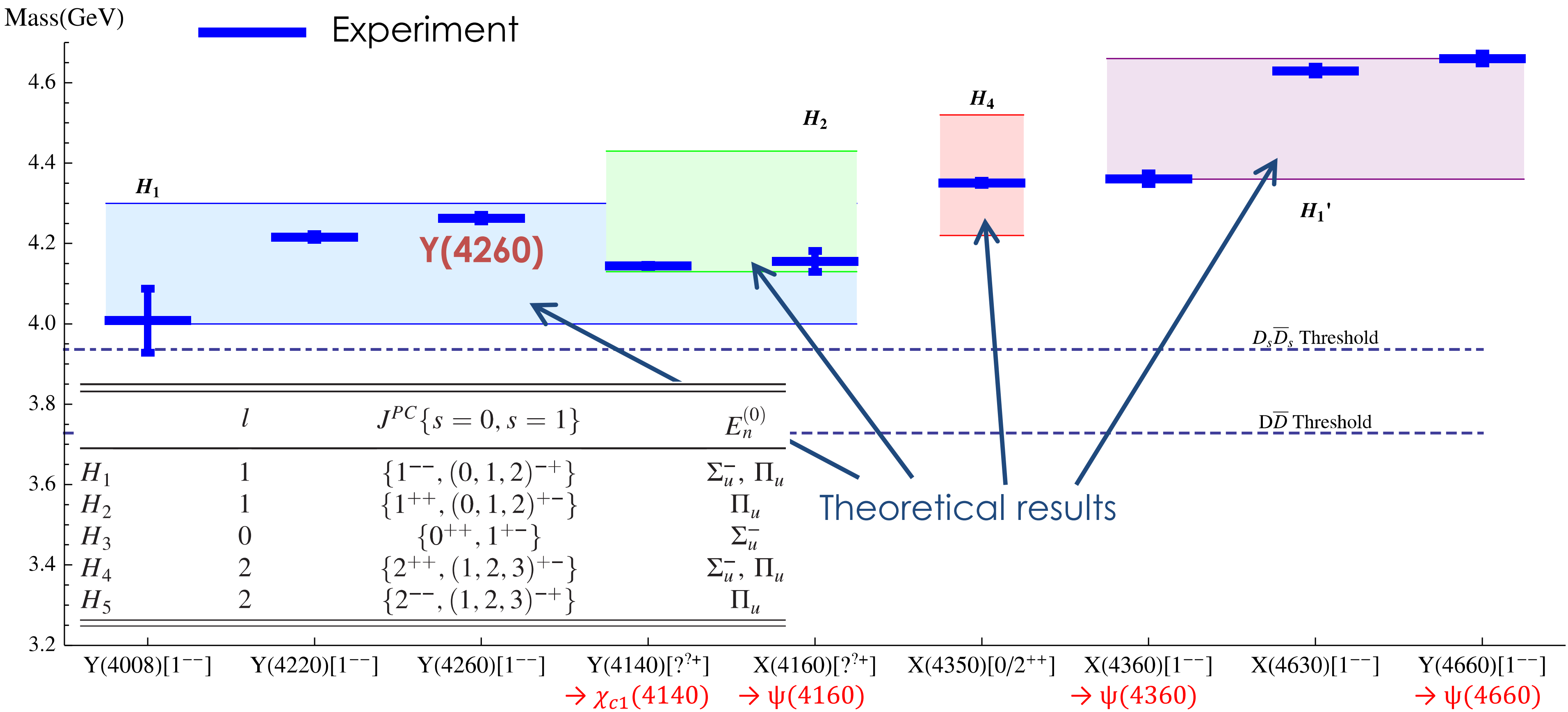


3. Heavy exotic hadrons -X, Y, Z hadrons-

ψ(4230) aka Y(4260)

Hybrid state???

M. Berwein, N. Brambilla, J.T. Castela, A. Vairo, Phys. Rev. D92, 114019 (2015)



	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

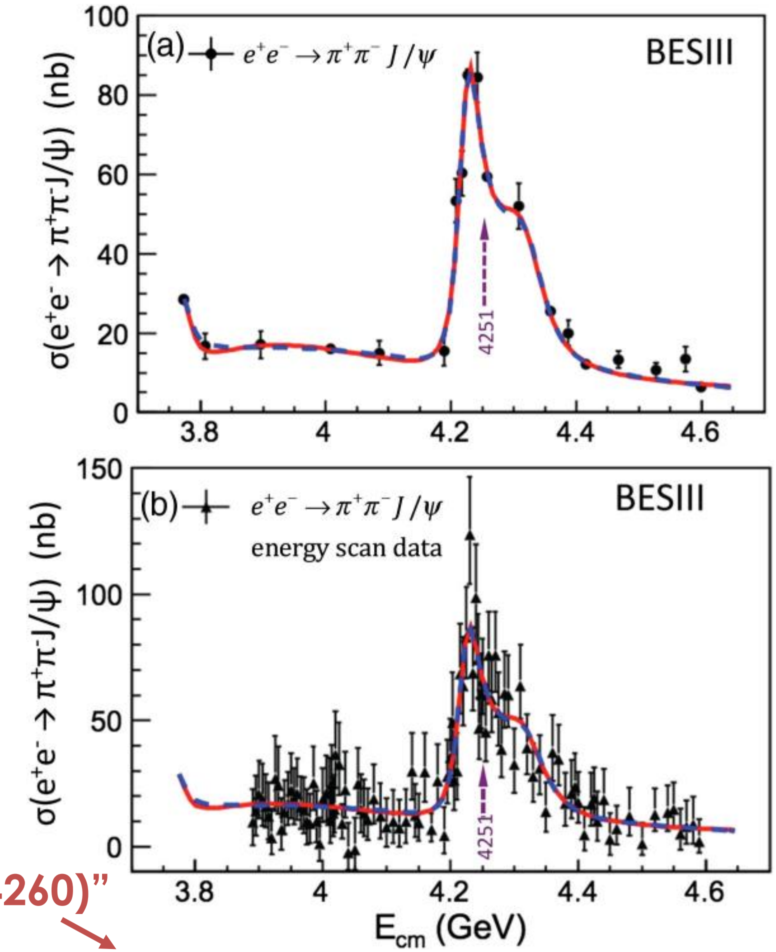
: Suppression to DD^{bar} decay (?)

See for early applications to Y(4260):
F. E. Close and P. R. Page, Phys. Lett. B 628, 215 (2005)
E. Kou and O. Pene, Phys. Lett. B 631, 164 (2005)
Cf. review: C. A. Meyer, E. S. Swanson, Prog. Part. Nucl. Phys. 82, 21 (2015)

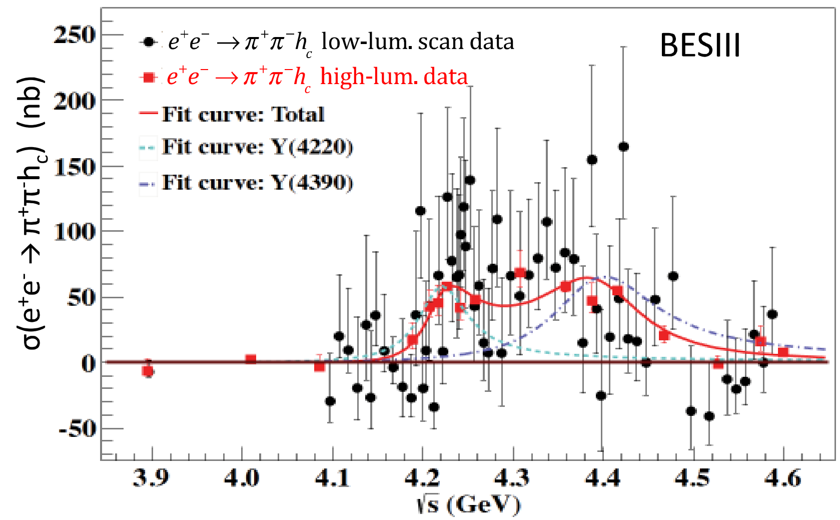
3. Heavy exotic hadrons -X, Y, Z hadrons-
ψ(4230) aka Y(4260)

No single Breit-Wigner shape **(reanalysis by higher statistics)**

BESIII, Phys. Rev. Lett. 118, 092001 (2017)



BESIII, Phys. Rev. Lett. 118, 092002 (2017)



$$M_1 = 4218 \pm 4 \text{ MeV}, \qquad \Gamma_1 = 66 \pm 9 \text{ MeV},$$
$$M_2 = 4392 \pm 6 \text{ MeV}, \qquad \Gamma_2 = 140 \pm 16 \text{ MeV},$$

Newest "Y(4260)" up to date

$$\boxed{\begin{aligned} M(Y(4220)) &= 4222 \pm 3 \text{ MeV}, \\ \Gamma(Y(4220)) &= 48 \pm 7 \text{ MeV}, \end{aligned}}$$

Olsen et al. Rev. Mod. Phys. 90, 015003 (2018)

"Y(4260)"

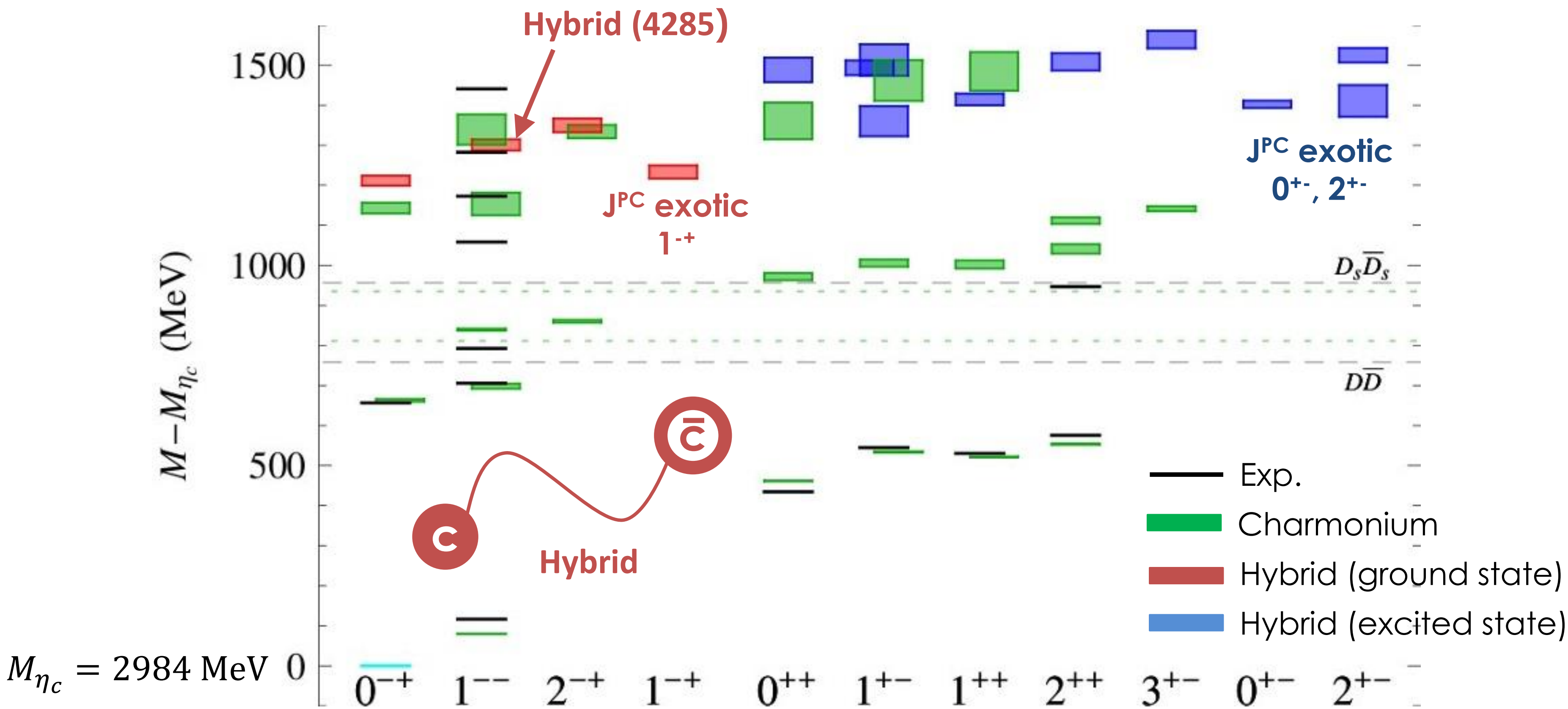
$$M_1 = 4222 \pm 4 \text{ MeV}, \qquad \Gamma_1 = 44 \pm 5 \text{ MeV},$$

"Y(4360)"

$$M_2 = 4320 \pm 13 \text{ MeV}, \qquad \Gamma_2 = 101^{+27}_{-22} \text{ MeV},$$

3. Heavy exotic hadrons -X, Y, Z hadrons-
ψ(4230) aka Y(4260)

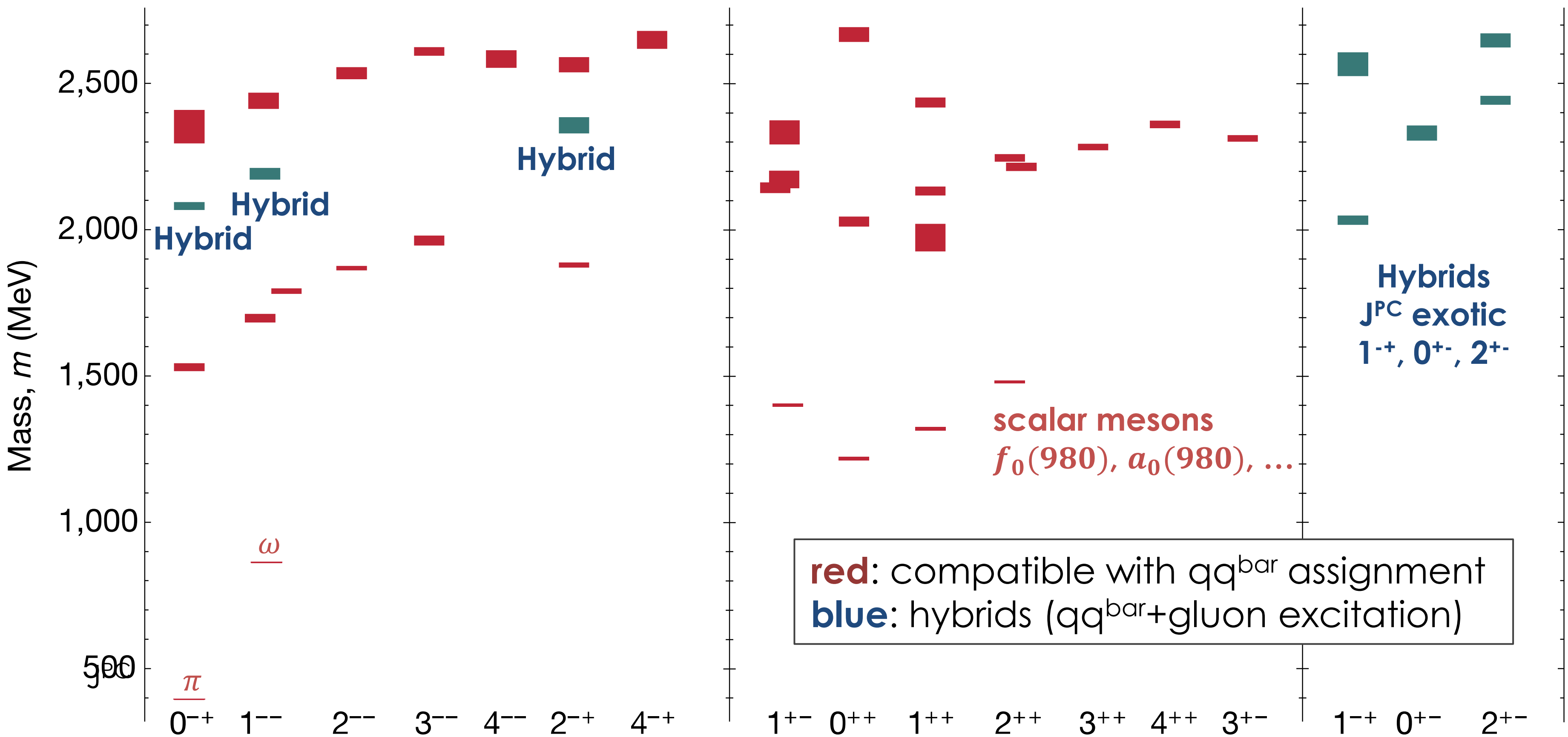
Lattice QCD ($m_\pi \approx 400$ MeV) Liu et al. JHEP07(2012)125



Mass of the hybrid (4285) seems consistent with $\psi(4230)$ aka $\psi(4260)$!

→ We should explore the other hybrids including J^{PC} exotics!!

Lattice computation of light meson spectrum
@m_π=392 MeV



J.J. Dudek, R.G. Edwards, P. Guo, C.E. Thomas, Phys. Rev. D 88, 094505 (2013)
Cf. M.R. Stephaerd, J.J. Dudeck, R E. Mirchell, Nat. Phys. 534, 487 (2016)

3. Heavy exotic hadrons -X, Y, Z hadrons-

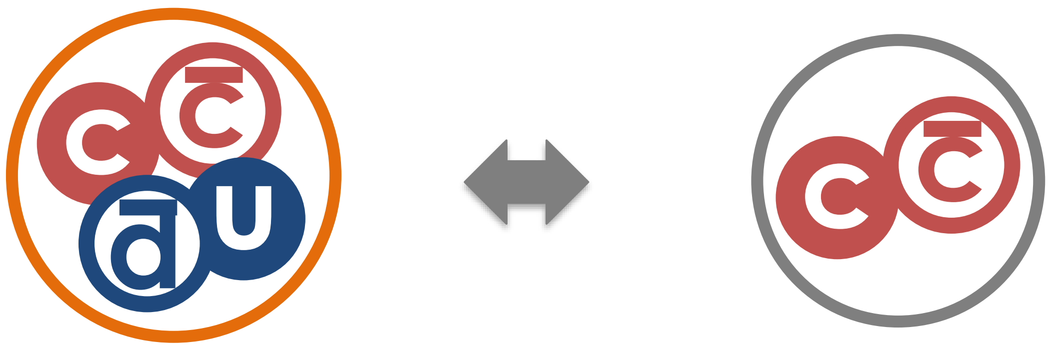
$$Z_c(4430)^+$$

3. Heavy exotic hadrons -X, Y, Z hadrons-

$Z_c(4430)^+$

Charged charmonium

First observation of genuinely “four-quark”



Charged (+)
Four quarks at least

Charge neutral

Note: cc^{bar} should be contained, because the final state of $Z_c(4430)$ includes a charmonum.

3. Heavy exotic hadrons -X, Y, Z hadrons-

$Z_c(4430)^+$

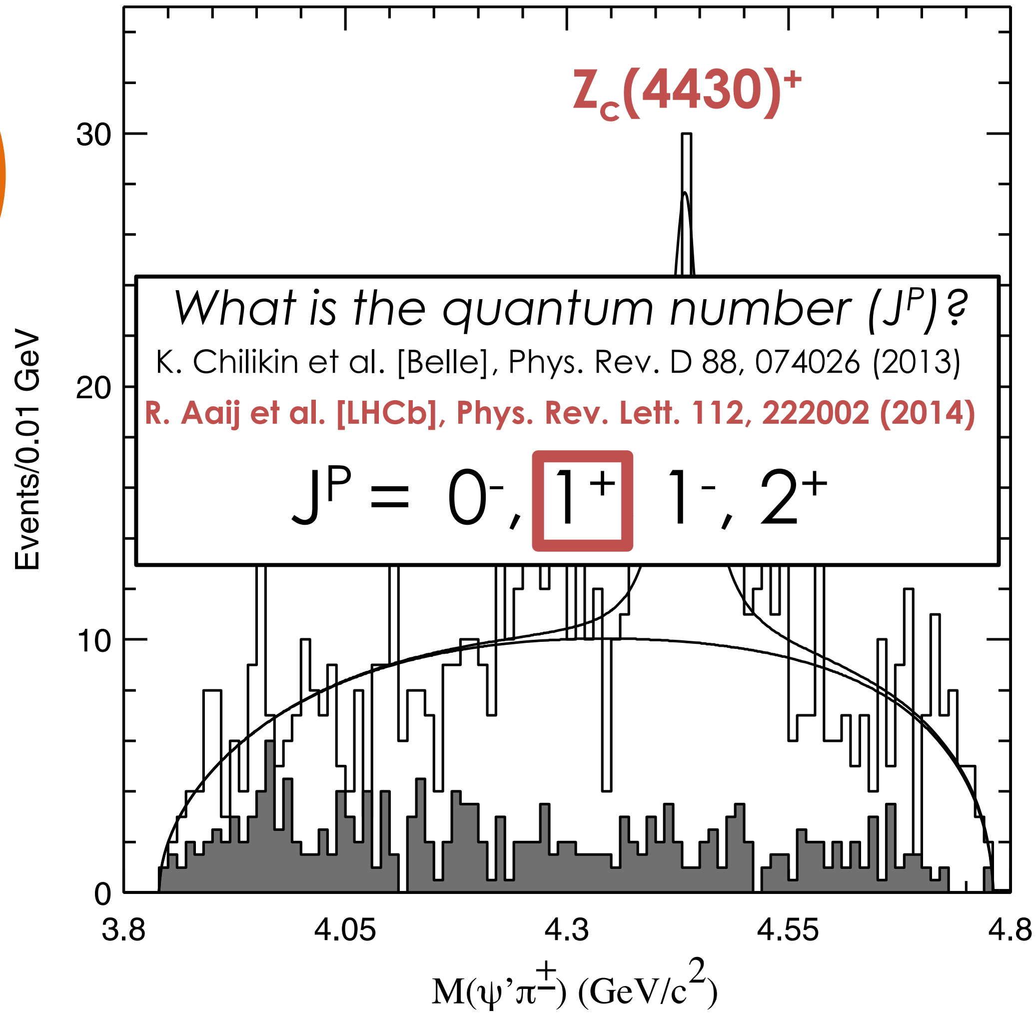
Charged charmonium

$$B \to \underline{\psi(2S)\pi^+ K}$$

S. K. Choi et al. [Belle], Phys. Rev. Lett. 100, 142001 (2008)

B. Aubert et al. [BaBar], Phys. Rev. D 79, 112001 (2009)

Charged (+)



What is the quantum number (J^P)?

K. Chilikin et al. [Belle], Phys. Rev. D 88, 074026 (2013)

R. Aaij et al. [LHCb], Phys. Rev. Lett. 112, 222002 (2014)

$J^P = 0^-, \boxed{1^+}\ 1^-, 2^+$

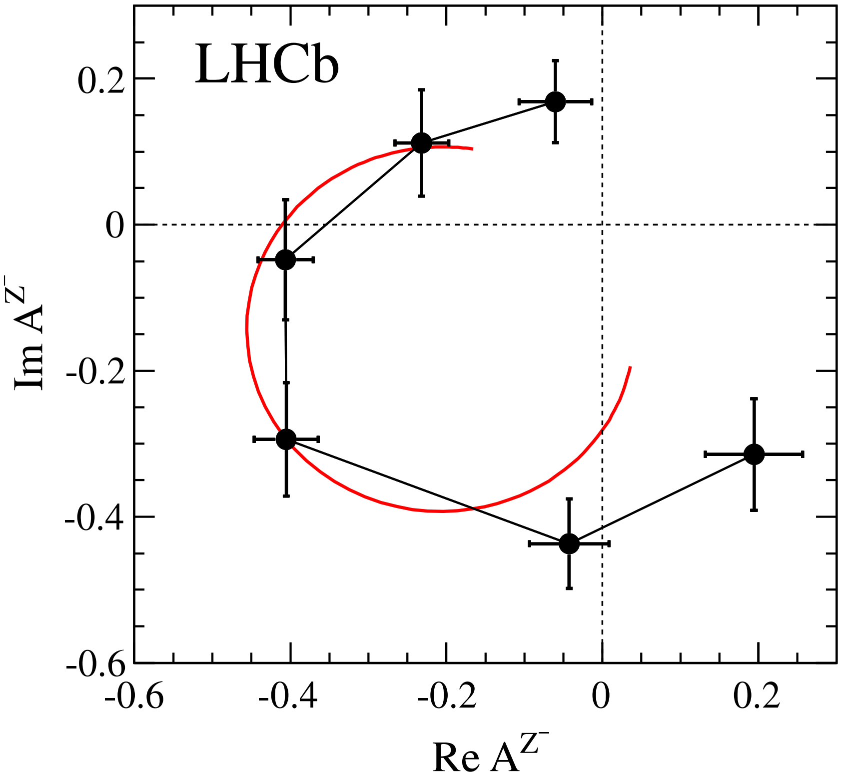
3. Heavy exotic hadrons -X, Y, Z hadrons-

$Z_c(4430)^+$

Does this peak really indicate a resonance?

Argand plot ($\psi'\pi^+$)

R. Aaij et al. [LHCb], Phys. Rev. Lett. 112, 222002 (2014)



Yes, this is consistent to be a resonance!!

(Necessary condition for being a resonance: If resonance, then circle in Argand plot.)

3. Heavy exotic hadrons -X, Y, Z hadrons- $Z_c(4430)^+$

Does this peal really indicate a resonance?

Argand plot (review)

Slide by Klaus Peters (GSI) Charm 2006

Introducing Partial Waves

JOHANN WOLFGANG GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Klaus Peters - PWA Primer

- Schrödinger's Equation

$$\boxed{-\frac{\hbar}{2\mu}\nabla^2\Psi(\vec{r}) + V(\vec{r})\Psi(\vec{r}) = E\Psi(\vec{r})} \qquad V(\vec{r}) = 0$$

$$\vec{k} = \frac{\vec{p}}{\hbar} = \mu\frac{\vec{v}}{\hbar} \qquad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$\Psi_i(r,\vartheta,\varphi) = e^{\imath kz}$$

$$\boxed{|i\rangle = \Psi_i = \sum_{l=0}^{\infty} U_l(r)P_l(\cos\vartheta)}$$

Angular Amplitude

$$\Psi_S = \Psi_f - \Psi_i = \frac{1}{k}\sum_{l=0}^{\infty}(2l+1)\frac{\eta_l e^{2\imath\delta_l}-1}{2\imath}P_l(\cos\vartheta)\frac{e^{\imath kr}}{r}$$: scattering wave

$$\boxed{T_l = \frac{\eta_l e^{2\imath\delta_l}-1}{2\imath}}$$

Dynamic Amplitude

3. Heavy exotic hadrons -X, Y, Z hadrons- $Z_c(4430)^+$

Does this peal really indicate a resonance?

Argand plot (review)

Slide by Klaus Peters (GSI)
Charm 2006

Standard Breit-Wigner

$$T_l = \frac{\eta_l e^{2\imath\delta_l} - 1}{2\imath}$$

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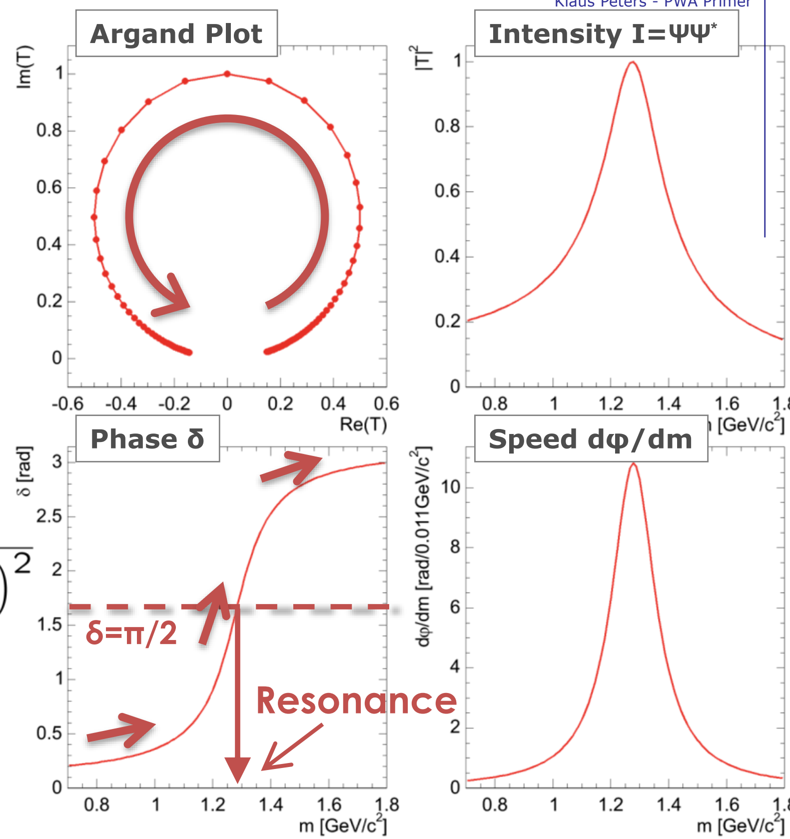
Klaus Peters - PWA Primer

- Full circle in the Argand Plot
- Phase motion from 0 to π

$$T(m) = \frac{\frac{\Gamma}{2}}{m_0 - m - \imath\frac{\Gamma}{2}}$$

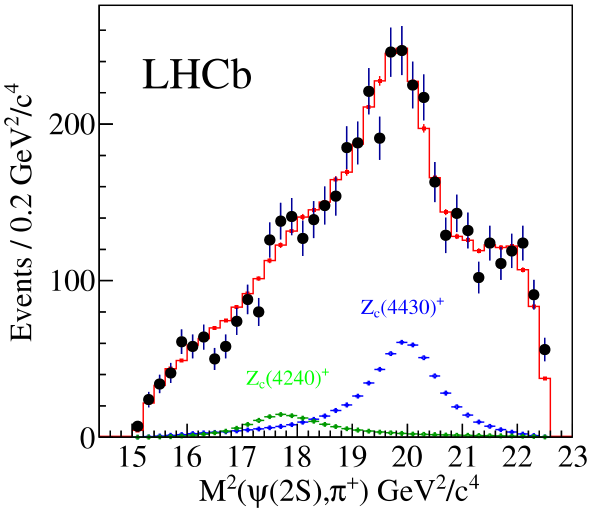
$$I(m) = |T(m)|^2 = \frac{\left(\frac{\Gamma}{2}\right)^2}{(m_0 - m)^2 + \left(\frac{\Gamma}{2}\right)^2}$$



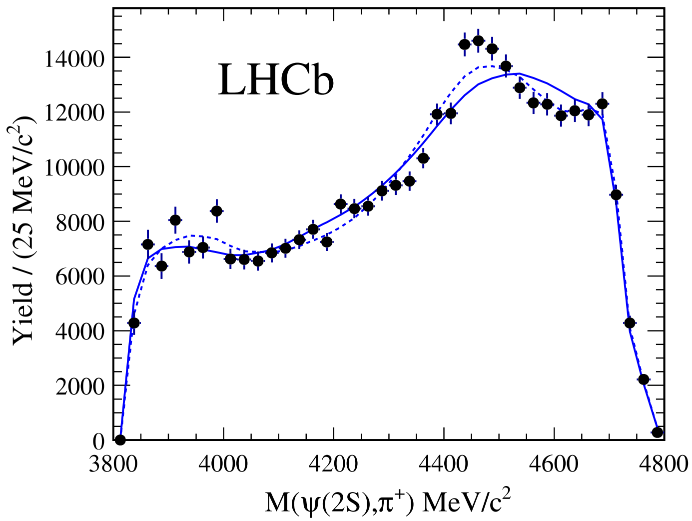
3. Heavy exotic hadrons -X, Y, Z hadrons-

Other charged states: $Z_c(4200)^+$, $Z_c(4050)^+$, $Z_c(4250)^+$

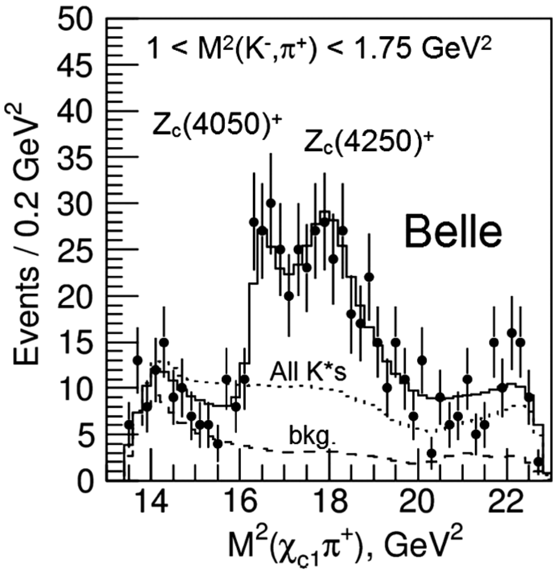
LHCb, Phys. Rev. Lett. 112, 222002 (2014)

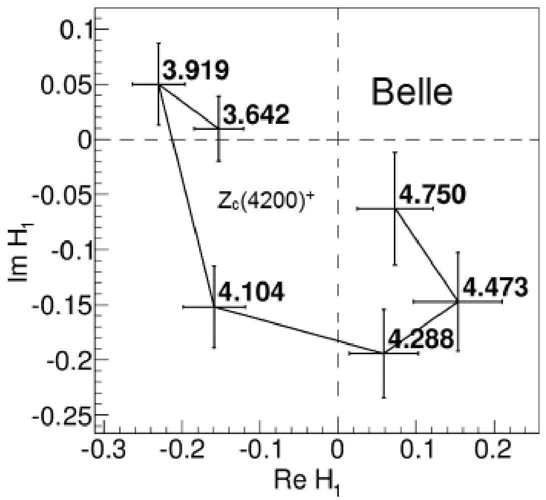


LHCb, Phys. Rev. D 92, 112009 (2015)



Belle, Phys. Rev. D 78, 072004 (2008)





Argand's plot for Zc(4200)
Belle, Phys. Rev. D 90, 112009 (2014)
Cf. Olsen et al. Rev. Mod. Phys. 90, 015003 (2018)

What exactly are they?

3. Heavy exotic hadrons -X, Y, Z hadrons-

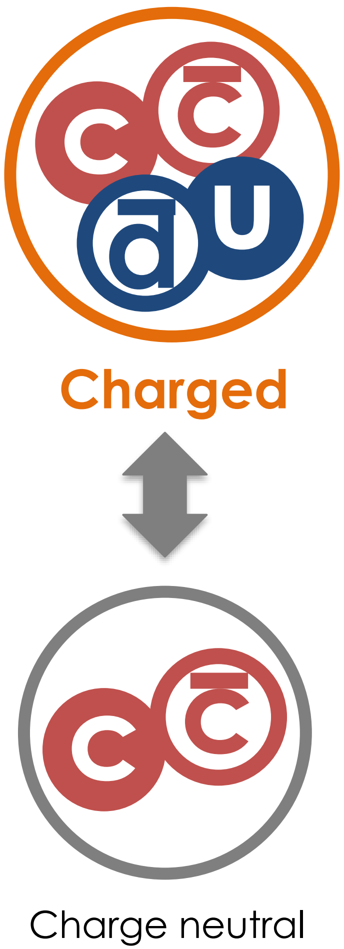
$$Z_c(3900)^+$$

3. Heavy exotic hadrons -X, Y, Z hadrons-

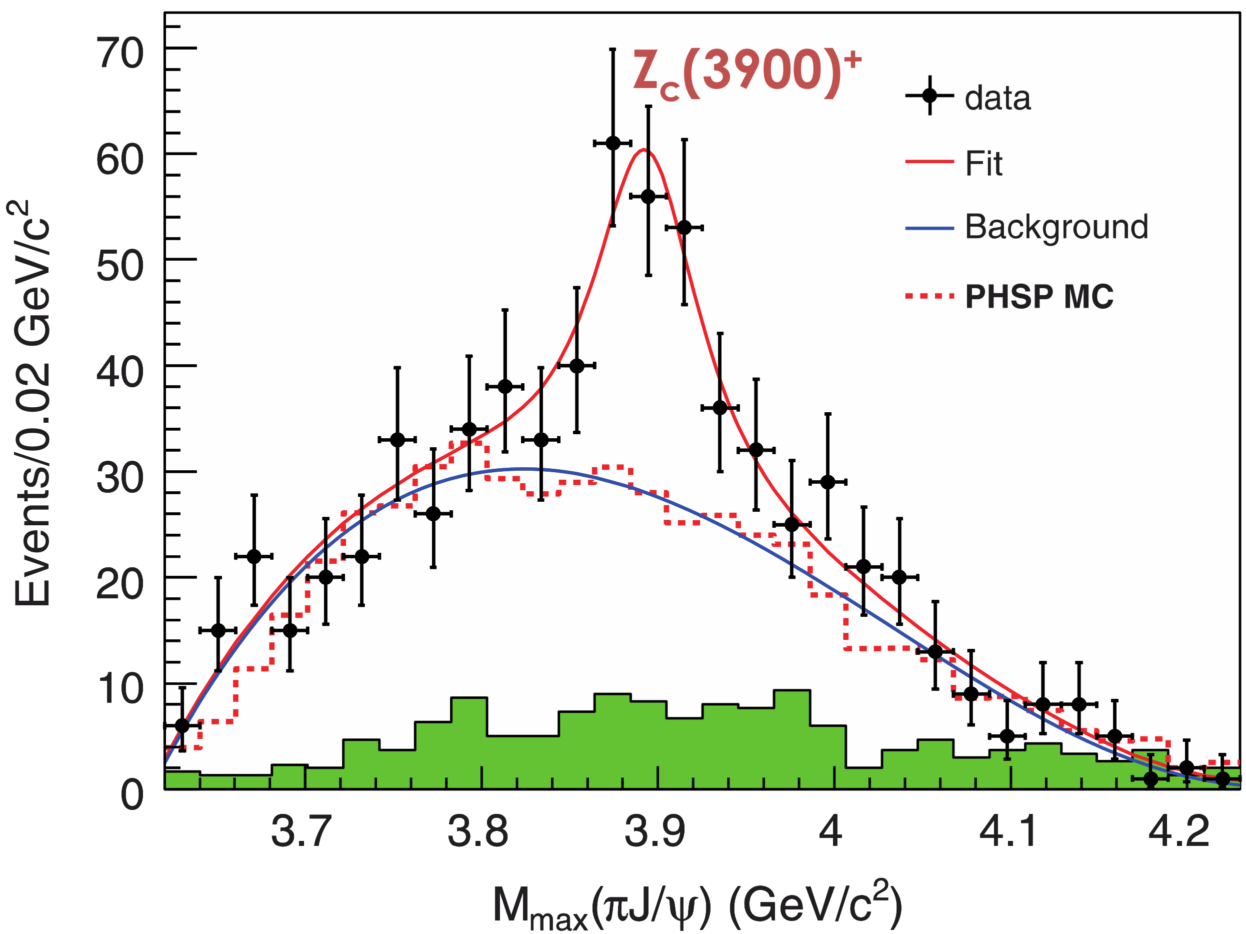
$Z_c(3900)^+$

Charged charmonium

Z. Q. Liu et al. [Belle],
Phys. Rev. Lett. 110, 252002 (2013)
Cf. M. Ablikim et al. [BESSIII],
Phys. Rev. Lett. 110, 252001 (2013)



$$Y(4260) \to \underline{J/\psi\pi^+}\pi^-$$

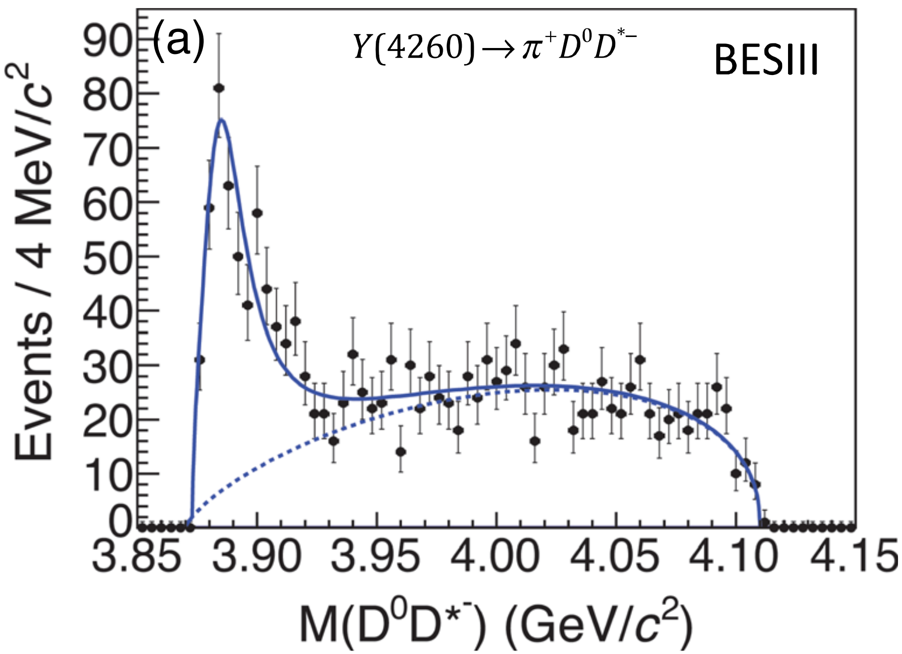


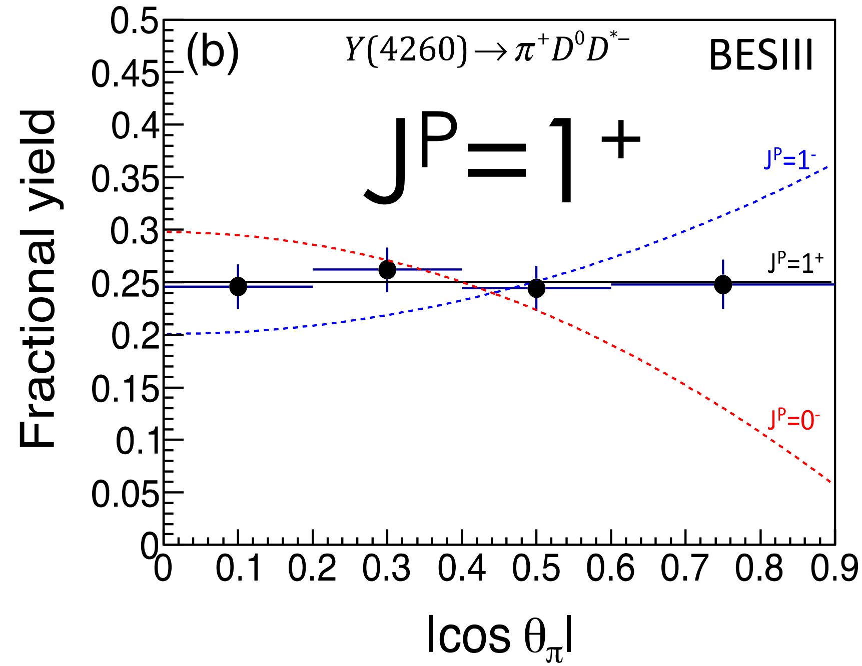
3. Heavy exotic hadrons -X, Y, Z hadrons-

$Z_c(3900)^+$

Charged charmonium

BESIII, Phys. Rev. Lett. 112, 022001 (2014)





Neutral partner: $Z_c(3900)^0$

BESIII, Phys. Rev. Lett. 115, 112003 (2015)

$$e^+e^- \to \pi^0\pi^0 J/\psi$$

BESIII, Phys. Rev. Lett. 115, 112002 (2015)

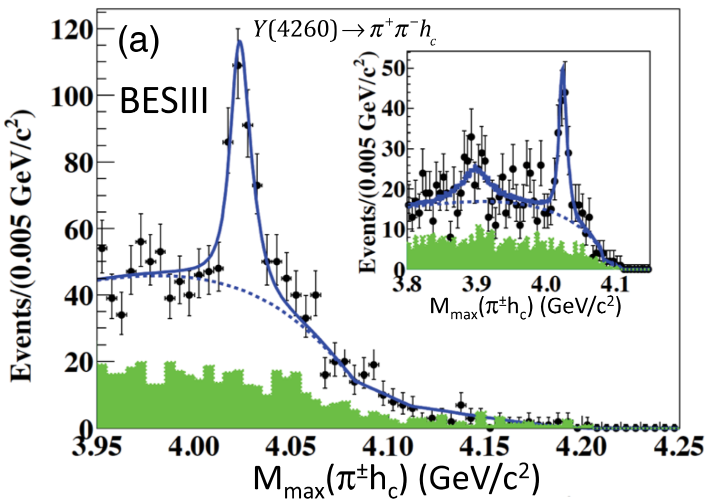
$$e^+e^- \to \pi^0 (D\bar{D}^*)^0$$
$$\hookrightarrow \; D^+\bar{D}^{*-} \quad D^0\bar{D}^{*0}$$

Cf. Olsen et al. Rev. Mod. Phys. 90, 015003 (2018)

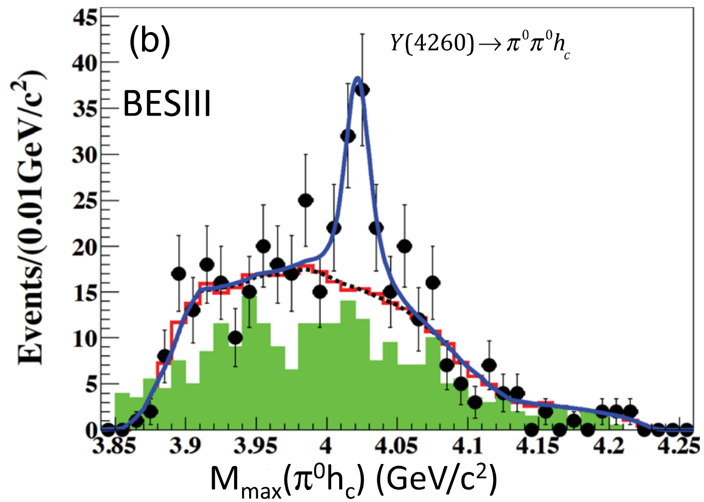
3. Heavy exotic hadrons -X, Y, Z hadrons- related state: $Z_c(4020)^+$

Charged charmonium

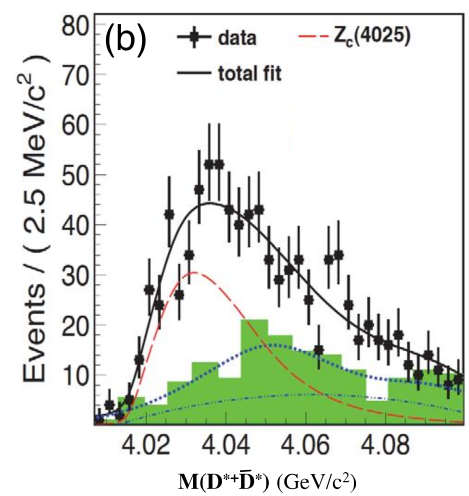
BESIII, PRL111, 242001 (2013)



BESIII, PRL113, 212002 (2014)



BESIII, PRL112, 132001 (2014)



Bottom version

$Z_c(3900)^+$

$Z_c(4020)^+$

paired states ?

(See next page.)

$Z_b(10610)^+$

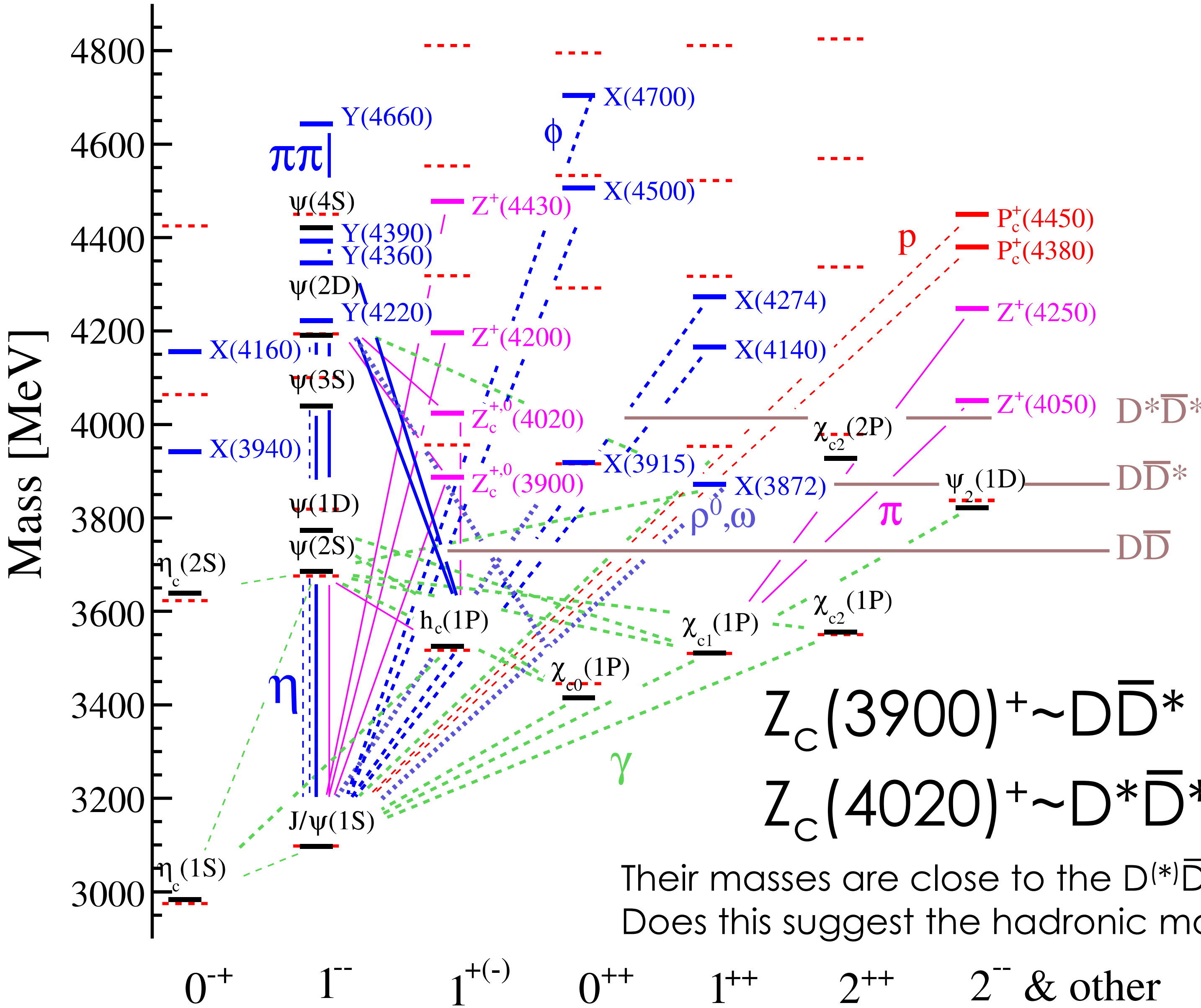
$Z_b(10650)^+$

We will discuss Z_b's in detail later.

Cf. Olsen et al. Rev. Mod. Phys. 90, 015003 (2018)

3. Heavy exotic hadrons -X, Y, Z hadrons-

Brief summary of charged Z_c's



$Z_c(3900)^+ \sim D\bar{D}^*$

$Z_c(4020)^+ \sim D^*\bar{D}^*$

Their masses are close to the $D^{(*)}\bar{D}^*$ thresholds.
Does this suggest the hadronic molecules?

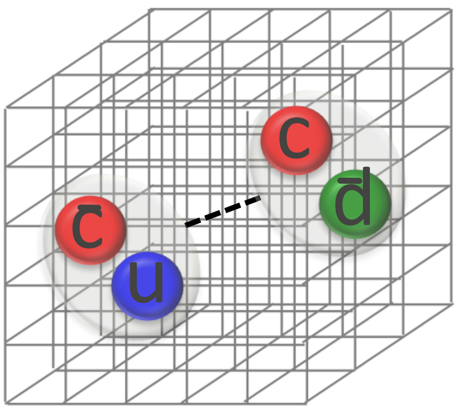
From Olsen et al. Rev. Mod. Phys. 90, 015003 (2018)

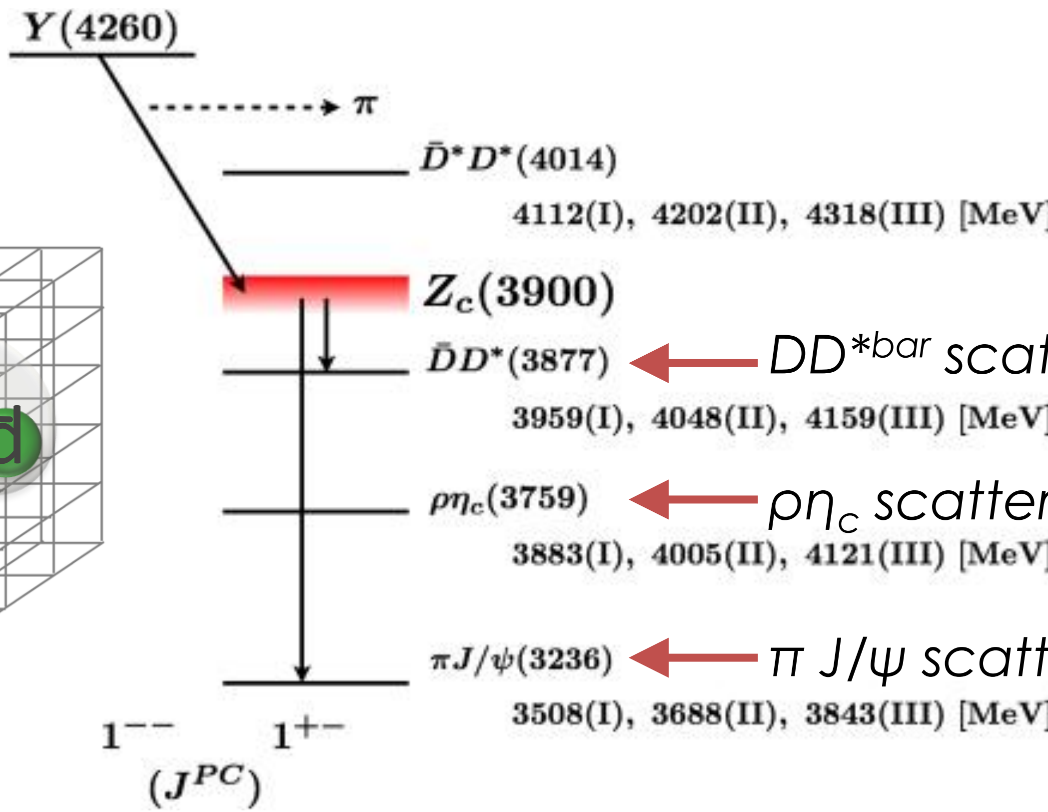
3. Heavy exotic hadrons -X, Y, Z hadrons-

$Z_c(3900)^+$

Lattice QCD

S. Prelovsek and L. Leskovec, Phys. Lett. B 727, 172 (2013)
S. Prelovsek, C. B. Lang, L. Leskovec, and D. Mohler, Phys. Rev. D 91, 014504 (2015)
Y. Chen et al., Phys. Rev. D 89, 094506 (2014)
Y. Ikeda et al., Phys. Rev. Lett. 117, 242001 (2016)





DD^{*bar} scattering

$\rho\eta_c$ scattering

$\pi\ J/\psi$ scattering

3. Heavy exotic hadrons -X, Y, Z hadrons-

$Z_c(3900)^+$

Lattice QCD

S. Prelovsek and L. Leskovec, Phys. Lett. B 727, 172 (2013)
S. Prelovsek, C. B. Lang, L. Leskovec, and D. Mohler, Phys. Rev. D 91, 014504 (2015)
Y. Chen et al., Phys. Rev. D 89, 094506 (2014)
Y. Ikeda et al., Phys. Rev. Lett. 117, 242001 (2016)

"HAL QCD" method

N. Ishii, S. Aoki, and T. Hatsuda, Phys. Rev. Lett. 99, 022001 (2007)
S. Aoki, T. Hatsuda, and N. Ishii, Prog. Theor. Phys. 123, 89 (2010)

① Calculate Nambu-Bethe-Salpeter wave function $\psi_n^\alpha(\vec{r})$ (correlation function)

$$C^{\alpha\beta}(\vec{r},t) \equiv \sum_{\vec{x}} \langle 0|\phi_1^\alpha(\vec{x}+\vec{r},t)\phi_2^\alpha(\vec{x},t)\overline{\mathcal{J}}^\beta|0\rangle / \sqrt{Z_1^\alpha Z_2^\alpha}$$

$$\alpha = (\pi J/\psi, \rho\eta_c, \overline{D}D^*)$$ **hadron basis**

$$\longrightarrow C^{\alpha\beta}(\vec{r},t) = \sum_n \psi_n^\alpha(\vec{r}) A_n^\beta e^{-W_n t}$$

② Schrödinger equation (inverse problem)

$$R^{\alpha\beta}(\vec{r},t) \equiv C^{\alpha\beta}(\vec{r},t)e^{(m_1^\alpha+m_2^\alpha)t}$$

$$\left(-\frac{\partial}{\partial t} - H_0^\alpha\right)R^{\alpha\beta}(\vec{r},t) = \sum_\gamma \Delta^{\alpha\gamma}\int d\vec{r'}\,U^{\alpha\gamma}(\vec{r},\vec{r'})R^{\gamma\beta}(\vec{r'},t)$$

$$H_0^\alpha = -\nabla^2/2\mu^\alpha \quad \mu^\alpha = m_1^\alpha m_2^\alpha/(m_1^\alpha+m_2^\alpha) \quad U^{\alpha\beta}(\vec{r},\vec{r'}) = V^{\alpha\beta}(\vec{r})\delta(\vec{r}-\vec{r'}) + O(\nabla^2)$$

$$\Delta^{\alpha\gamma} = e^{(m_1^\alpha+m_2^\alpha)t}/e^{(m_1^\gamma+m_2^\gamma)t}$$

inter-hadron potential

3. Heavy exotic hadrons -X, Y, Z hadrons-

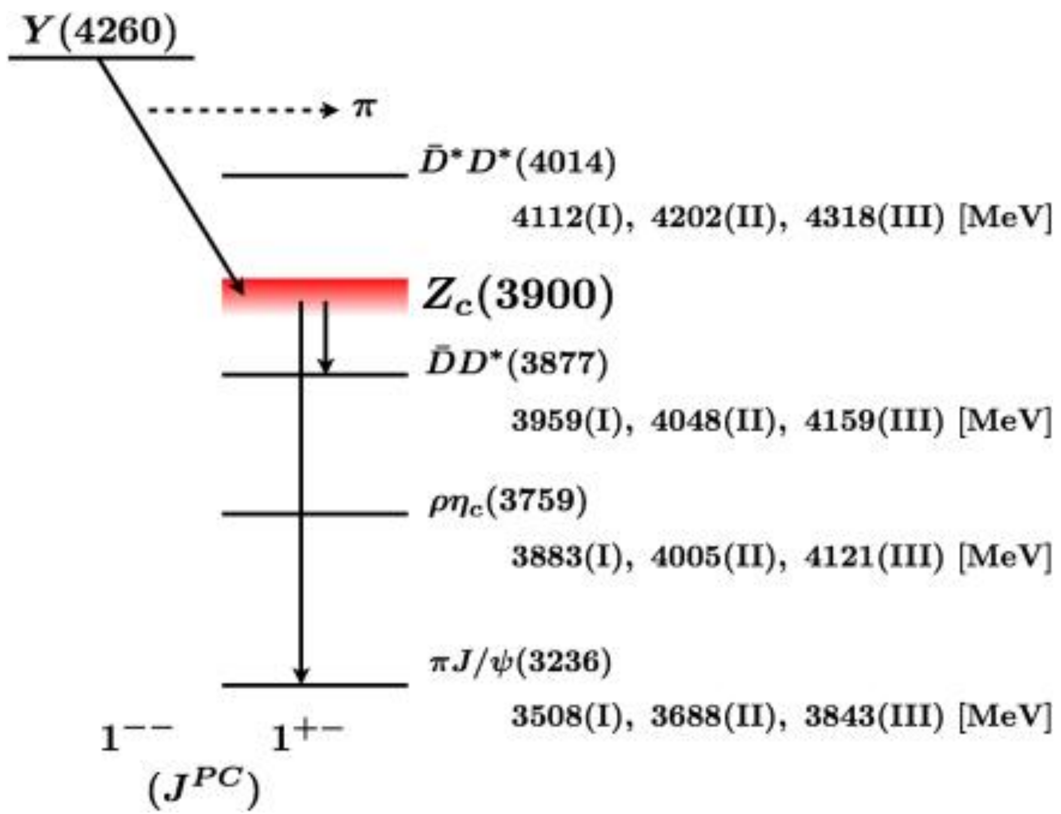
$Z_c(3900)^+$

Lattice QCD

Y. Ikeda et al., Phys. Rev. Lett. 117, 242001 (2016)

	m_π	m_ρ	m_{η_c}	$m_{J/\psi}$	$m_{\bar{D}}$	m_{D^*}
Expt.	140	775	2984	3097	1870	2007
Case I	411(1)	896(8)	2988(1)	3097(1)	1903(1)	2056(3)
Case II	570(1)	1000(5)	3005(1)	3118(1)	1947(1)	2101(2)
Case III	701(1)	1097(4)	3024(1)	3143(1)	2000(1)	2159(2)

Correct thresholds



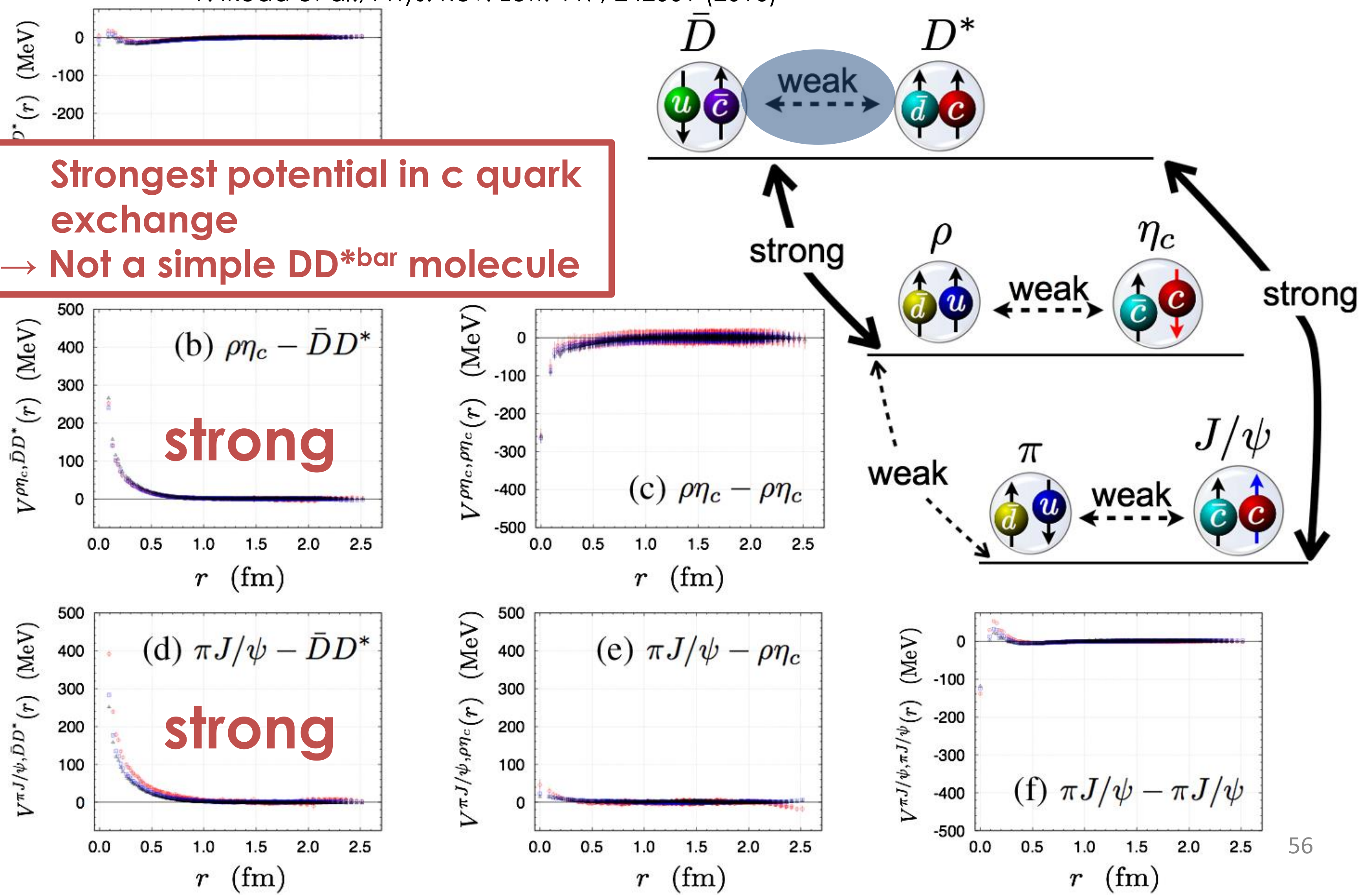
3. Heavy exotic hadrons -X, Y, Z hadrons-

$Z_c(3900)^+$

Lattice QCD

Y. Ikeda et al., Phys. Rev. Lett. 117, 242001 (2016)

Strongest potential in c quark exchange
→ Not a simple DD^{*bar} molecule



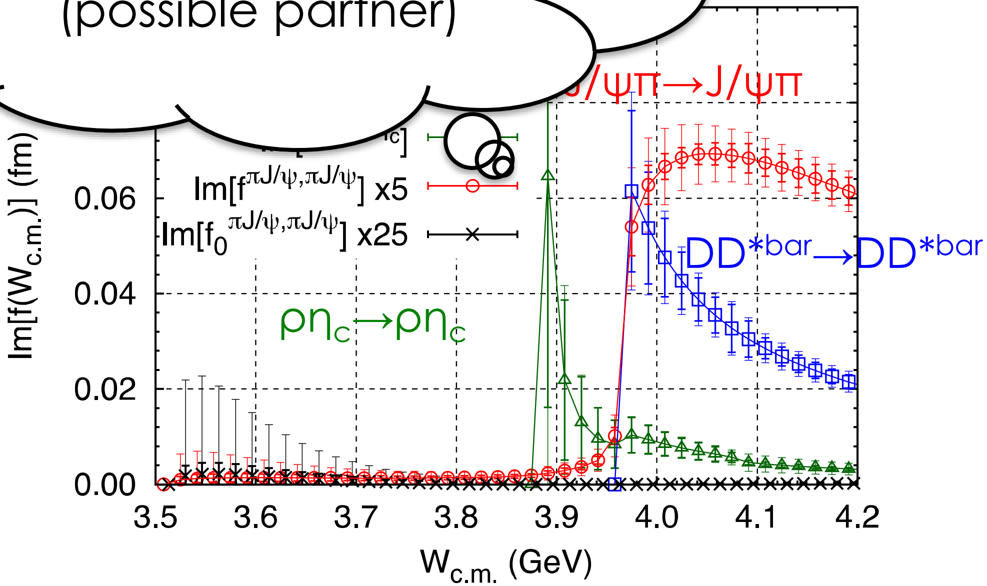
3. Heavy exotic hadrons -X, Y, Z hadrons-

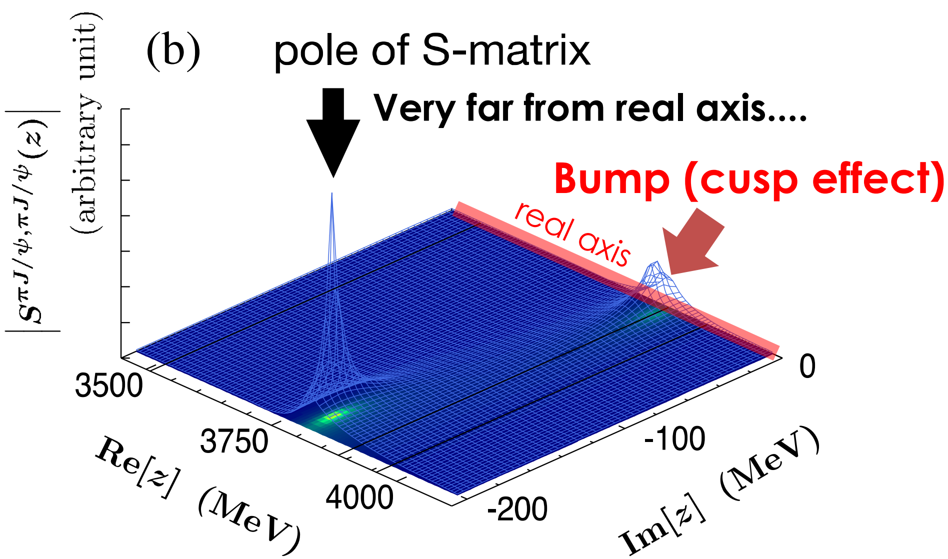
$Z_c(3900)^+$

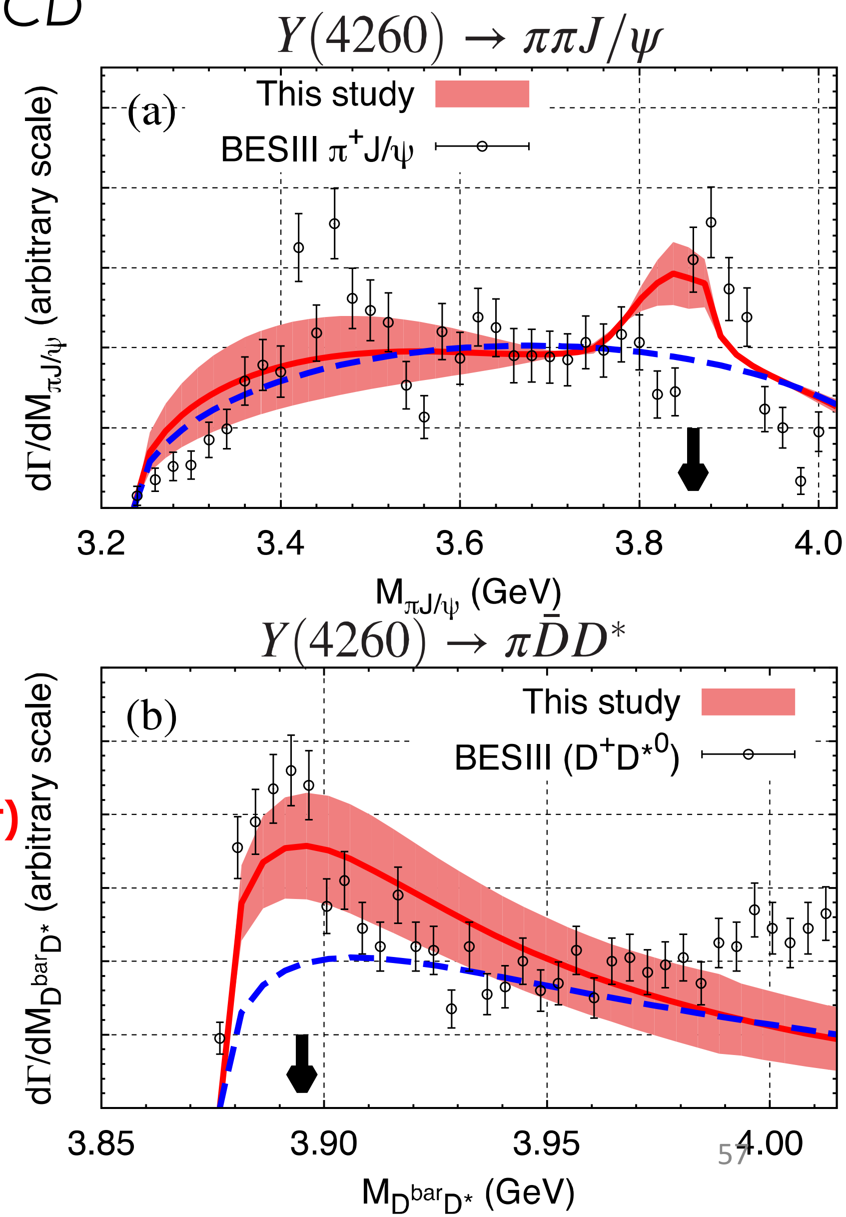
Lattice QCD

How about $Z_c(4020)^+$?
(possible partner)

Y. Ikeda et al.,
Phys. Rev. Lett. 117, 242001 (2016)



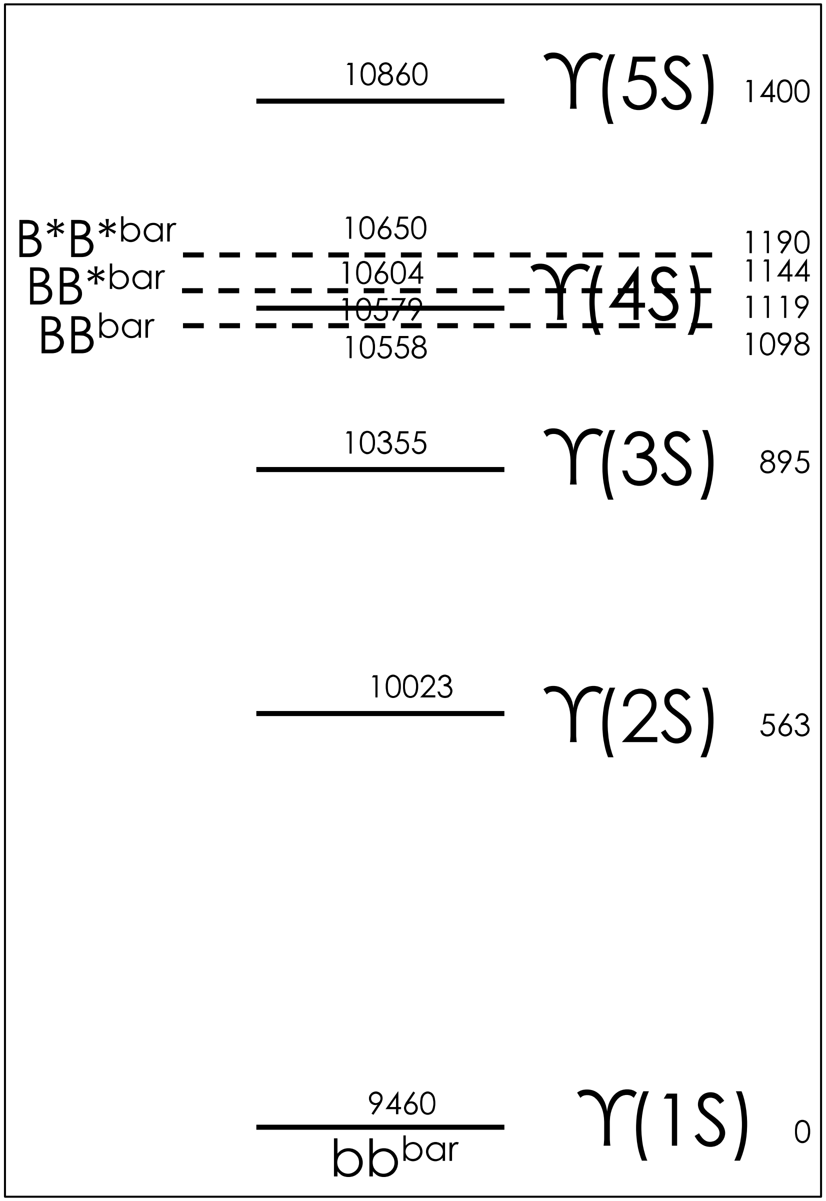




Bottom

Tetraquark

3. Heavy exotic hadrons -X, Y, Z hadrons-



Belle can search Y(5S)

Belle's main search: Y(4S) as BB^{bar} factory

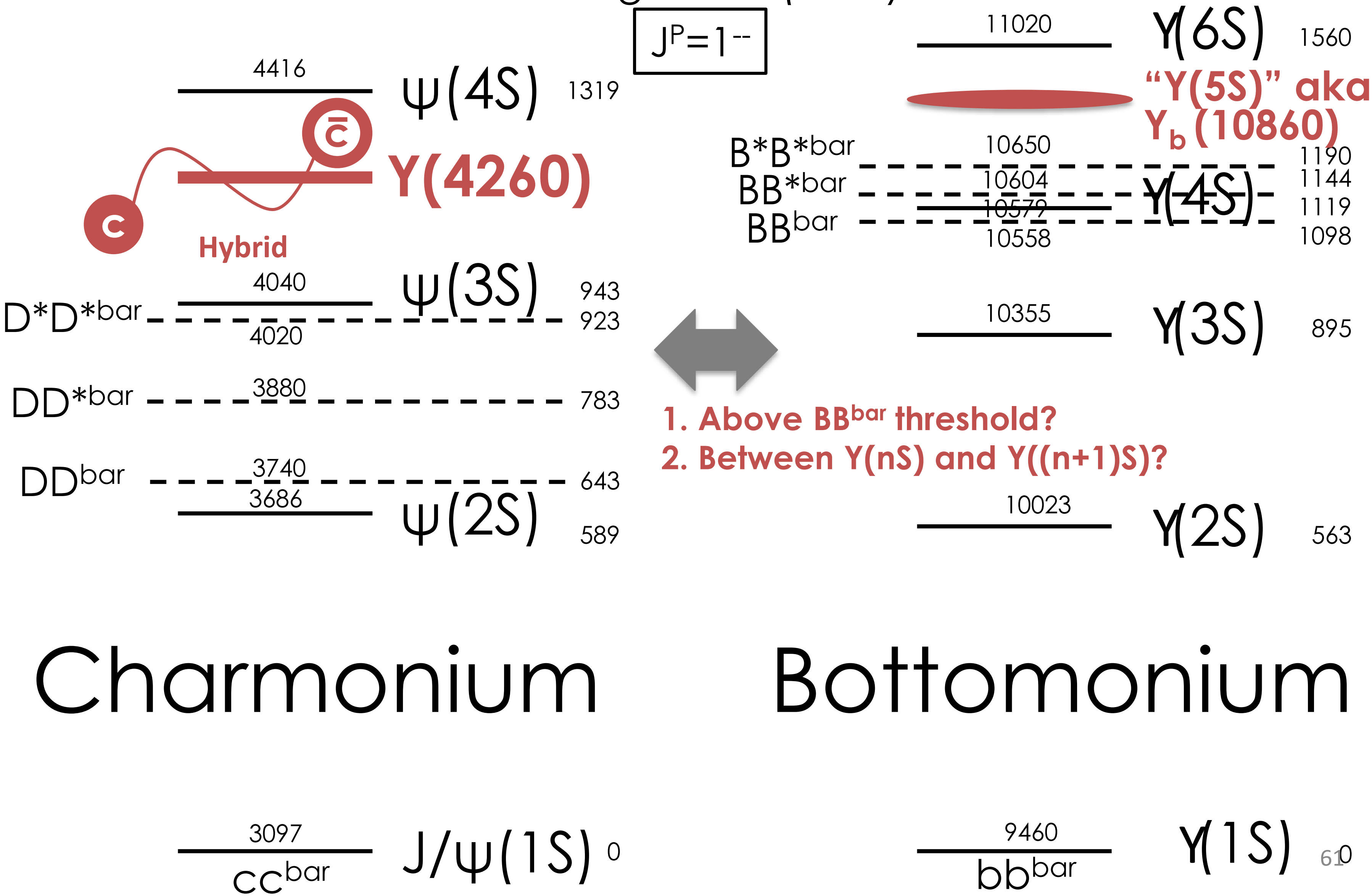
$\Upsilon_b(10860)$

3. Heavy exotic hadrons -X, Y, Z hadrons-

$Y_b(10860)$

Analogue of Y(4260)?

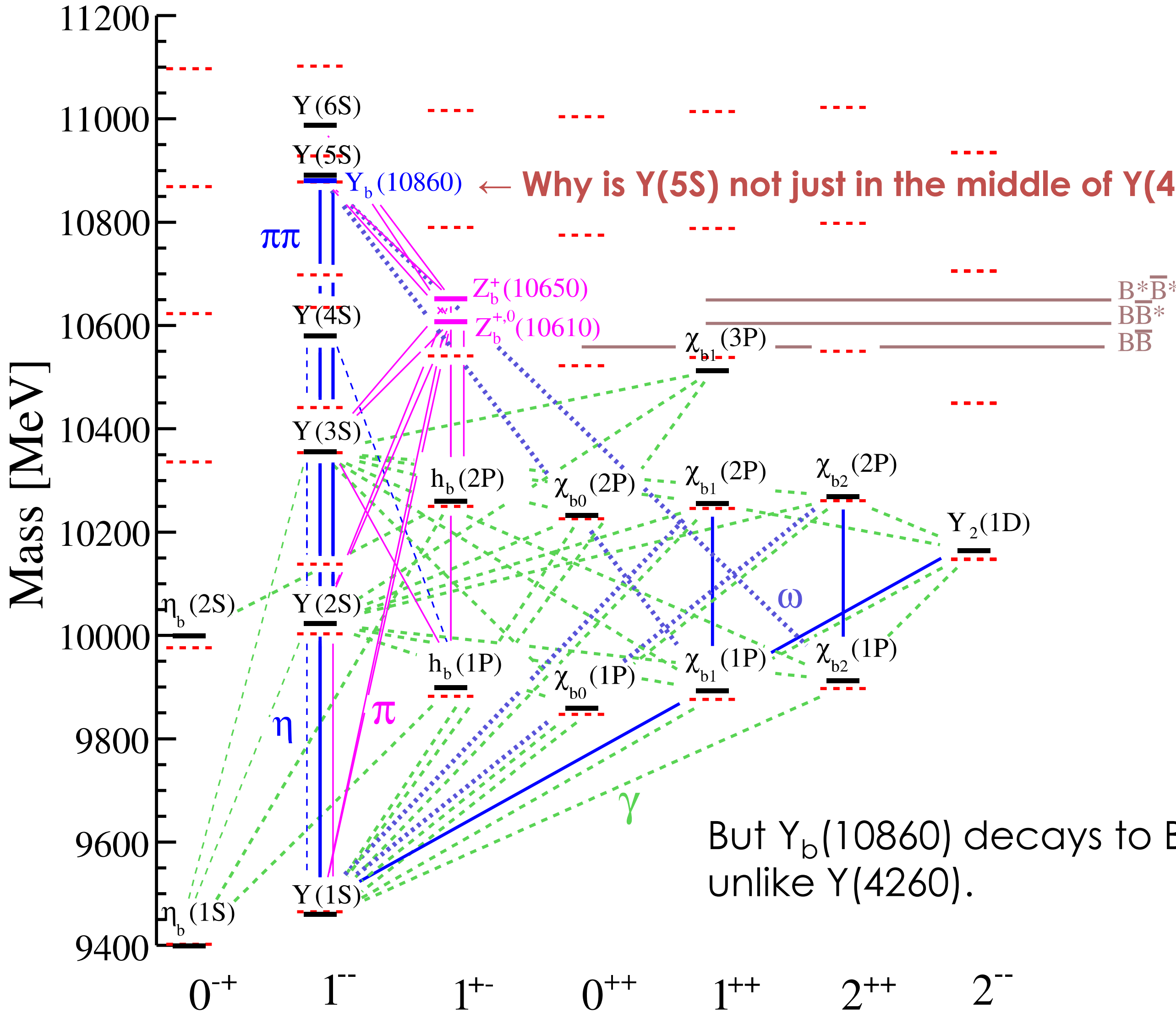
K.-F. Chen et al. [Belle], Phys. Rev. D 82, 091106 (2010)



3. Heavy exotic hadrons -X, Y, Z hadrons-

$Y_b(10860)$

Analogue of Y(4260)?



← **Why is Υ(5S) not just in the middle of Υ(4S) and Υ(6S)?**

But $Y_b(10860)$ decays to BB^{bar} (B.R.76%) unlike Y(4260).

From Olsen et al. Rev. Mod. Phys. 90, 015003 (2018)

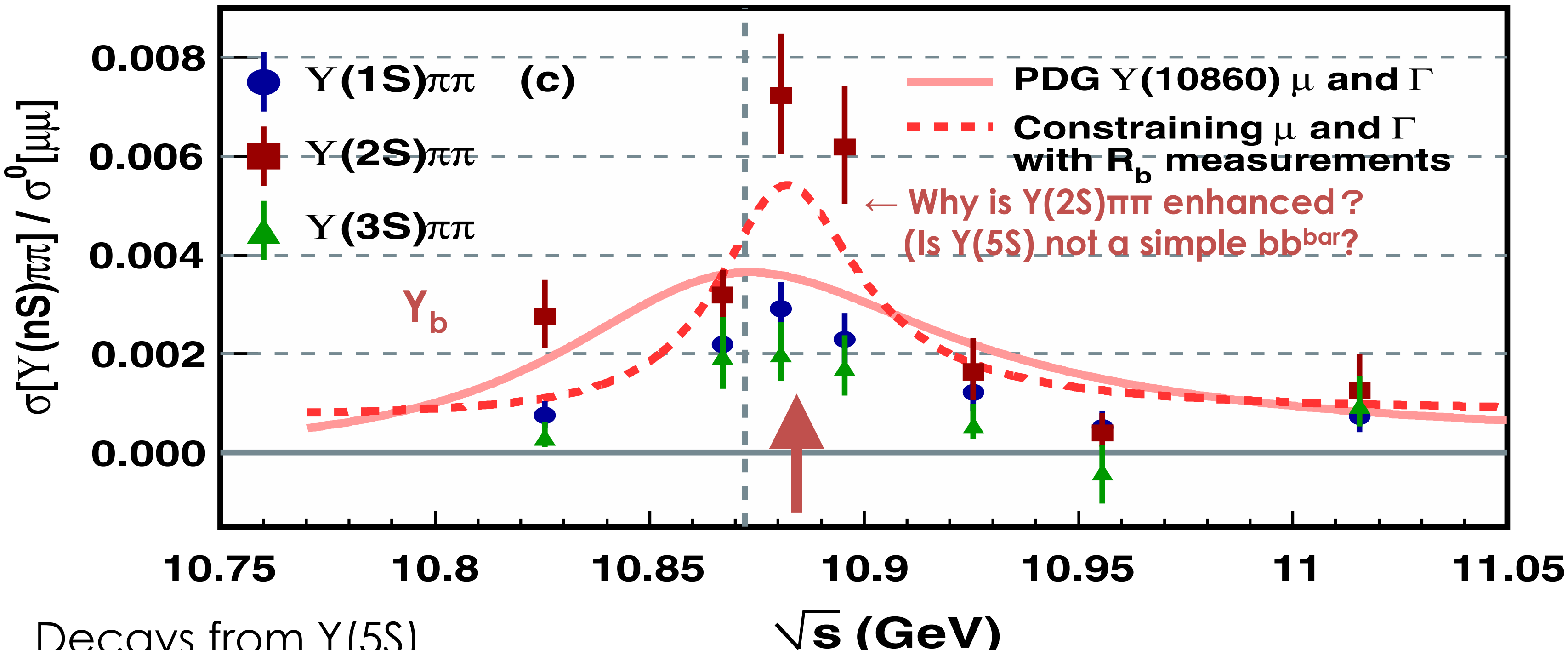
3. Heavy exotic hadrons -X, Y, Z hadrons-

$Y_b(10860)$

Analogue of Y(4260)?

K.-F. Chen et al. [Belle], Phys. Rev. D 82, 091106 (2010)

$e^+e^- \to \Upsilon(nS)\pi^+\pi^- \quad (n = 1, 2, 3)$ σ^0: $e^+e^- \to \mu^+\mu^-$



Decays from Υ(5S)

$\Upsilon(1S)\pi^+\pi^-\ \sigma_{\rm peak}$ (pb)	$2.78^{+0.42}_{-0.34} \pm 0.23$
$\Upsilon(2S)\pi^+\pi^-\ \sigma_{\rm peak}$ (pb)	$4.82^{+0.77}_{-0.62} \pm 0.66$
$\Upsilon(3S)\pi^+\pi^-\ \sigma_{\rm peak}$ (pb)	$1.71^{+0.35}_{-0.31} \pm 0.24$
μ (MeV/c^2)	$10888.4^{+2.7}_{-2.6} \pm 1.2$
Γ (MeV/c^2)	$30.7^{+8.3}_{-7.0} \pm 3.1$

← **Enhanced**

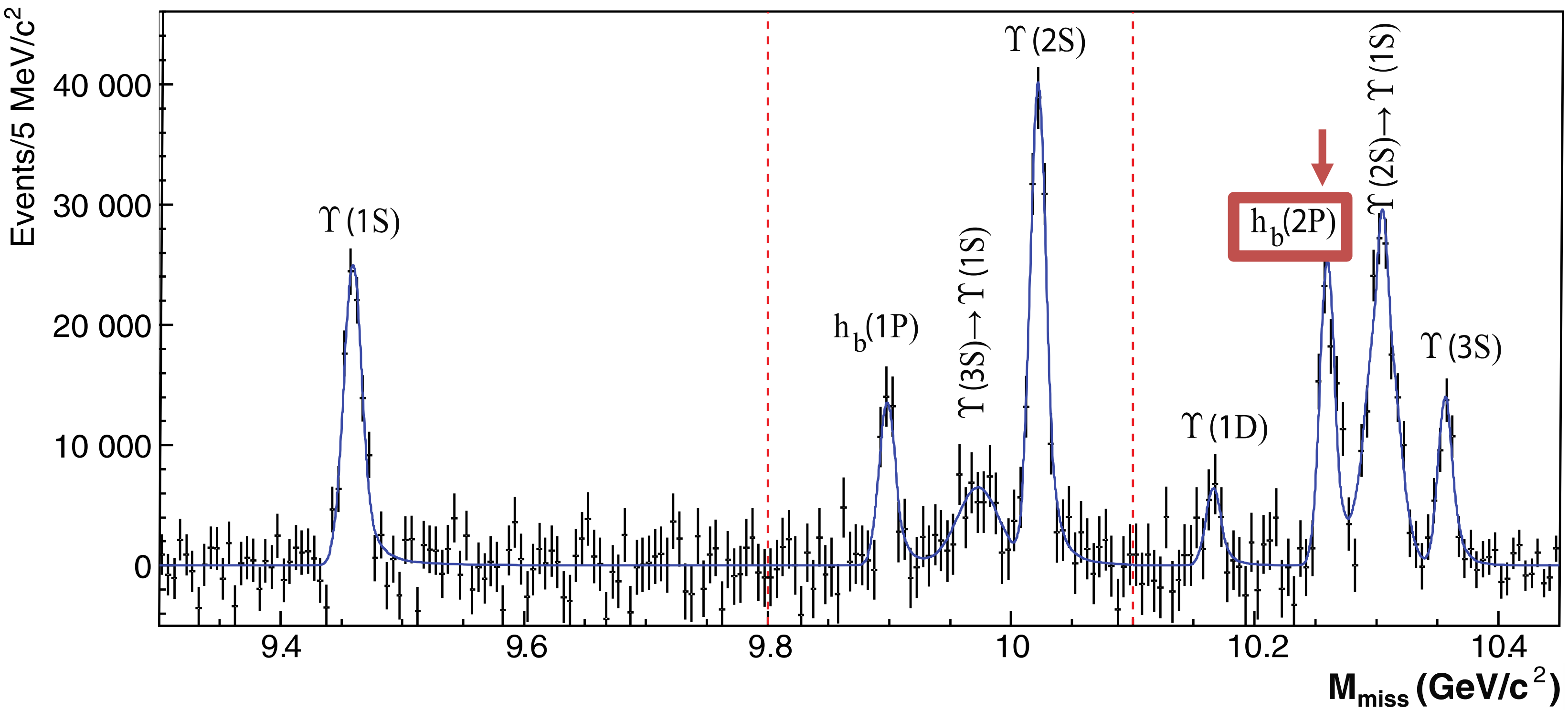
There should be some other components except for bb^{bar} (hybrid, or mixture with bb^{bar}?)
Note: $Y_b(10860)$ decays to BB^{bar} in contrast to Y(4260).

3. Heavy exotic hadrons -X, Y, Z hadrons-
$h_b(2P)$

by-product...

I. Adachi et al. [Belle],
Phys. Rev. Lett. 108, 032001 (2012)

$$e^+e^- \to \pi^+\pi^-\Upsilon(nS)[\to \mu^+\mu^-]$$



(accidental) first discovery of $h_b(2P)$

3. Heavy exotic hadrons -X, Y, Z hadrons-

$$Z_b(10610)^+$$

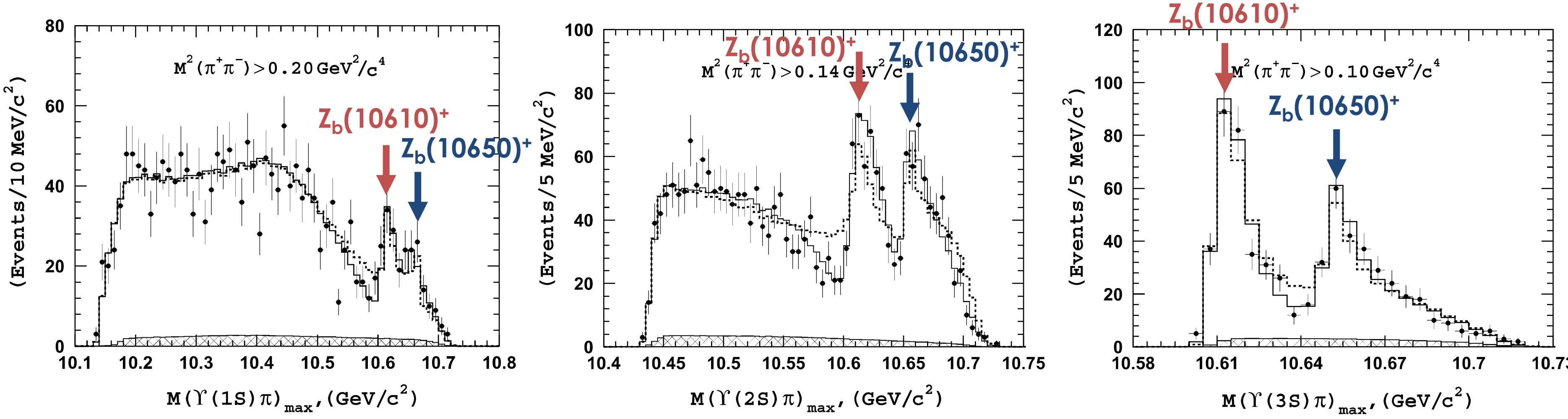
$$Z_b(10650)^+$$

3. Heavy exotic hadrons -X, Y, Z hadrons-
$Z_b(10610)^+$ & $Z_b(10650)^+$

Charged bottomonium

A. Bondar et al. [Belle], Phys. Rev. Lett. 108, 122001 (2012)

$$\Upsilon(5S) \to \Upsilon(nS)\pi^+\pi^- \ (n = 1, 2, 3)$$



———— $J^P=1^+$ favored

- - - - - - $J^P=2^+$ unfavored

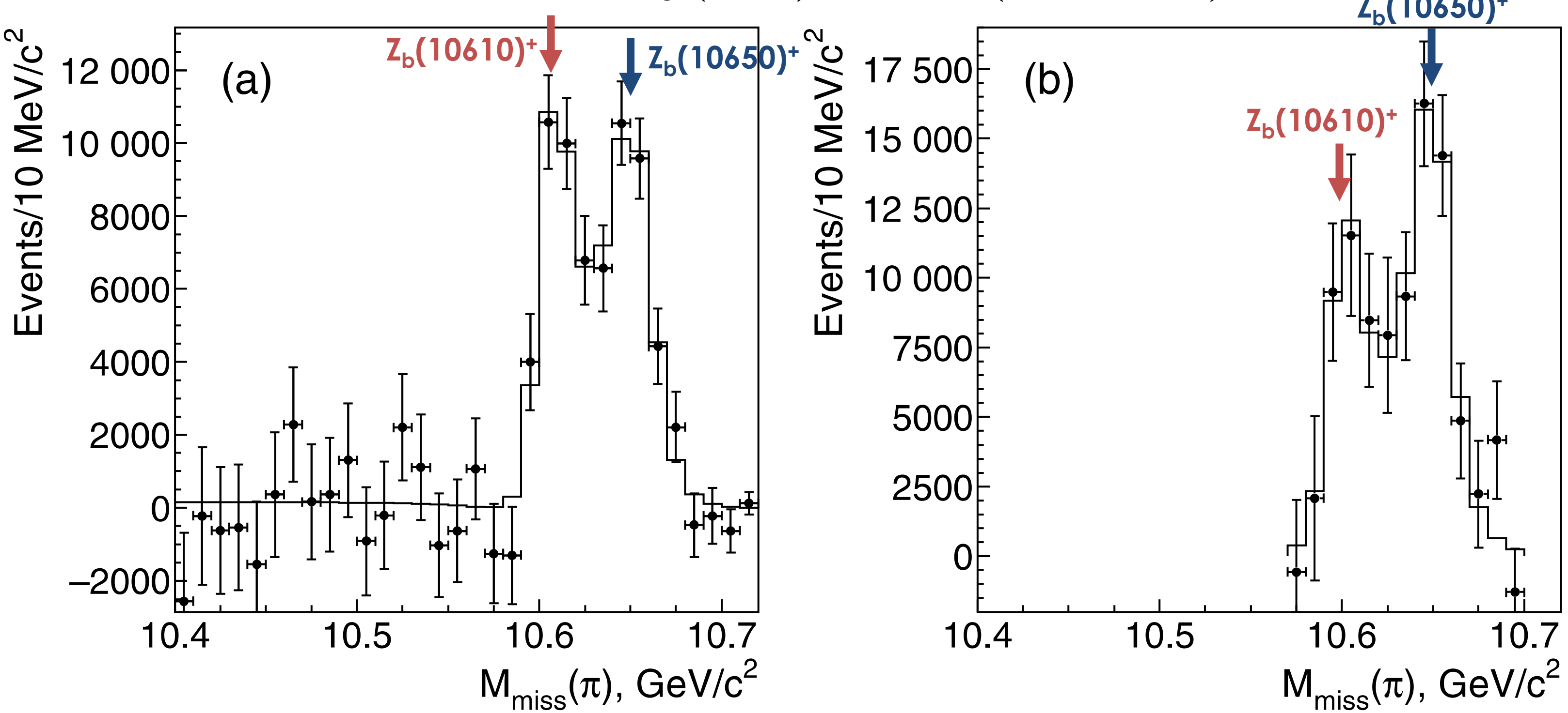
3. Heavy exotic hadrons -X, Y, Z hadrons-

$Z_b(10610)^+$ & $Z_b(10650)^+$

Charged bottomonium

A. Bondar et al. [Belle], Phys. Rev. Lett. 108, 122001 (2012)

$$\Upsilon(5S) \to h_b(mP)\pi^+\pi^- \ (m = 1, 2)$$



3. Heavy exotic hadrons -X, Y, Z hadrons-

$Z_b(10610)^+$ & $Z_b(10650)^+$

Charged bottomonium

A. Bondar et al. [Belle], Phys. Rev. Lett. 108, 122001 (2012)

summary table

Parameter		$\Upsilon(1S)\pi^+$	$\Upsilon(2S)\pi^+$	$\Upsilon(3S)\pi^+$
$Z_b(10610)^+$	M (MeV/c^2)	$10608.5 \pm 3.4^{+3.7}_{-1.4}$	$10608.1 \pm 1.2^{+1.5}_{-0.2}$	$10607.4 \pm 1.5^{+0.8}_{-0.2}$
	Γ (MeV/c^2)	$18.5 \pm 5.3^{+6.1}_{-2.3}$	$20.8 \pm 2.5^{+0.3}_{-2.1}$	$18.7 \pm 3.4^{+2.5}_{-1.3}$
$Z_b(10650)^+$	M (MeV/c^2)	$10656.7 \pm 5.0^{+1.1}_{-3.1}$	$10650.7 \pm 1.5^{+0.5}_{-0.2}$	$10651.2 \pm 1.0^{+0.4}_{-0.3}$
	Γ (MeV/c^2)	$12.1^{+11.3+2.7}_{-4.8-0.6}$	$14.2 \pm 3.7^{+0.9}_{-0.4}$	$9.3 \pm 2.2^{+0.3}_{-0.5}$
Relative phase (deg)		$67 \pm 36^{+24}_{-52}$	$-10 \pm 13^{+34}_{-12}$	$-5 \pm 22^{+15}_{-33}$

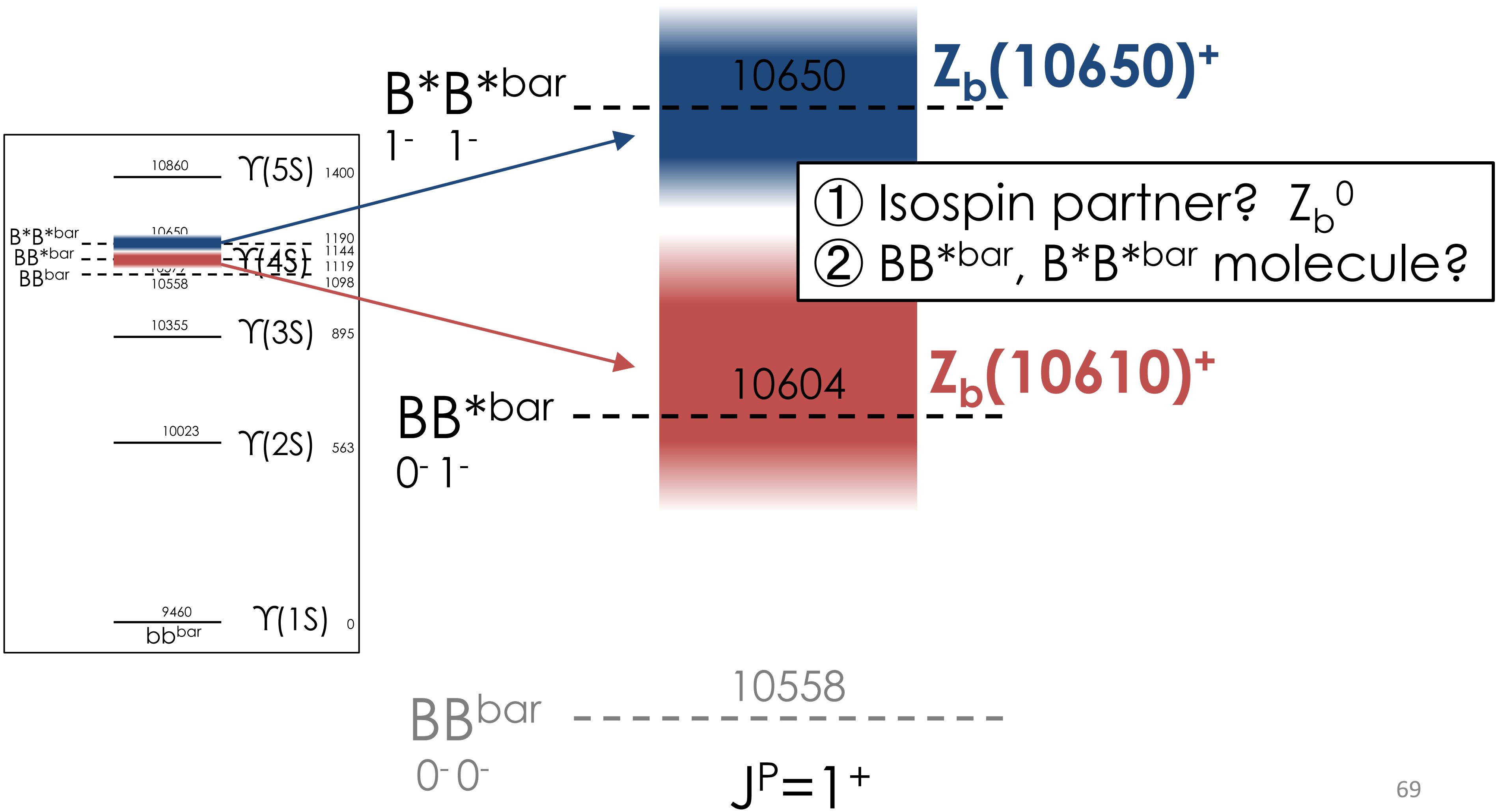
Parameter		$h_b(1P)\pi^+$	$h_b(2P)\pi^+$
$Z_b(10610)^+$	M (MeV/c^2)	$10605 \pm 2^{+3}_{-1}$	10599^{+6+5}_{-3-4}
	Γ (MeV/c^2)	$11.4 \pm^{+4.4+2.1}_{-3.9-1.2}$	13^{+10+9}_{-3-4}
$Z_b(10650)^+$	M (MeV/c^2)	$10654 \pm 3^{+1}_{-2}$	10651^{+2+3}_{-3-2}
	Γ (MeV/c^2)	$20.9 \pm^{+5.4+2.1}_{-4.7-5.7}$	$19 \pm 7^{+11}_{-7}$
Relative phase (deg)		187^{+44+3}_{-57-12}	$181^{+65+74}_{-105-109}$

3. Heavy exotic hadrons -X, Y, Z hadrons-

$Z_b(10610)^+$ & $Z_b(10650)^+$

Charged bottomonium

A. Bondar et al. [Belle], Phys. Rev. Lett. 108, 122001 (2012)



3. Heavy exotic hadrons -X, Y, Z hadrons-

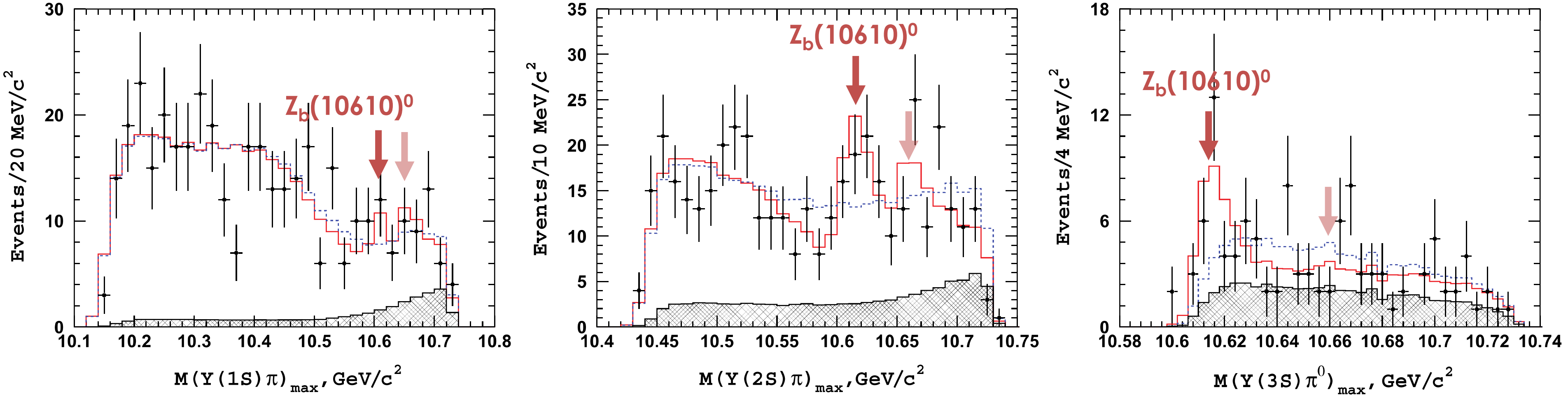
$Z_b(10610)^+$ & $Z_b(10650)^+$

Charged bottomonium

P. Krokovny et al. [Belle], Phys. Rev. D 88, 052016 (2013)

① Isospin partner? Z_b^0

$$\Upsilon(5S) \to \Upsilon(nS)\pi^0\pi^0 \ (n = 1, 2, 3)$$



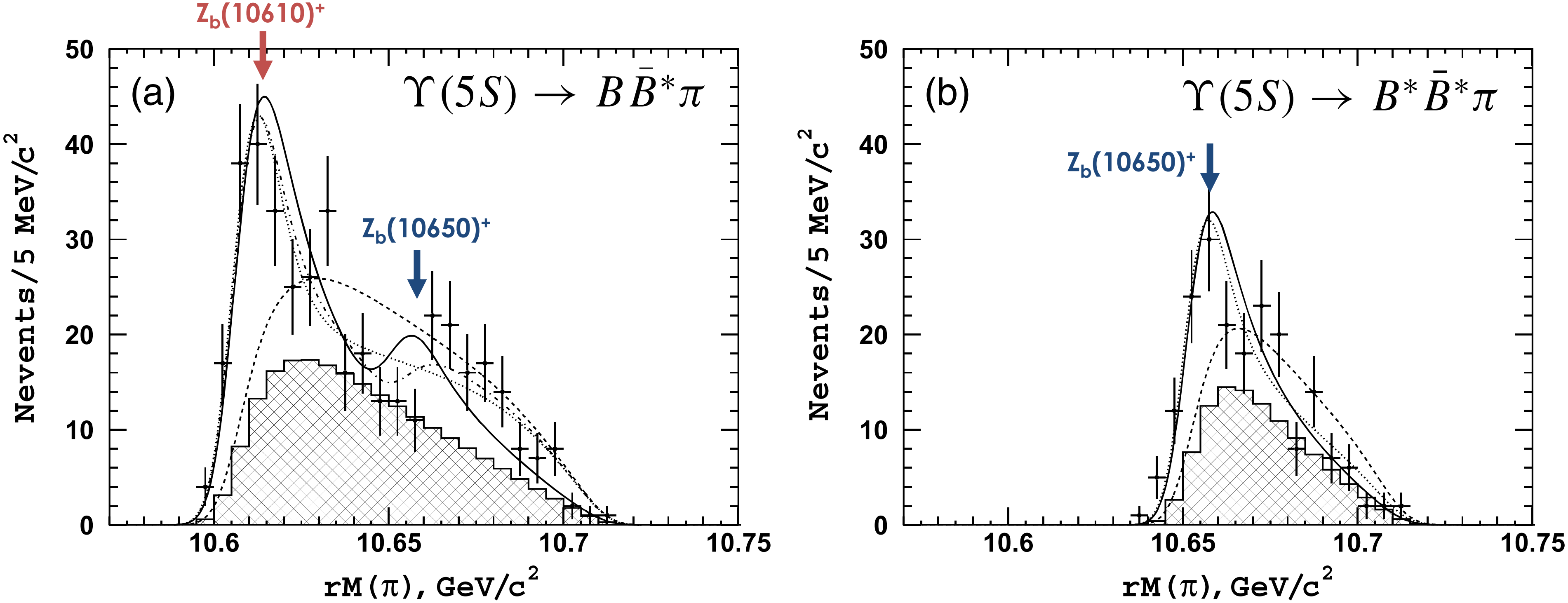
C-parity of $Z_b(10610)^0$: **C=-1** ($\psi(J^{PC}=1^{--})$&$\pi^0(J^{PC}=0^{-+})$)

$Z_b(10610)^0$ was discovered, but $Z_b(10650)^0$ could not be seen due to low statistics...

3. Heavy exotic hadrons -X, Y, Z hadrons-
$Z_b(10610)^+$ & $Z_b(10650)^+$
Charged bottomonium

I. Adachi et al., [Belle], arXiv:1209.6450 [hep-ex]

② BB^{*bar}, B^*B^{*bar} molecule?



$Z_b(10610)^+$ and $Z_b(10650)^+$ seem to contain much component of BB^{*bar} and B^*B^{*bar}.

3. Heavy exotic hadrons -X, Y, Z hadrons-

$Z_b(10610)^+$ & $Z_b(10650)^+$

Charged bottomonium

I. Adachi et al., [Belle], arXiv:1209.6450 [hep-ex]

② BB*bar, B*B*bar molecule?

branching fraction %

Channel	$Z_b(10610)^+$	$Z_b(10650)^+$
$\Upsilon(1S)\pi^+$	0.61 ± 0.28	0.19 ± 0.09
$\Upsilon(2S)\pi^+$	4.19 ± 1.51	1.54 ± 0.69
$\Upsilon(3S)\pi^+$	2.49 ± 0.96	1.81 ± 0.75
$h_b(1P)\pi^+$	4.40 ± 2.17	10.3 ± 5.5
$h_b(2P)\pi^+$	6.26 ± 3.76	19.0 ± 9.3
$B^+\bar{B}^{*0} + \bar{B}^0 B^{*+}$	$\boxed{82.0 \pm 3.5}$	—
$B^{*+}\bar{B}^{*0}$	—	$\boxed{67.2 \pm 7.1}$

$Z_b(10610)^+$ and $Z_b(10650)^+$ seem to contain much component of BB*bar and B*B*bar.

→ Molecule picture?

3. Heavy exotic hadrons -X, Y, Z hadrons-

$Z_b(10610)^+$ & $Z_b(10650)^+$

hadronic molecule interpretation

Though $Z_b(10610)^+$ and $Z_b(10650)^+$ may not be simple hadronic molecules, this picture provides us with a good starting point to under stand those properties.

Hint from experiments:
Are there equal weights for S=1 and S=0?

$$\mathrm{Br}[Z_b \to \Upsilon\pi] \approx \mathrm{Br}[Z_b \to h_b\pi]$$

$\uparrow\uparrow$ $Q\bar{Q}$ spins $\uparrow\downarrow$

If Z_b are B^*B^{*bar} and BB^{*bar} molecules ($Qq^{bar}Q^{bar}q$),

$$|Z_b(10650)\rangle \simeq \left|B^*\bar{B}^*\right\rangle = \frac{1}{\sqrt{2}}\left|0_H^- \otimes 1_l^-\right\rangle + \frac{1}{\sqrt{2}}\left|1_H^- \otimes 0_l^-\right\rangle$$

QQ^{bar} qq^{bar}

$$|Z_b(10610)\rangle \simeq \left|B\bar{B}^*\right\rangle = \frac{1}{\sqrt{2}}\left|0_H^- \otimes 1_l^-\right\rangle - \frac{1}{\sqrt{2}}\left|1_H^- \otimes 0_l^-\right\rangle$$

heavy quark spins 1 and 0 should exist with same fraction!

A. E. Bondar, A. Garmash, A. I. Milstein, R. Mizuk, M. B. Voloshin, Phys. Rev. D 84, 054010 (2011)

3. Heavy exotic hadrons -X, Y, Z hadrons- $Z_b(10610)^+$ & $Z_b(10650)^+$

hadronic molecule interpretation

Many papers…

A. E. Bondar, A. Garmash, A. I. Milstein, R. Mizuk, and M. B. Voloshin, Phys. Rev. D 84, 054010 (2011)
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S. Ohkoda, Y. Yamaguchi, S. Yasui, K. Sudoh, and A. Hosaka, Phys. Rev. D 86, 014004 (2012)
Y. Yang, J. Ping, C. Deng, and H.-S. Zong, J. Phys. G 39, 105001 (2012)
C.-Y. Cui, Y.-L. Liu, and M.-Q. Huang, Phys. Rev. D 85, 074014 (2012)
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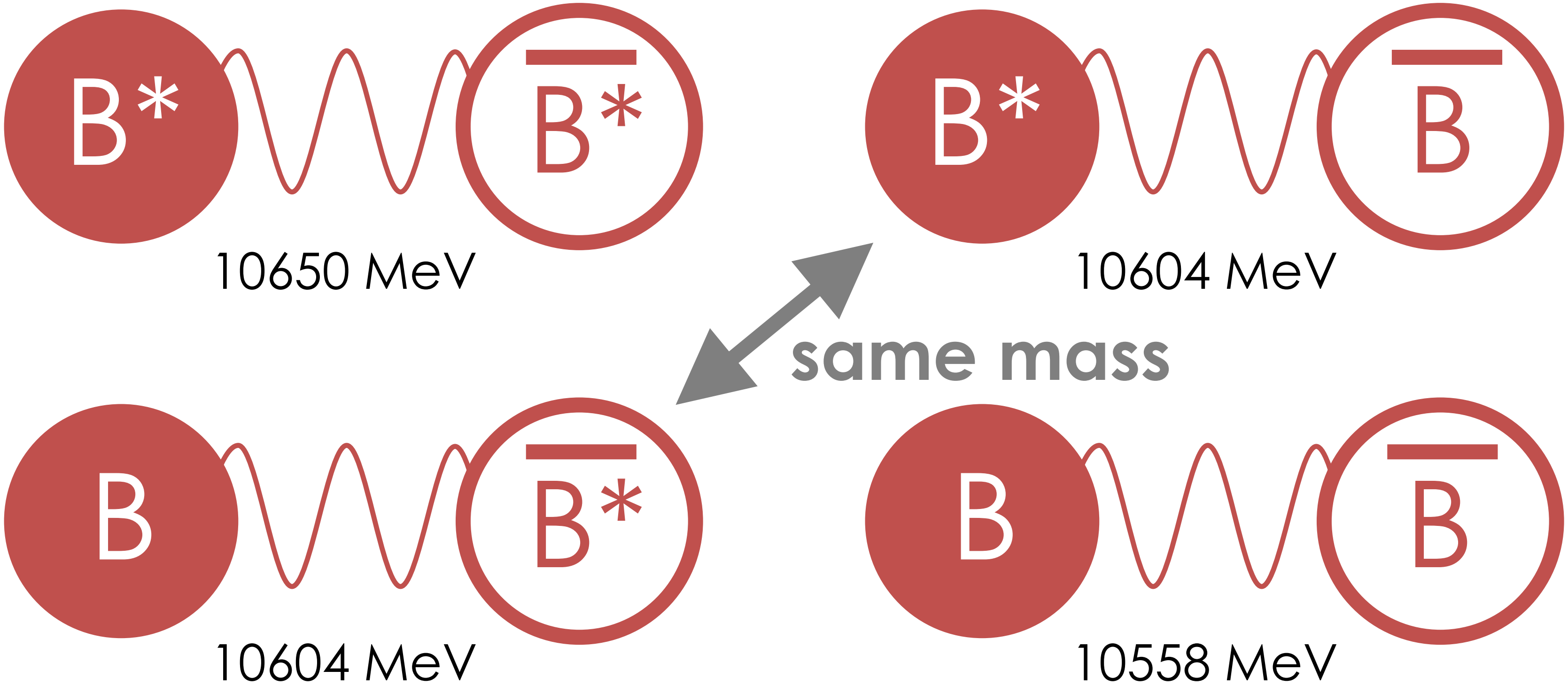
Y. Dong, A. Faessler, T. Gutsche, and V. E. Lyubovitskij, J. Phys. G 40, 015002 (2013)
Z.-G. Wang, Eur. Phys. J. C 74, 2963 (2014)
Z.-G. Wang and T. Huang, Eur. Phys. J. C 74, 2891 (2014)
Z.-G. Wang and T. Huang, Nucl. Phys. A 930, 63 (2014)
S. Ohkoda, S. Yasui, and A. Hosaka, Phys. Rev. D 89, 074029 (2014)
J. M. Dias, F. Aceti, and E. Oset, Phys. Rev. D 91, 076001 (2015)
... and more

3. Heavy exotic hadrons -X, Y, Z hadrons-

$Z_b(10610)^+$ & $Z_b(10650)^+$

hadronic molecule interpretation



3. Heavy exotic hadrons -X, Y, Z hadrons-

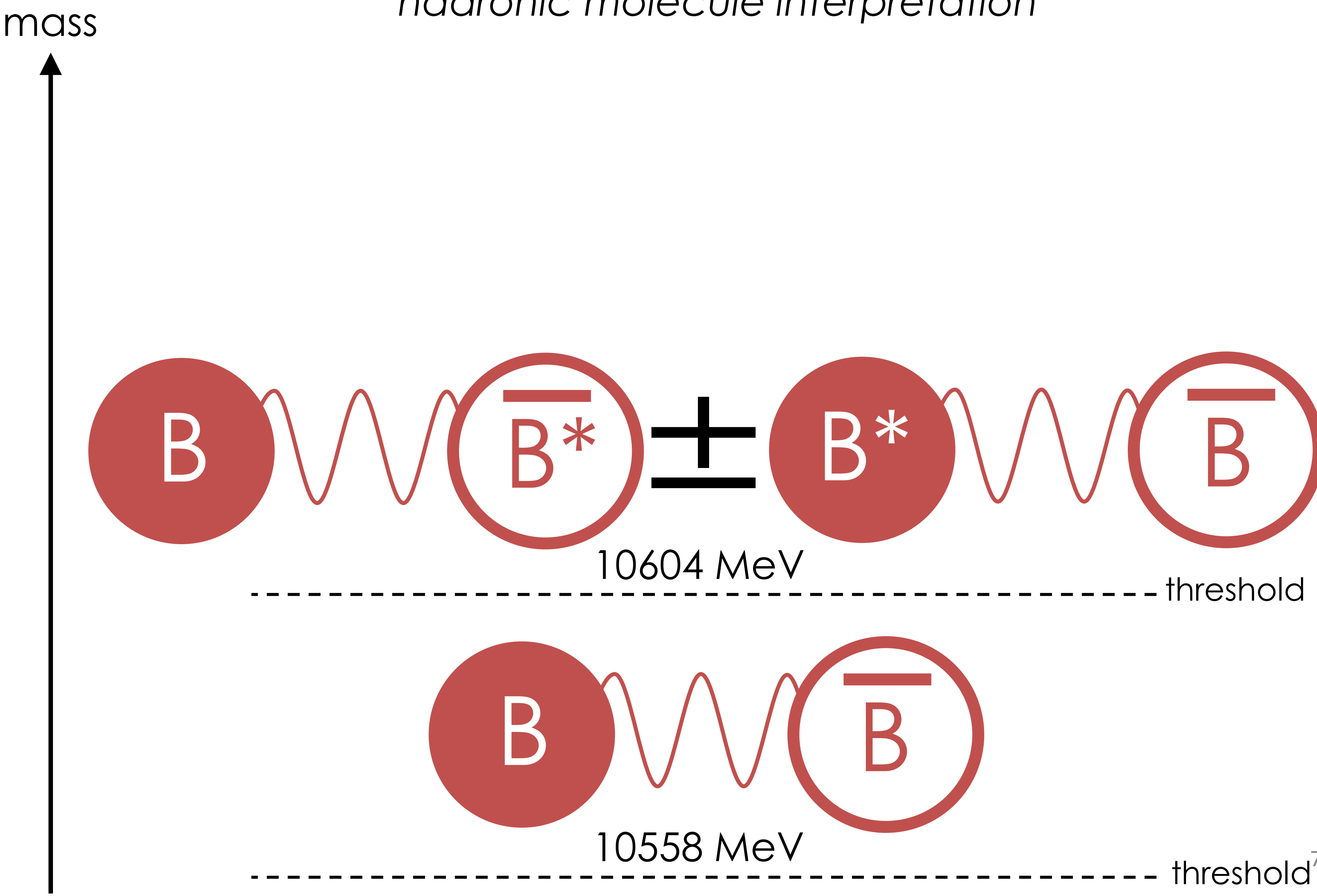
$Z_b(10610)^+$ & $Z_b(10650)^+$

hadronic molecule interpretation

3. Heavy exotic hadrons -X, Y, Z hadrons-

$Z_b(10610)^+$ & $Z_b(10650)^+$

hadronic molecule interpretation



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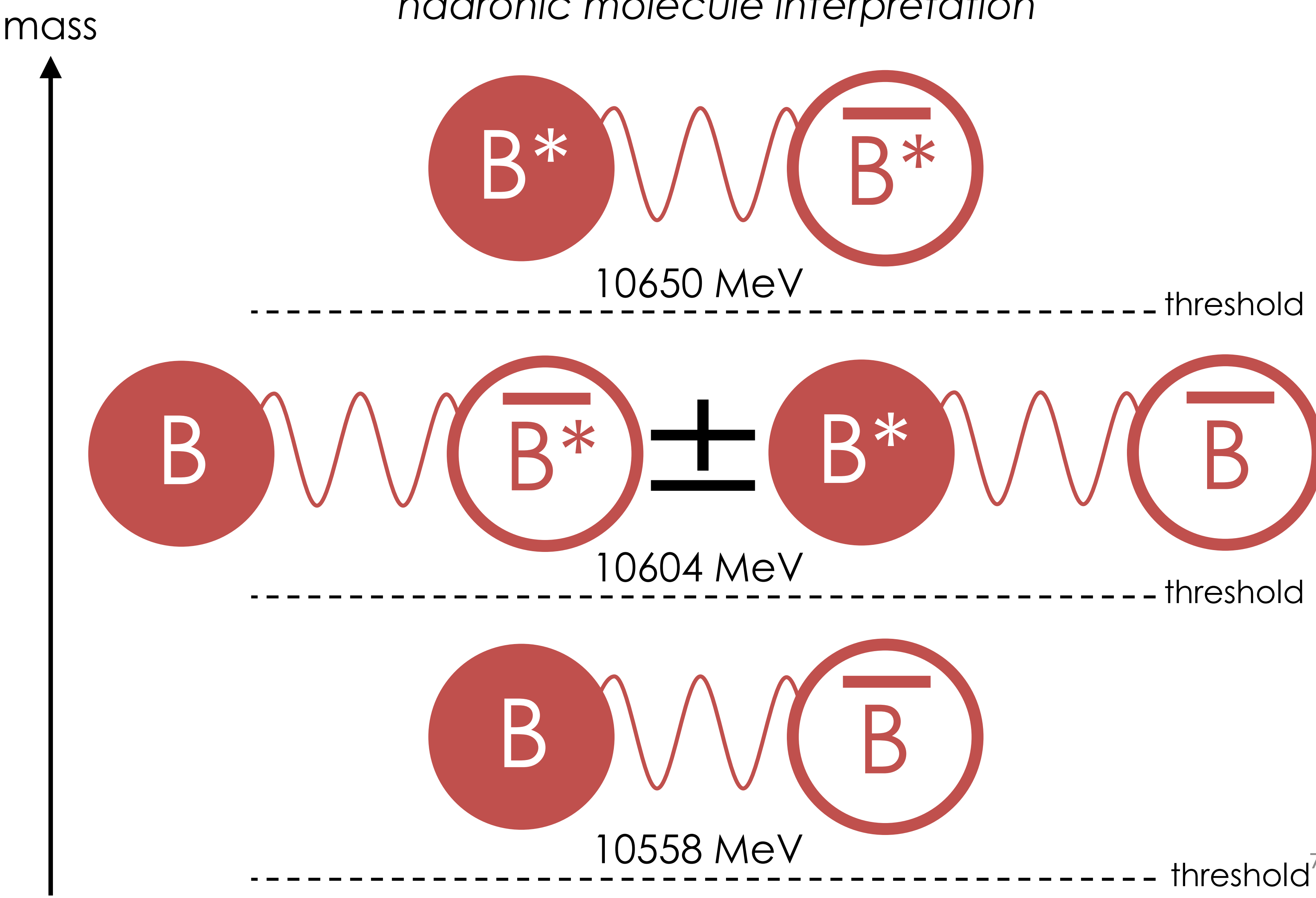
hadronic molecule interpretation



3. Heavy exotic hadrons -X, Y, Z hadrons-

$Z_b(10610)^+$ & $Z_b(10650)^+$

hadronic molecule interpretation

S. Ohkoda, et al.,
Phys. Rev. D86, 014004 (2012)

Classification of $B^{(*)}B^{(*)bar}$ states

C-parity is defined only for I_z=0 (I=1)

J^{PC} $(J \leq 2)$	Components	
0^{+-}	—	
0^{++}	$B\bar{B}(^1S_0)$, $B^*\bar{B}^*(^1S_0)$, $B^*\bar{B}^*(^5D_0)$	
0^{--}	$\frac{1}{\sqrt{2}}\left(B\bar{B}^* + B^*\bar{B}\right)(^3P_0)$	W_{b0}
0^{-+}	$\frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)(^3P_0)$, $B^*\bar{B}^*(^3P_0)$	
1^{+-}	$\frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)(^3S_1)$, $\frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)(^3D_1)$, $B^*\bar{B}^*(^3S_1)$, $B^*\bar{B}^*(^3D_1)$	Z_b
1^{++}	$\frac{1}{\sqrt{2}}\left(B\bar{B}^* + B^*\bar{B}\right)(^3S_1)$, $\frac{1}{\sqrt{2}}\left(B\bar{B}^* + B^*\bar{B}\right)(^3D_1)$, $B^*\bar{B}^*(^5D_1)$	
1^{--}	$B\bar{B}(^1P_1)$, $\frac{1}{\sqrt{2}}\left(B\bar{B}^* + B^*\bar{B}\right)(^3P_1)$, $B^*\bar{B}^*(^1P_1)$, $B^*\bar{B}^*(^5P_1)$, $B^*\bar{B}^*(^5F_1)$	W_{b1}, W'_{b1}
1^{-+}	$\frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)(^3P_1)$, $B^*\bar{B}^*(^3P_1)$	
2^{+-}	$\frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)(^3D_2)$, $B^*\bar{B}^*(^3D_2)$	
2^{++}	$B\bar{B}(^1D_2)$, $\frac{1}{\sqrt{2}}\left(B\bar{B}^* + B^*\bar{B}\right)(^3D_2)$, $B^*\bar{B}^*(^1D_2)$, $B^*\bar{B}^*(^5S_2)$, $B^*\bar{B}^*(^5D_2)$, $B^*\bar{B}^*(^5G_2)$	
2^{-+}	$\frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)(^3P_2)$, $\frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)(^3F_2)$, $B^*\bar{B}^*(^3P_2)$, $B^*\bar{B}^*(^3F_2)$	
2^{--}	$\frac{1}{\sqrt{2}}\left(B\bar{B}^* + B^*\bar{B}\right)(^3P_2)$, $\frac{1}{\sqrt{2}}\left(B\bar{B}^* + B^*\bar{B}\right)(^3F_2)$, $B^*\bar{B}^*(^5P_2)$, $B^*\bar{B}^*(^5F_2)$	W_{b2}, W'_{b2}

3. Heavy exotic hadrons -X, Y, Z hadrons-

$Z_b(10610)^+$ & $Z_b(10650)^+$

hadronic molecule interpretation

C-parity of $B^{(*)}B^{(*)bar}$

$$CBC^{-1}=B^{bar} \qquad CB^*C^{-1}=B^{*bar}$$

① BB^{bar}: $CBB^{bar}C^{-1} = (CBC^{-1})(CB^{bar}C^{-1}) = B^{bar}B = BB^{bar}$

② $BB^{*bar} \pm B^*B^{bar}$:

$C(BB^{*bar} \pm B^*B^{bar})C^{-1}$

$= (CBC^{-1})(CB^{*bar}C^{-1}) \pm (CB^*C^{-1})(CB^{bar}C^{-1})$

$= B^{bar}B^* \pm B^{*bar}(B)$

$= \pm(BB^{*bar} \pm B^*B^{bar})$

③ B^*B^{*bar}: $C(B^*B^{*bar})_{S=0,1,2}C^{-1} = (-1)^S B^*B^{*bar}$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M \rangle = (-1)^{J-j_1-j_2} \langle j_2 j_1 m_2 m_1 | j_2 j_1 J M \rangle$$

$J = 0,1,2$

$j_1, j_2 = 1$

3. Heavy exotic hadrons -X, Y, Z hadrons-

$Z_b(10610)^+$ & $Z_b(10650)^+$

hadronic molecule interpretation

$J^{PC}=1^{+-}$, $I=1$ from experiments

	spin^P	angular momentum	J^P	C-parity	
$B\text{B}^{\text{bar}}$ $0^-\ 0^-$	0^+	S wave	0^+	+1	
$B\text{B}^{*\text{bar}} \pm B^*\text{B}^{\text{bar}}$ $0^-\ 1^-$ **minus**	1^+	S wave	1^+	± 1	✔
	1^+	D wave	$1^+, 2^+, 3^+$	± 1	✔
$B^*\text{B}^{*\text{bar}}$ $1^-\ 1^-$	0^+	S wave	0^+	+1	
	1^+	S wave	1^+	-1	✔
	1^+	D wave	$1^+, 2^+, 3^+$	-1	✔
	2^+	S wave	2^+	+1	
	2^+	D wave	$0^+, 1^+, 2^+, 3^+, 4^+$	+1	

$$\boxed{\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})(^3S_1),\ \frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})(^3D_1),\ B^*\bar{B}^*(^3S_1),\ B^*\bar{B}^*(^3D_1)}$$

3. Heavy exotic hadrons -X, Y, Z hadrons-

$Z_b(10610)^+$ & $Z_b(10650)^+$

hadronic molecule interpretation

If $J^{PC}=1^{++}$, $I=1$...

	spin^P	angular momentum	J^P	C-parity	
$B\bar{B}$ (bar) $0^-\ 0^-$	0^+	S wave	0^+	+1	
$B\bar{B}^* \pm B^*\bar{B}$ (BB*bar ± B*Bbar) $0^-\ 1^-$ plus	1^+	S wave	1^+	±1	✔
	1^+	D wave	$1^+,2^+,3^+$	±1	✔
$B^*\bar{B}^*$ (B*B*bar) $1^-\ 1^-$	0^+	S wave	0^+	+1	
	1^+	S wave	1^+	-1	
	1^+	D wave	$1^+,2^+,3^+$	-1	
	2^+	S wave	2^+	+1	
	2^+	D wave	$0^+,1^+,2^+,3^+,4^+$	+1	✔

$$\boxed{\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})(^3S_1),\ \frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})(^3D_1),\ B^*\bar{B}^*(^5D_1)}$$

3. Heavy exotic hadrons -X, Y, Z hadrons-

$Z_b(10610)^+$ & $Z_b(10650)^+$

hadronic molecule interpretation

S. Ohkoda, et al., Phys. Rev. D86, 014004 (2012)

Classification of $B^{(*)}B^{(*)bar}$ states

C-parity is defined only for I_z=0 (I=1)

J^{PC} $(J \leq 2)$	Components	
0^{+-}	—	
0^{++}	$B\bar{B}(^1S_0)$, $B^*\bar{B}^*(^1S_0)$, $B^*\bar{B}^*(^5D_0)$	
0^{--}	$\frac{1}{\sqrt{2}}\left(B\bar{B}^* + B^*\bar{B}\right)(^3P_0)$	W_{b0}
0^{-+}	$\frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)(^3P_0)$, $B^*\bar{B}^*(^3P_0)$	
1^{+-}	$\frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)(^3S_1)$, $\frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)(^3D_1)$, $B^*\bar{B}^*(^3S_1)$, $B^*\bar{B}^*(^3D_1)$	Z_b
1^{++}	$\frac{1}{\sqrt{2}}\left(B\bar{B}^* + B^*\bar{B}\right)(^3S_1)$, $\frac{1}{\sqrt{2}}\left(B\bar{B}^* + B^*\bar{B}\right)(^3D_1)$, $B^*\bar{B}^*(^5D_1)$	
1^{--}	$B\bar{B}(^1P_1)$, $\frac{1}{\sqrt{2}}\left(B\bar{B}^* + B^*\bar{B}\right)(^3P_1)$, $B^*\bar{B}^*(^1P_1)$, $B^*\bar{B}^*(^5P_1)$, $B^*\bar{B}^*(^5F_1)$	W_{b1}, W'_{b1}
1^{-+}	$\frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)(^3P_1)$, $B^*\bar{B}^*(^3P_1)$	
2^{+-}	$\frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)(^3D_2)$, $B^*\bar{B}^*(^3D_2)$	
2^{++}	$B\bar{B}(^1D_2)$, $\frac{1}{\sqrt{2}}\left(B\bar{B}^* + B^*\bar{B}\right)(^3D_2)$, $B^*\bar{B}^*(^1D_2)$, $B^*\bar{B}^*(^5S_2)$, $B^*\bar{B}^*(^5D_2)$, $B^*\bar{B}^*(^5G_2)$	
2^{-+}	$\frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)(^3P_2)$, $\frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)(^3F_2)$, $B^*\bar{B}^*(^3P_2)$, $B^*\bar{B}^*(^3F_2)$	
2^{--}	$\frac{1}{\sqrt{2}}\left(B\bar{B}^* + B^*\bar{B}\right)(^3P_2)$, $\frac{1}{\sqrt{2}}\left(B\bar{B}^* + B^*\bar{B}\right)(^3F_2)$, $B^*\bar{B}^*(^5P_2)$, $B^*\bar{B}^*(^5F_2)$	W_{b2}, W'_{b2}

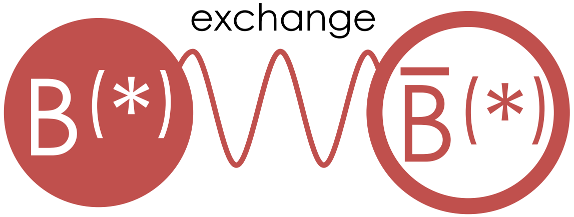
3. Heavy exotic hadrons -X, Y, Z hadrons-

$Z_b(10610)^+$ & $Z_b(10650)^+$

hadronic molecule interpretation

S. Ohkoda, et al.,
Phys. Rev. D86, 014004 (2012)

light meson
exchange



The Schrödinger equation

$$\boxed{H\psi = E\psi}$$

$J^{PC}=1^{+-}$, I=1

$$H = \begin{array}{c} \begin{array}{cccc} \boxed{\frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)({}^3S_1)} & \boxed{\frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)({}^3D_1)} & \boxed{B^*\bar{B}^*({}^3S_1)} & \boxed{B^*\bar{B}^*({}^3D_1)} \end{array} \\ \begin{pmatrix} K_1+V_{11} & V_{12} & V_{13} & V_{14} \\ V_{21} & K_2+V_{22} & V_{23} & V_{24} \\ V_{31} & V_{32} & K_3+V_{33} & V_{34} \\ V_{41} & V_{42} & V_{43} & K_4+V_{44} \end{pmatrix} \end{array}$$

K_i : kinetic term, V_{ij} : potential term (i,j=1,2,3,4)

What is the potential component V_{ij}?

(light meson exchange potentials)

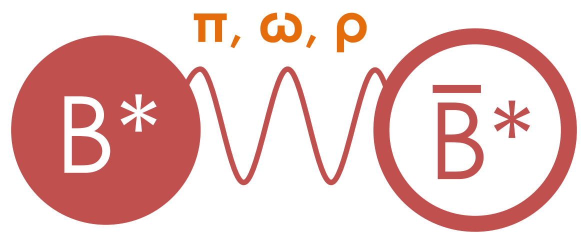
3. Heavy exotic hadrons -X, Y, Z hadrons-

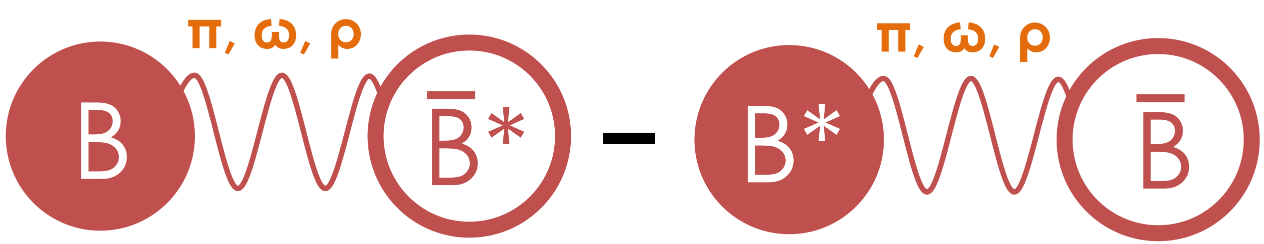
$Z_b(10610)^+$ & $Z_b(10650)^+$

hadronic molecule interpretation

π, ω, ρ meson exchange potentials

S. Ohkoda, et al., Phys. Rev. D86, 014004 (2012)





What are the π, ω, ρ potentials?

3. Heavy exotic hadrons -X, Y, Z hadrons-
$Z_b(10610)^+$ & $Z_b(10650)^+$

hadronic molecule interpretation

$$F_{\mu\nu}(\rho) = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu + [\rho_\mu, \rho_\nu]$$

$$\hat{\pi} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} & \pi^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} \end{pmatrix} \quad A_\mu \simeq \frac{i}{f_\pi} \partial_\mu \hat{\pi}$$

$$\hat{\rho}_\mu = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} \end{pmatrix}_\mu$$

$$\mathcal{L}_{\pi HH} = g\, \mathrm{tr} \bar{H}_a H_b \gamma_\nu \gamma_5 A^\nu_{ba}$$

$$\mathcal{L}_{vHH} = -i\beta\, \mathrm{tr} \bar{H}_a H_b v^\mu (\rho_\mu)_{ba} + i\lambda\, \mathrm{tr} \bar{H}_a H_b \sigma_{\mu\nu} F_{\mu\nu}(\rho)_{ba},$$

$$H_a = \frac{1 + \not{v}}{2} [P^*_{a\mu} \gamma^\mu - P_a \gamma_5]$$

$$\rho_\mu = i \frac{g_V}{\sqrt{2}} \hat{\rho}_\mu$$

$$\bar{H}_a = \gamma_0 H^\dagger_a \gamma_0$$

HQS + chiral symmetry

3. Heavy exotic hadrons -X, Y, Z hadrons-

$Z_b(10610)^+$ & $Z_b(10650)^+$

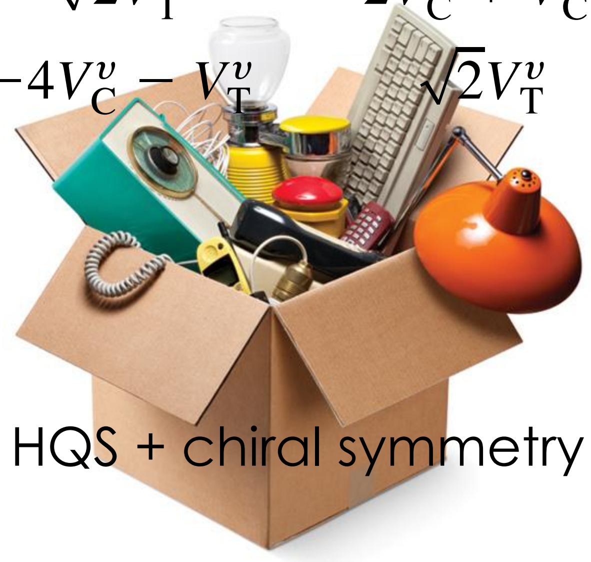
hadronic molecule interpretation

$$V^{\pi}_{1^{+-}} = \begin{pmatrix} \mathrm{V_C} & -\sqrt{2}\mathrm{V_T} & -2\mathrm{V_C} & -\sqrt{2}\mathrm{V_T} \\ -\sqrt{2}\mathrm{V_T} & \mathrm{V_C}+\mathrm{V_T} & -\sqrt{2}\mathrm{V_T} & -2\mathrm{V_C}+\mathrm{V_T} \\ -2\mathrm{V_C} & -\sqrt{2}\mathrm{V_T} & \mathrm{V_C} & -\sqrt{2}\mathrm{V_T} \\ -\sqrt{2}\mathrm{V_T} & -2\mathrm{V_C}+\mathrm{V_T} & -\sqrt{2}\mathrm{V_T} & \mathrm{V_C}+\mathrm{V_T} \end{pmatrix}$$

$$V^{v}_{1^{+-}} = \begin{pmatrix} 2V^v_{\mathrm{C}} + V^{v\prime}_{\mathrm{C}} & \sqrt{2}V^v_{\mathrm{T}} & -4V^v_{\mathrm{C}} & \sqrt{2}V^v_{\mathrm{T}} \\ \sqrt{2}V^v_{\mathrm{T}} & 2V^v_{\mathrm{C}} - V^v_{\mathrm{T}} + V^{v\prime}_{\mathrm{C}} & \sqrt{2}V^v_{\mathrm{T}} & -4V^v_{\mathrm{C}} - V^v_{\mathrm{T}} \\ -4V^v_{\mathrm{C}} & \sqrt{2}V^v_{\mathrm{T}} & 2V^v_{\mathrm{C}} + V^{v\prime}_{\mathrm{C}} & \sqrt{2}V^v_{\mathrm{T}} \\ \sqrt{2}V^v_{\mathrm{T}} & -4V^v_{\mathrm{C}} - V^v_{\mathrm{T}} & \sqrt{2}V^v_{\mathrm{T}} & 2V^v_{\mathrm{C}} - V^v_{\mathrm{T}} + V^{v\prime}_{\mathrm{C}} \end{pmatrix}$$

$$V_C \propto \pm\frac{e^{-mr}}{r}$$ central potential

$$V_T \propto \pm\frac{e^{-mr}}{r}\left(1+\frac{3}{mr}+\frac{3}{m^2r^2}\right)$$ tensor potential



HQS + chiral symmetry

3. Heavy exotic hadrons -X, Y, Z hadrons-

Review: $\pi B^{(*)}B^*$ coupling from effective theory

② Constructing effective Lagrangian (leading order of $m_Q \to \infty$)

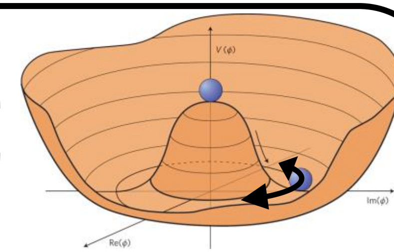
$$\mathcal{L}_{\text{heavy-light}} = \mathrm{Tr}\bar{H}_v v\cdot iDH_v + g\mathrm{Tr}\bar{H}_v H_v \gamma_\mu \gamma_5 A^\mu + \mathcal{O}(1/M)$$

chiral covariant derivative: $D^\mu H_v = \partial^\mu H_v - iV^\mu H_v$

invariant under **HQS** and **chiral symmetry**

Non-linear chiral transformation

Non-linear rep. of π field: $\xi = \exp(i\phi/\sqrt{2}f_\pi)$ $\phi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$



Vector current: $V^\mu(x) = \frac{i}{2}(\xi^\dagger\partial^\mu\xi + \xi\partial^\mu\xi^\dagger)$
(even # of π)

$$V^\mu(x) \to U_q V^\mu(x) U_q^\dagger + iU_q\partial^\mu U_q^\dagger$$

Axial-vector current: $A^\mu(x) = \frac{i}{2}(\xi^\dagger\partial^\mu\xi - \xi\partial^\mu\xi^\dagger)$
(odd # of π)

$$A^\mu \to U_q A^\mu U_q^\dagger$$

Example of vertex structure (axial-vector coupling) $A^\mu \simeq -\partial^\mu\phi/\sqrt{2}f_\pi$

We will see details later.

time ↑

B* — B: $q^i P_v^{*i\dagger} P_v$ — π — q^i

B — B*: $q^i P_v^{\dagger} P_v^{*i}$ — π — q^i

B* — B*: $q^i \varepsilon^{ijk} P_v^{*j\dagger} P_v^{*k}$ — π — q^i

$(i, j, k = 1, 2, 3)$

3. Heavy exotic hadrons -X, Y, Z hadrons-

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$J^{PC}=1^{+-}, I=1$

hadronic molecule interpretation

$$\boxed{\frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)(^3S_1),\ \frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)(^3D_1),\ B^*\bar{B}^*(^3S_1),\ B^*\bar{B}^*(^3D_1)}$$

2 questions

① Why do we consider BB*bar(B*Bbar) and B*B*bar simultaneously?

BB*bar ± B*Bbar sector

$$\frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)(^3S_1)$$

$$\frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)(^3D_1)$$

HQS

B*B*bar sector

$$B^*\bar{B}^*(^3S_1)$$

$$B^*\bar{B}^*(^3D_1)$$

$$\frac{1}{\sqrt{2}}\left(B\bar{B}_i^* - B_i^*\bar{B}\right)$$

$$\frac{1}{\sqrt{2}}\left(B_i^*\bar{B}_j^* - B_j^*B_i^*\right)$$

Answer: Heavy quark spin (HQS) symmetry make them mixed.

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hadronic molecule interpretation

Transformation for heavy quark spin rotation

B meson: $B \to B + \delta B \qquad \delta B = -\frac{1}{2}\theta_i B_i^*$

B* meson: $B_i^* \to B_i^* + \delta B_i^* \quad \delta B_i^* = \frac{1}{2}\varepsilon_{ijk}\theta_j B_k^* - \frac{1}{2}\theta_i B$

① $BB^{*bar} \pm B^*B^{bar}$ sector:

$$
\begin{aligned}
&\delta \frac{1}{\sqrt{2}}\left(B\bar{B}_i^* - B_i^*\bar{B}\right) \\
= &\ \frac{1}{\sqrt{2}}\left(\delta B\bar{B}_i^* + B\delta\bar{B}_i^* - \delta B_i^*\bar{B} - B_i^*\delta\bar{B}\right) \\
= &\ \frac{1}{\sqrt{2}}\left(\left(-\frac{1}{2}\theta_j B_j^*\right)\bar{B}_i^* + B\left(\frac{1}{2}\varepsilon_{ijk}\theta_j\bar{B}_k^* - \frac{1}{2}\theta_i\bar{B}\right) - \left(\frac{1}{2}\varepsilon_{ijk}\theta_j B_k^* - \frac{1}{2}\theta_i B\right)\bar{B} - B_i^*\left(-\frac{1}{2}\theta_j\bar{B}_j^*\right)\right) \\
= &\ \frac{1}{\sqrt{2}}\left(-\frac{1}{2}\theta_j B_j^*\bar{B}_i^* + B\frac{1}{2}\varepsilon_{ijk}\theta_j\bar{B}_k^* - \frac{1}{2}\varepsilon_{ijk}\theta_j B_k^*\bar{B} + \frac{1}{2}\theta_j B_i^*\bar{B}_j^*\right) \\
= &\ \frac{1}{\sqrt{2}}\left(-\frac{1}{2}\theta_j\left(B_j^*\bar{B}_i^* - B_i^*\bar{B}_j^*\right) + \frac{1}{2}\varepsilon_{ijk}\theta_j\left(B\bar{B}_k^* - B_k^*\bar{B}\right)\right) \\
= &\ -\frac{1}{2}\theta_j\frac{1}{\sqrt{2}}\left(B_j^*\bar{B}_i^* - B_i^*\bar{B}_j^*\right) + \frac{1}{2}\varepsilon_{ijk}\theta_j\frac{1}{\sqrt{2}}\left(B\bar{B}_k^* - B_k^*\bar{B}\right)
\end{aligned}
$$

② B*B*bar :

$$
\begin{aligned}
&\delta \frac{1}{\sqrt{2}}\left(B_i^* \bar{B}_j^* - B_j^* B_i^*\right) \\
=\ & \frac{1}{\sqrt{2}}\left(\delta B_i^* \bar{B}_j^* + B_i^* \delta \bar{B}_j^* - \delta B_j^* B_i^* - B_j^* \delta B_i^*\right) \\
=\ & \frac{1}{\sqrt{2}}\Bigg(\left(\frac{1}{2}\varepsilon_{ikl}\theta_k B_l^* - \frac{1}{2}\theta_i B\right)\bar{B}_j^* + B_i^*\left(\frac{1}{2}\varepsilon_{jkl}\theta_k \bar{B}_l^* - \frac{1}{2}\theta_j \bar{B}\right) \\
&\qquad - \left(\frac{1}{2}\varepsilon_{jkl}\theta_k B_l^* - \frac{1}{2}\theta_j B\right)\bar{B}_i^* - B_j^*\left(\frac{1}{2}\varepsilon_{ikl}\theta_k \bar{B}_l^* - \frac{1}{2}\theta_i \bar{B}\right)\Bigg) \\
=\ & \frac{1}{\sqrt{2}}\Bigg(\frac{1}{2}\varepsilon_{ikl}\theta_k B_l^* \bar{B}_j^* - \frac{1}{2}\theta_i B \bar{B}_j^* + \frac{1}{2}\varepsilon_{jkl}\theta_k B_i^* \bar{B}_l^* - \frac{1}{2}\theta_j B_i^* \bar{B} \\
&\qquad - \frac{1}{2}\varepsilon_{jkl}\theta_k B_l^* \bar{B}_i^* + \frac{1}{2}\theta_j B \bar{B}_i^* - \frac{1}{2}\varepsilon_{ikl}\theta_k B_j^* \bar{B}_l^* + \frac{1}{2}\theta_i B_j^* \bar{B}\Bigg) \\
=\ & \frac{1}{\sqrt{2}}\Bigg(\frac{1}{2}\varepsilon_{ikl}\theta_k B_l^* \bar{B}_j^* - \frac{1}{2}\varepsilon_{ikl}\theta_k B_j^* \bar{B}_l^* + \frac{1}{2}\varepsilon_{jkl}\theta_k B_i^* \bar{B}_l^* - \frac{1}{2}\varepsilon_{jkl}\theta_k B_l^* \bar{B}_i^* \\
&\qquad - \frac{1}{2}\theta_i B \bar{B}_j^* - \frac{1}{2}\theta_j B_i^* \bar{B} + \frac{1}{2}\theta_j B \bar{B}_i^* + \frac{1}{2}\theta_i B_j^* \bar{B}\Bigg) \\
=\ & \frac{1}{2}\varepsilon_{ikl}\theta_k \frac{1}{\sqrt{2}}\left(B_l^* \bar{B}_j^* - B_j^* \bar{B}_l^*\right) + \frac{1}{2}\varepsilon_{jkl}\theta_k \frac{1}{\sqrt{2}}\left(B_i^* \bar{B}_l^* - B_l^* \bar{B}_i^*\right) \\
& - \frac{1}{2}\theta_i \frac{1}{\sqrt{2}}\left(B \bar{B}_j^* - B_j^* \bar{B}\right) + \frac{1}{2}\theta_j \frac{1}{\sqrt{2}}\left(B \bar{B}_i^* - B_i^* \bar{B}\right)
\end{aligned}
$$

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$J^{PC}=1^{+-}, I=1$

hadronic molecule interpretation

$$\boxed{\frac{1}{\sqrt{2}}\left(B\bar{B}^{*}-B^{*}\bar{B}\right)({}^3S_1),\ \frac{1}{\sqrt{2}}\left(B\bar{B}^{*}-B^{*}\bar{B}\right)({}^3D_1),\ B^{*}\bar{B}^{*}({}^3S_1),\ B^{*}\bar{B}^{*}({}^3D_1)}$$

2 questions

② Why do we consider S-wave and D-wave simultaneously?

BB*bar ± B*Bbar sector

$$\frac{1}{\sqrt{2}}\left(B\bar{B}^{*}-B^{*}\bar{B}\right)({}^3S_1)$$

$$\frac{1}{\sqrt{2}}\left(B\bar{B}^{*}-B^{*}\bar{B}\right)({}^3D_1)$$

HQS

B*B*bar sector

$$B^{*}\bar{B}^{*}({}^3S_1)$$

$$B^{*}\bar{B}^{*}({}^3D_1)$$

$$\frac{1}{\sqrt{2}}\left(B\bar{B}_i^{*}-B_i^{*}\bar{B}\right)$$

$$\frac{1}{\sqrt{2}}\left(B_i^{*}\bar{B}_j^{*}-B_j^{*}B_i^{*}\right)$$

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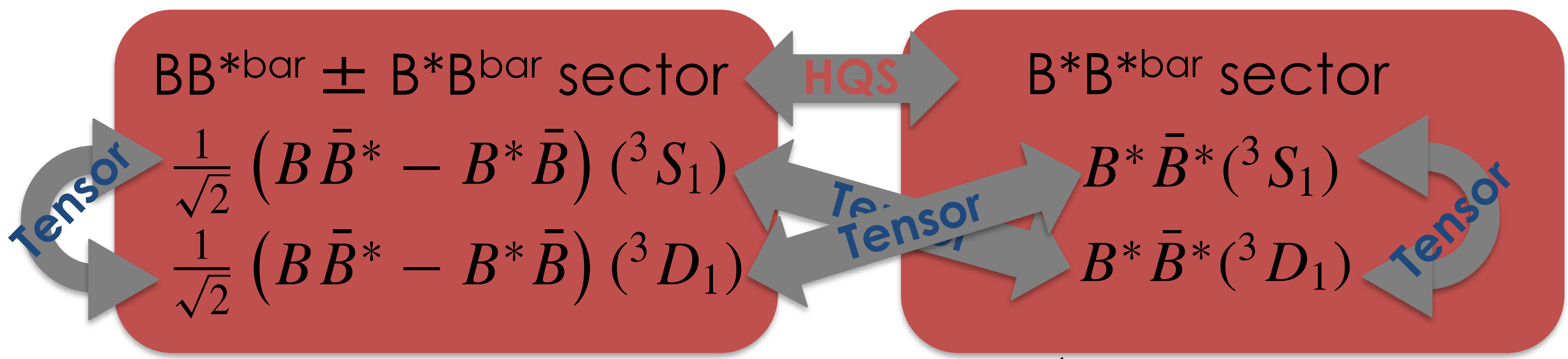

$$\frac{1}{\sqrt{2}}\left(B\bar{B}_i^* - B_i^*\bar{B}\right) \qquad \frac{1}{\sqrt{2}}\left(B_i^*\bar{B}_j^* - B_j^*B_i^*\right)$$

Answer: Tensor potential mixes L and L±2 components.

3. Heavy exotic hadrons -X, Y, Z hadrons-

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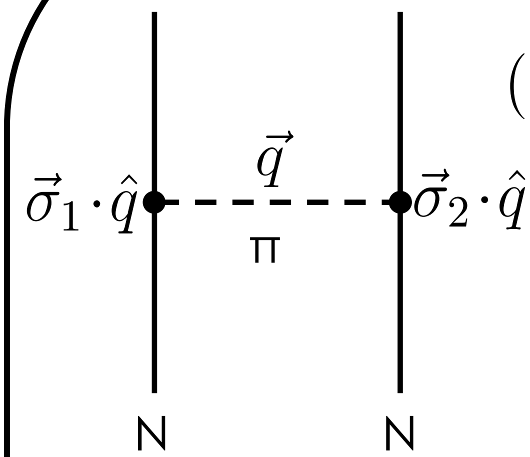
hadronic molecule interpretation

Tensor operator

① OPEP for NN: $\mathcal{L}_{\text{Yukawa}} = -\frac{g_A}{2F}\bar{N}\gamma^\mu\gamma_5\partial_\mu\phi N$



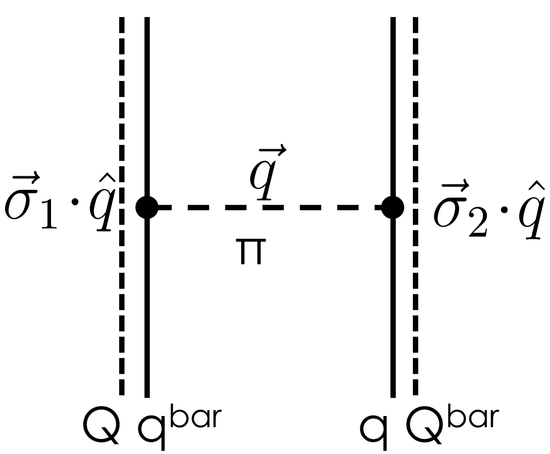
$$(\vec{\sigma}_1\cdot\hat{q})(\vec{\sigma}_2\cdot\hat{q}) = \frac{1}{3}\left(\vec{\sigma}_1\cdot\vec{\sigma}_2 + S_{12}(\hat{q})\right) \qquad \hat{q} = \frac{\vec{q}}{|\vec{q}|}$$

Tensor: $S_{12}(\hat{q}) = 3(\vec{\sigma}_1\cdot\hat{q})(\vec{\sigma}_2\cdot\hat{q}) - \vec{\sigma}_1\cdot\vec{\sigma}_2$

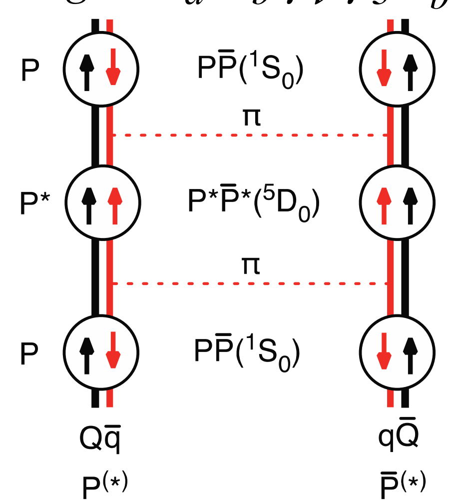
$$\to S_{12}(\hat{q}) = \sqrt{\frac{24\pi}{5}}\sum_{m=-2}^{2}(-1)^m(\vec{\sigma}_1\times\vec{\sigma}_2)^{(2)}_{-m}Y_{2m}(\hat{q})$$

S-wave and D-wave mixing (spin=1)

② OPEP for $B^{(*)}B^{(*)bar}$: $\mathcal{L}_{\pi HH} = g\,\mathrm{tr}\bar{H}_a H_b\gamma_\nu\gamma_5 A^\nu_{ba}$



Example

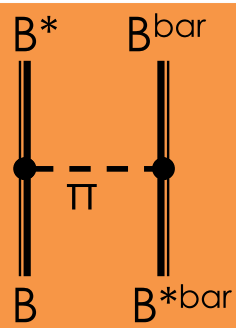


Light spin **triplet**, **S-wave**

Light spin **triplet**, **D-wave**

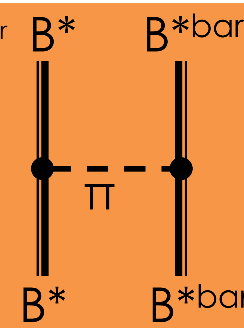
Light spin **triplet**, **S-wave**

(1) BB*bar→B*Bbar



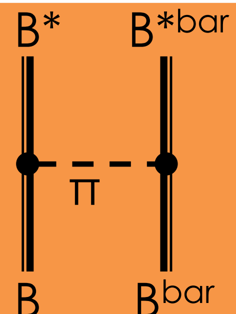
$$-\left(\sqrt{2}\frac{g}{f_\pi}\right)^2 \frac{1}{3}\left[\vec{\varepsilon}_1^{\,*}\cdot\vec{\varepsilon}_2\, C_\pi(r) + S_{\varepsilon_1^*,\varepsilon_2}\, T_\pi(r)\right]\vec{\tau}_1\cdot\vec{\tau}_2$$

(2) B*B*bar→B*B*bar



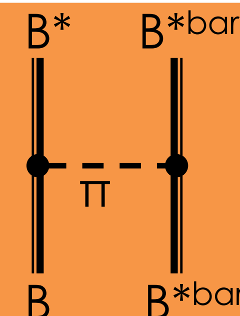
$$-\left(\sqrt{2}\frac{g}{f_\pi}\right)^2 \frac{1}{3}\left[\vec{T}_1\cdot\vec{T}_2\, C_\pi(r) + S_{T_1,T_2}\, T_\pi(r)\right]\vec{\tau}_1\cdot\vec{\tau}_2$$

(3) BBbar→B*B*bar



$$-\left(\sqrt{2}\frac{g}{f_\pi}\right)^2 \frac{1}{3}\left[\vec{\varepsilon}_1^{\,*}\cdot\vec{\varepsilon}_2^{\,*}\, C_\pi(r) + S_{\varepsilon_1^*,\varepsilon_2^*}\, T_\pi(r)\right]\vec{\tau}_1\cdot\vec{\tau}_2$$

(4) BB*bar→B*B*bar



$$\left(\sqrt{2}\frac{g}{f_\pi}\right)^2 \frac{1}{3}\left[\vec{\varepsilon}_1^{\,*}\cdot\vec{T}_2\, C_\pi(r) + S_{\varepsilon_1^*,T_2}\, T_\pi(r)\right]\vec{\tau}_1\cdot\vec{\tau}_2$$

Polarization vector (B*): $\vec{\varepsilon}^{\,(\pm)} = \left(\mp 1/\sqrt{2}, \pm i/\sqrt{2}, 0\right)$ $\vec{\varepsilon}^{\,(0)} = (0, 0, 1)$

Spin 1 operator (B*): $\vec{T} = (T^1, T^2, T^3)$ $T^1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ $T^2 = \frac{i}{\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$ $T^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

Tensor operator:

$$S_{\varepsilon_1^*,\varepsilon_2^*} = 3\left(\vec{\varepsilon}^{\,(\lambda_1)*}\cdot\hat{r}\right)\left(\vec{\varepsilon}^{\,(\lambda_2)*}\cdot\hat{r}\right) - \vec{\varepsilon}^{\,(\lambda_1)*}\cdot\vec{\varepsilon}^{\,(\lambda_2)*} \qquad S_{T_1,T_2} = 3\left(\vec{T}_1\cdot\hat{r}\right)\left(\vec{T}_2\cdot\hat{r}\right) - \vec{T}_1\cdot\vec{T}_2$$

$$S_{\varepsilon_1^*,\varepsilon_2} = 3\left(\vec{\varepsilon}^{\,(\lambda_1)*}\cdot\hat{r}\right)\left(\vec{\varepsilon}^{\,(\lambda_2)}\cdot\hat{r}\right) - \vec{\varepsilon}^{\,(\lambda_1)*}\cdot\vec{\varepsilon}^{\,(\lambda_2)} \qquad S_{\varepsilon_1^*,T_2} = 3\left(\vec{\varepsilon}^{\,(\lambda_1)*}\cdot\hat{r}\right)\left(\vec{T}_2\cdot\hat{r}\right) - \vec{\varepsilon}^{\,(\lambda_1)*}\cdot\vec{T}_2$$

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hadronic molecule interpretation

Summary table: mixing effects (HQS, tensor potential)

Components	$\frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)(^3S_1)$	$\frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)(^3D_1)$	$B^*\bar{B}^*(^3S_1)$	$B^*\bar{B}^*(^3D_1)$
$\frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)(^3S_1)$		**Tensor**	**HQS**	**Tensor**
$\frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)(^3D_1)$	**Tensor**	**Tensor**	**Tensor**	**HQS** **Tensor**
$B^*\bar{B}^*(^3S_1)$	**HQS**	**Tensor**		**Tensor**
$B^*\bar{B}^*(^3D_1)$	**Tensor**	**HQS Tensor**	**Tensor**	**Tensor**

Central potential: $C_\pi(r) \simeq \frac{1}{r}e^{-m_\pi r}$

Tensor potential: $T_\pi(r) \simeq \left(1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right)\frac{1}{r}e^{-m_\pi r}$

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$$\frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)(^3S_1),\ \frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)(^3D_1),\ B^*\bar{B}^*(^3S_1),\ B^*\bar{B}^*(^3D_1)$$

$$\boxed{H\psi = E\psi}$$

① Kinetic term

$$K_{1^{+-}} = \mathrm{diag}\left(-\frac{1}{2\tilde{m}_{\mathrm{BB}^*}}\triangle_0,\ -\frac{1}{2\tilde{m}_{\mathrm{BB}^*}}\triangle_2,\ -\frac{1}{2\tilde{m}_{\mathrm{B^*B^*}}}\triangle_0 + \Delta m_{\mathrm{BB}^*},\ -\frac{1}{2\tilde{m}_{\mathrm{B^*B^*}}}\triangle_2 + \Delta m_{\mathrm{BB}^*}\right)$$

② OPEP

$$V^{\pi}_{1^{+-}} = \begin{pmatrix} \mathrm{V_C} & -\sqrt{2}\mathrm{V_T} & -2\mathrm{V_C} & -\sqrt{2}\mathrm{V_T} \\ -\sqrt{2}\mathrm{V_T} & \mathrm{V_C} + \mathrm{V_T} & -\sqrt{2}\mathrm{V_T} & -2\mathrm{V_C} + \mathrm{V_T} \\ -2\mathrm{V_C} & -\sqrt{2}\mathrm{V_T} & \mathrm{V_C} & -\sqrt{2}\mathrm{V_T} \\ -\sqrt{2}\mathrm{V_T} & -2\mathrm{V_C} + \mathrm{V_T} & -\sqrt{2}\mathrm{V_T} & \mathrm{V_C} + \mathrm{V_T} \end{pmatrix}$$

Tensor potential

③ Vector-meson exchange potential

$$V^{v}_{1^{+-}} = \begin{pmatrix} 2V^v_{\mathrm{C}} + V^{v\prime}_{\mathrm{C}} & \sqrt{2}V^v_{\mathrm{T}} & -4V^v_{\mathrm{C}} & \sqrt{2}V^v_{\mathrm{T}} \\ \sqrt{2}V^v_{\mathrm{T}} & 2V^v_{\mathrm{C}} - V^v_{\mathrm{T}} + V^{v\prime}_{\mathrm{C}} & \sqrt{2}V^v_{\mathrm{T}} & -4V^v_{\mathrm{C}} - V^v_{\mathrm{T}} \\ -4V^v_{\mathrm{C}} & \sqrt{2}V^v_{\mathrm{T}} & 2V^v_{\mathrm{C}} + V^{v\prime}_{\mathrm{C}} & \sqrt{2}V^v_{\mathrm{T}} \\ \sqrt{2}V^v_{\mathrm{T}} & -4V^v_{\mathrm{C}} - V^v_{\mathrm{T}} & \sqrt{2}V^v_{\mathrm{T}} & 2V^v_{\mathrm{C}} - V^v_{\mathrm{T}} + V^{v\prime}_{\mathrm{C}} \end{pmatrix}$$

3. Heavy exotic hadrons -X, Y, Z hadrons-

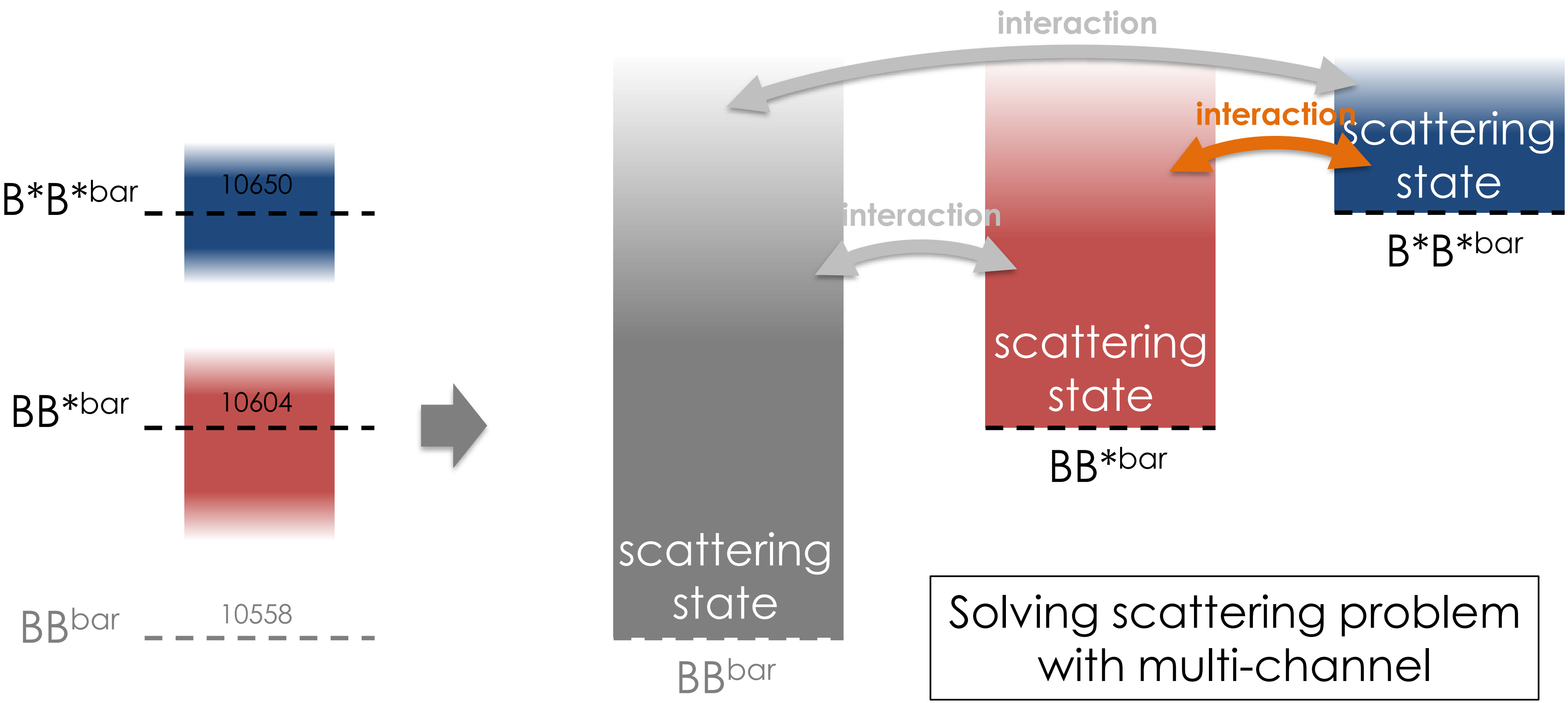
$Z_b(10610)^+$ & $Z_b(10650)^+$

$J^{PC}=1^{+-}, I=1$

hadronic molecule interpretation

$$\frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)(^3S_1),\ \frac{1}{\sqrt{2}}\left(B\bar{B}^* - B^*\bar{B}\right)(^3D_1),\ B^*\bar{B}^*(^3S_1),\ B^*\bar{B}^*(^3D_1)$$

$$\boxed{H\psi = E\psi}$$



Solving scattering problem with multi-channel

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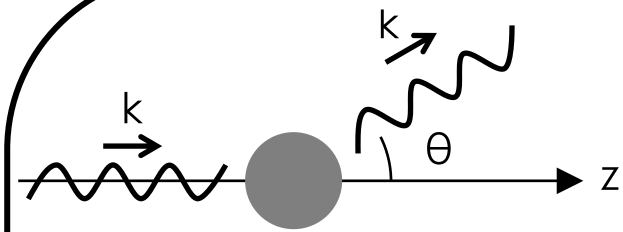
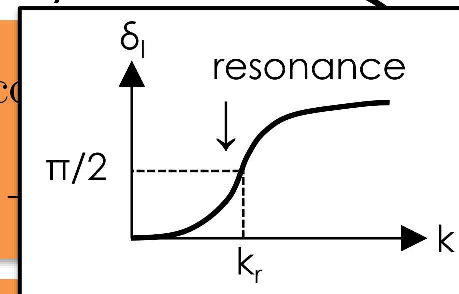
How to solve scattering problem? (review)



$$\varphi^{(+)}(r,\theta) = \sum_{\ell=0}^{\infty} i^\ell (2\ell+1) c_\ell \frac{\chi_\ell(r)}{kr} P_\ell(\mathrm{c}$$

asymptotic state: $\chi_\ell(r) \sim \sin\left(kr - \frac{\pi\ell}{2} -\right.$
(phase shift)



① Partial wave decomposition

$$\varphi^{(+)}(r,\theta) = e^{ikz} + f(\theta)\frac{e^{ikr}}{r}$$

$$f(\theta) = \sum_{\ell=0}^{\infty}(2\ell+1)\frac{e^{2i\delta_\ell}-1}{2ik}P_\ell(\cos\theta)$$

$$= \sum_{\ell=0}^{\infty}(2\ell+1)f_\ell(k)P_\ell(\cos\theta)$$

Partial wave amplitude $\quad f_\ell(k) = \dfrac{1}{k(\cot\delta_\ell - i)}$

② Resonance (definition by phase shift)

$$\cot\delta_\ell(k_r) = 0 \Leftrightarrow \delta_\ell(k_r) = \frac{\pi}{2}$$

$$\cot\delta_\ell = \cot\frac{\pi}{2} + \frac{\mathrm{d}}{\mathrm{d}E}\cot\delta_\ell\Big|_{\delta_\ell=\pi/2}(E-E_r) + \dots$$

decay width: $\dfrac{\mathrm{d}}{\mathrm{d}E}\cot\delta_\ell\Big|_{\delta_\ell=\pi/2} \equiv -\dfrac{2}{\Gamma} \qquad E_r = \dfrac{k_r^2}{2m}$

Pole as complex energy $\quad f_\ell(k) \sim -\dfrac{1}{k}\dfrac{\frac{\Gamma}{2}}{E - E_r + i\frac{\Gamma}{2}}$

☑ Matching (direct) method: finding $\delta_l(k)$

☑ Complex scaling method: finding $E_r + i\Gamma/2$

S. Aoyama T. Myo, K. Kato, K. Ikeda, Prog. Theor. Phys. 116, 1 (2006)

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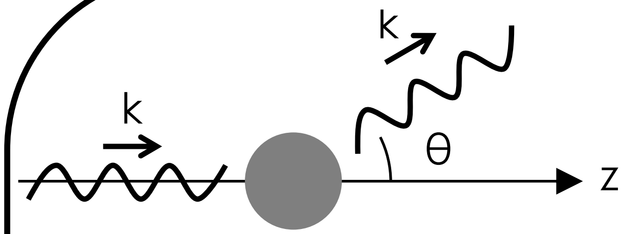
How to solve scattering problem? (review)



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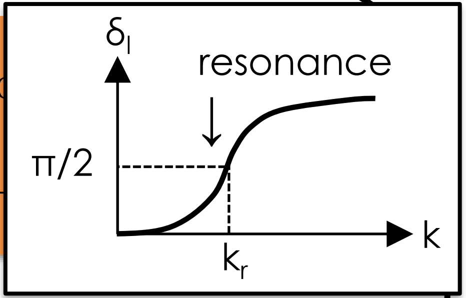

Typical mechanisms of resonances

① Centrifugal potential ② Feshbach resonance ③ *E*-dependent potential

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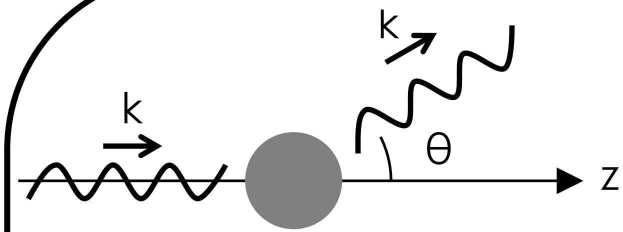

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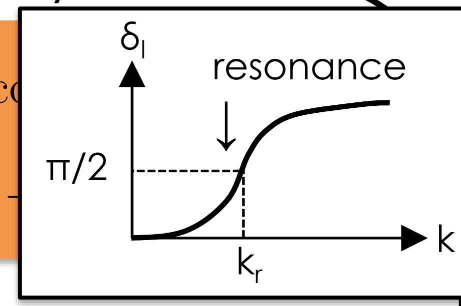

Typical mechanisms of resonances

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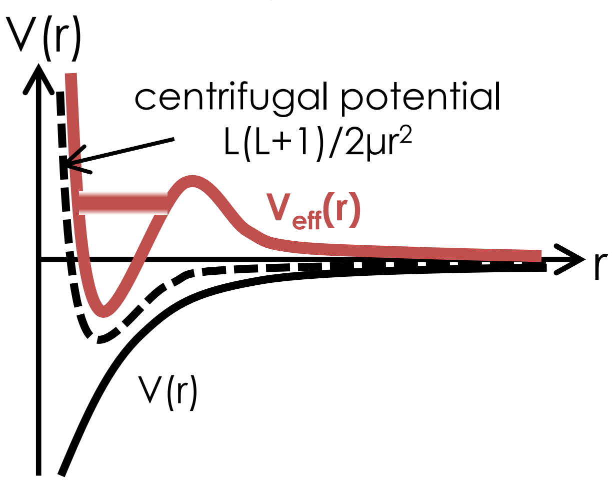

② Feshbach resonance

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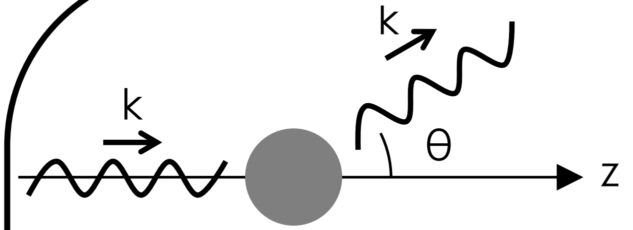

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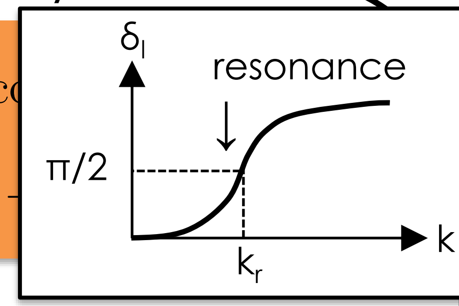

Typical mechanisms of resonances

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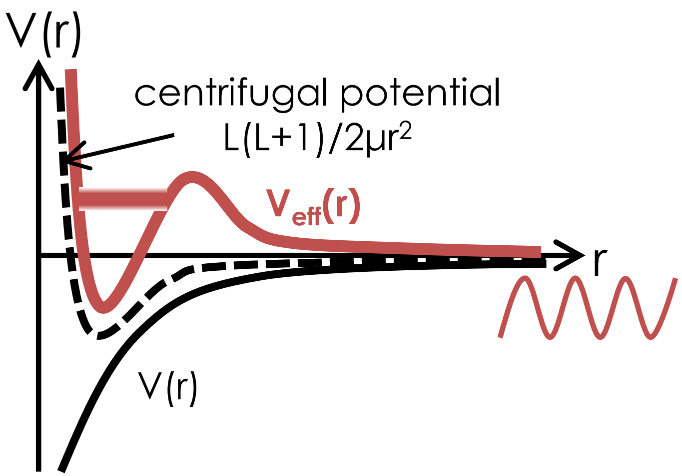
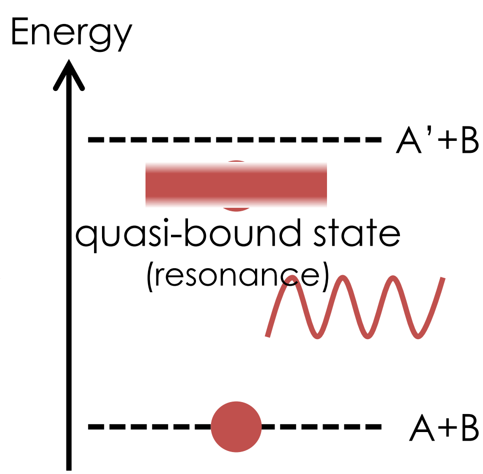


② Feshbach resonance



A+B → A'+B' → A+B

③ *E*-dependent potential

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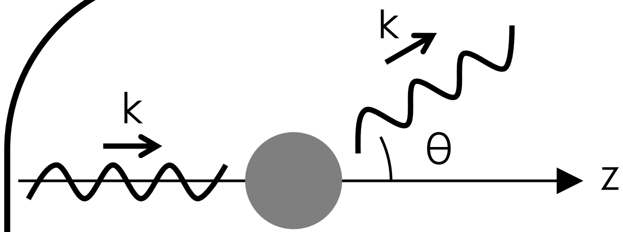

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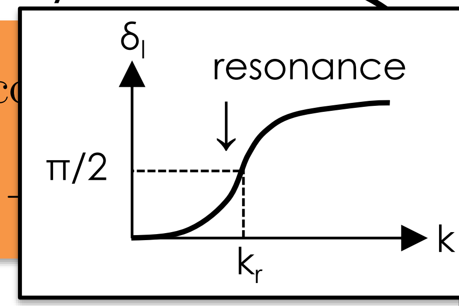
(phase shift)



Typical mechanisms of resonances

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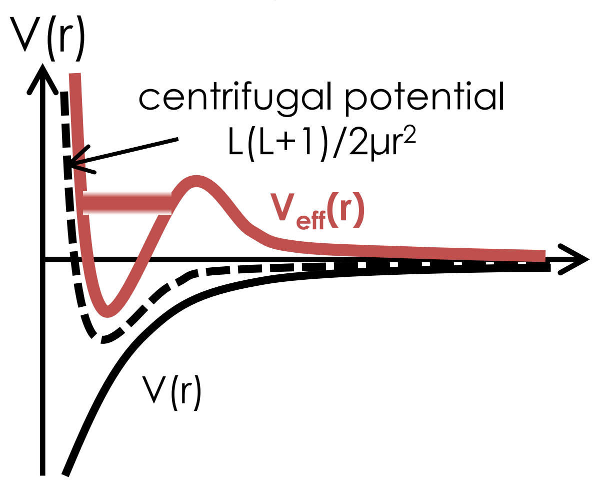
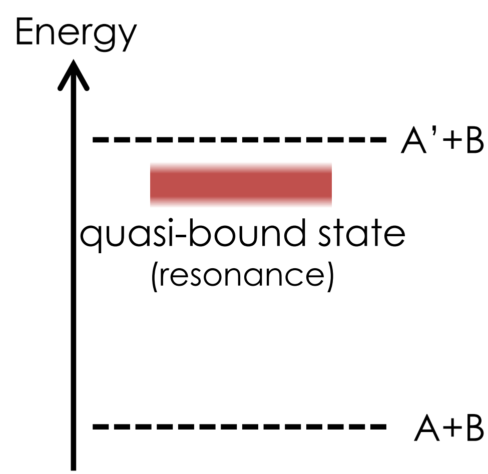


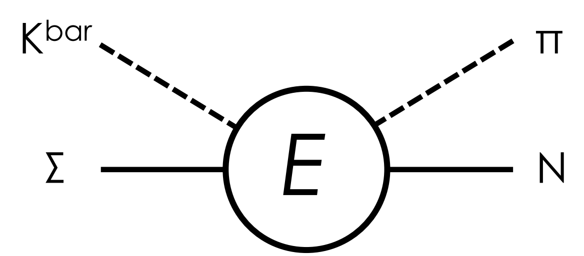
② Feshbach resonance



A+B → A'+B' → A+B

③ *E*-dependent potential

Λ(1405) baryon resonance

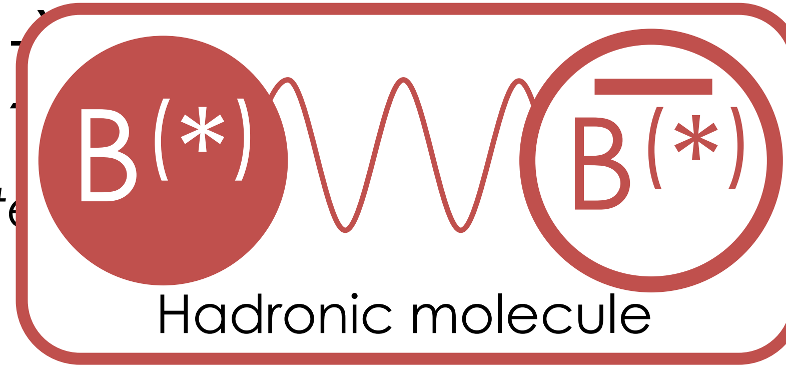


Weinberg-Tomozawa interaction

(Chiral symmetry for NG boson)

3. Heavy exotic hadrons -
$Z_b(10610)^+$ & $Z_b($

hadronic molecule inte



Numerical result

$I^G(J^{PC})$	Threshold	E [MeV] π potential	$\pi\rho\omega$ potential	Decay channels s wave	p wave
$1^+(0^{+-})$	...	...	...	...	$h_b + \pi$, $\chi_{b0,1,2} + \rho$
$1^-(0^{++})$	$B\bar{B}$	-6.5	no	$\eta_b + \pi$, $\Upsilon + \rho$	$h_b + \rho^*$, $\chi_{b1} + \pi$
$1^+(0^{--})$	$B\bar{B}^*$	-9.9	-9.8	$\chi_{b1} + \rho^*$	$\eta_b + \rho$, $\Upsilon + \pi$
$1^-(0^{-+})$	$B\bar{B}^*$	no	no	$h_b + \rho$, $\chi_{b0} + \pi$	$\Upsilon + \rho$
$1^+(1^{+-})$	$B\bar{B}^*$	-7.7	-8.5 ← **bound state**	$\Upsilon + \pi$	$h_b + \pi$, $\chi_{b1} + \rho^*$
			$50.4 - i15.1/2$ ← **resonant state**		
$1^-(1^{++})$	$B\bar{B}^*$	-16.7	-1.9	$\Upsilon + \rho$	$h_b + \rho^*$, $\chi_{b0,1} + \pi$
$1^+(1^{--})$	$B\bar{B}$	$7.0 - i37.9/2$	$7.1 - i37.4/2$	$h_b + \pi$, $\chi_{b0,1,2} + \rho^*$	$\eta_b + \rho$, $\Upsilon + \pi$
		$58.8 - i30.0/2$	$58.6 - i27.7/2$		
$1^-(1^{-+})$	$B\bar{B}^*$	no	no	$h_b + \rho$, $\chi_{b1} + \pi$	$\eta_b + \pi$, $\Upsilon + \rho$
$1^+(2^{+-})$	$B\bar{B}^*$	no	no	...	$h_b + \pi$, $\chi_{b0,1,2} + \rho$
$1^-(2^{++})$	$B\bar{B}$	$63.5 - i8.3/2$	$62.7 - i8.4/2$	$\Upsilon + \rho$	$h_b + \rho^*$, $\chi_{b1,2} + \pi$
$1^-(2^{-+})$	$B\bar{B}^*$	no	no	$h_b + \rho$	$\Upsilon + \rho$
$1^+(2^{--})$	$B\bar{B}^*$	$2.0 - i4.1/2$	$2.0 - i3.9/2$	$\chi_{b1} + \rho^*$	$\eta_b + \rho$, $\Upsilon + \pi$
		$44.2 - i2.5/2$	$44.1 - i2.8/2$		

S. Ohkoda, et al.,
Phys. Rev. D86, 014004 (2012)

3. Heavy exotic hadrons -X, Y, Z hadrons-

$Z_b(10610)^+$ & $Z_b(10650)^+$

hadronic molecule interpretation

S. Ohkoda, et al.,
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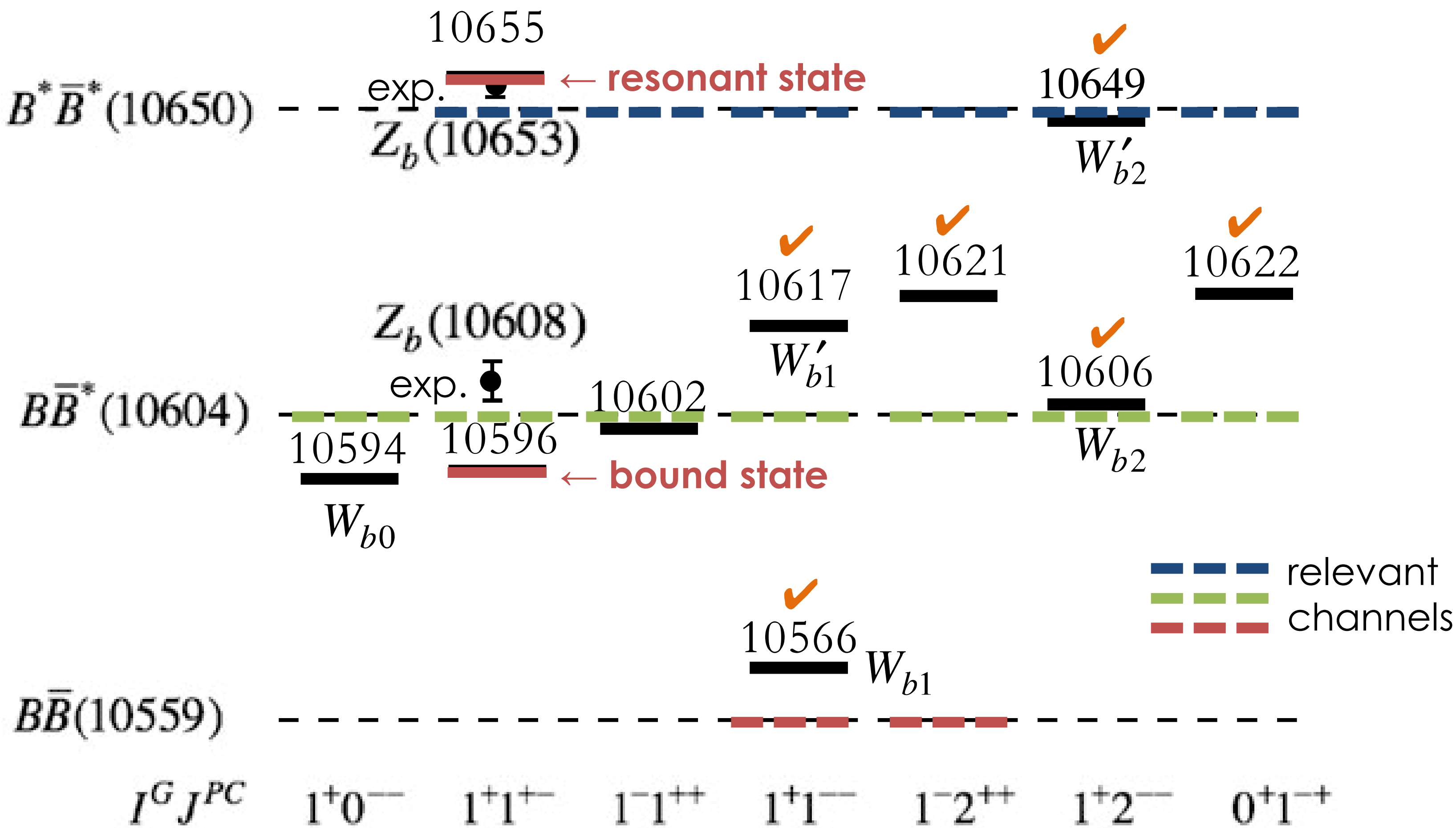
Numerical result



Feshbach resonances

3. Heavy exotic hadrons -X, Y, Z hadrons-

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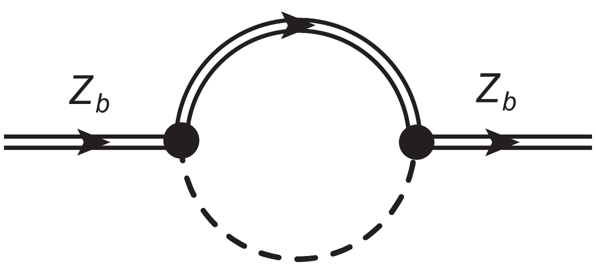
hadronic molecule interpretation

Other effects?

① $\Upsilon\pi$, $h_b\pi$ loop Method by M. R. Pennington and D. J. Wilson, Phys. Rev. D 76, 077502 (2007)



A. Loop propagator:

$$G_z(s) = \frac{i}{s - \mathcal{M}^2(s)} = \frac{i}{s - m_0^2 - \Pi(s)}$$
$$= \frac{i}{s - m_0^2 - \sum_{n=1} \Pi_n(s)},$$

$$\sum_{n=1} \Delta\Pi_n(s, s_0) = \mathcal{M}^2(s) - m_0^2 \equiv \delta M^2(s)$$

B. Dispersion relation:

$$\Delta\Pi_n(s, s_0) \equiv \Pi_n(s) - \Pi_n(s_0)$$
$$= \frac{(s - s_0)}{\pi} \int_{s_n}^{\infty} ds' \frac{\mathrm{Im}\Pi_n(s')}{(s' - s)(s' - s_0)}$$

C. Parametrization:

$$\mathrm{Im}\,\Pi_n(s) = -g_n^2 \left(\frac{2q_{\mathrm{cm}}}{\sqrt{s}}\right)^{2L+1} \exp\left(-\frac{q_{\mathrm{cm}}^2}{\Lambda^2}\right)$$

D. Numerical result

- g_n and g_m are determined from Zb→$\Upsilon\pi$, $h_b\pi$
- $s_0 \approx 9000$ MeV

	$\Upsilon(1S)\pi$	$\Upsilon(2S)\pi$	$\Upsilon(3S)\pi$	$h_b(1P)\pi$	$h_b(2P)\pi$	Total
M_{th}	9600	10 163	10 495	10 038	10 399	...
g_n	1986	844	956	7392	14 179	...
δM	6.3	0.5	−1.3	−0.1	−3.0	2.4

Mass is slightly shifted upwards.

3. Heavy exotic hadrons -X, Y, Z hadrons-

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hadronic molecule interpretation

Other effects?

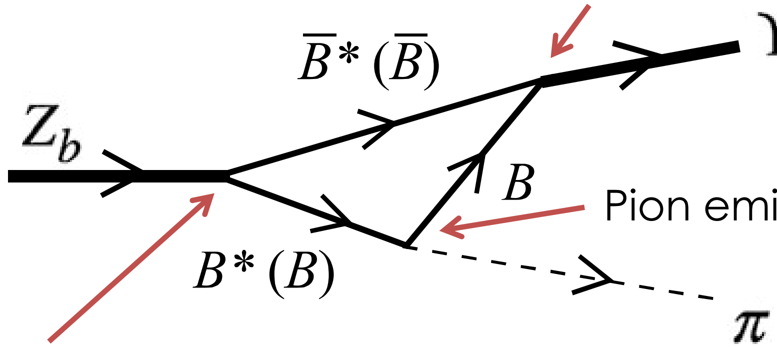
② Triangle diagram effect

(decay width)

S. Ohkoda, S. Yasui, and A. Hosaka, Phys. Rev. D 89, 074029 (2014)

BB^{*bar} or B^*B^{*bar} merging into the final state Υ

(vector meson dominance)



Pion emission by B^{*bar}

Z_b's dissociation into BB^{*bar} or B^*B^{*bar}

(form factor with cutoff Λ_Z)

$Z_b(10610)$ decay width [MeV]

Λ_Z	$\cdots$	1000	1050	1100	1150	Expt.
$\Upsilon(1S)\pi^+$	96.3	0.074	0.079	0.083	0.087	0.059 ± 0.017
$\Upsilon(2S)\pi^+$	20.0	0.47	0.50	0.52	0.55	0.81 ± 0.22
$\Upsilon(3S)\pi^+$	0.498	0.14	0.14	0.15	0.15	0.40 ± 0.10

$Z_b(10650)$ decay width [MeV]

Λ_Z	$\cdots$	1000	1050	1100	1150	Expt.
$\Upsilon(1S)\pi^+$	71.3	0.044	0.046	0.049	0.051	0.028 ± 0.008
$\Upsilon(2S)\pi^+$	17.6	0.31	0.33	0.34	0.36	0.28 ± 0.07
$\Upsilon(3S)\pi^+$	0.858	0.18	0.19	0.20	0.21	0.19 ± 0.05

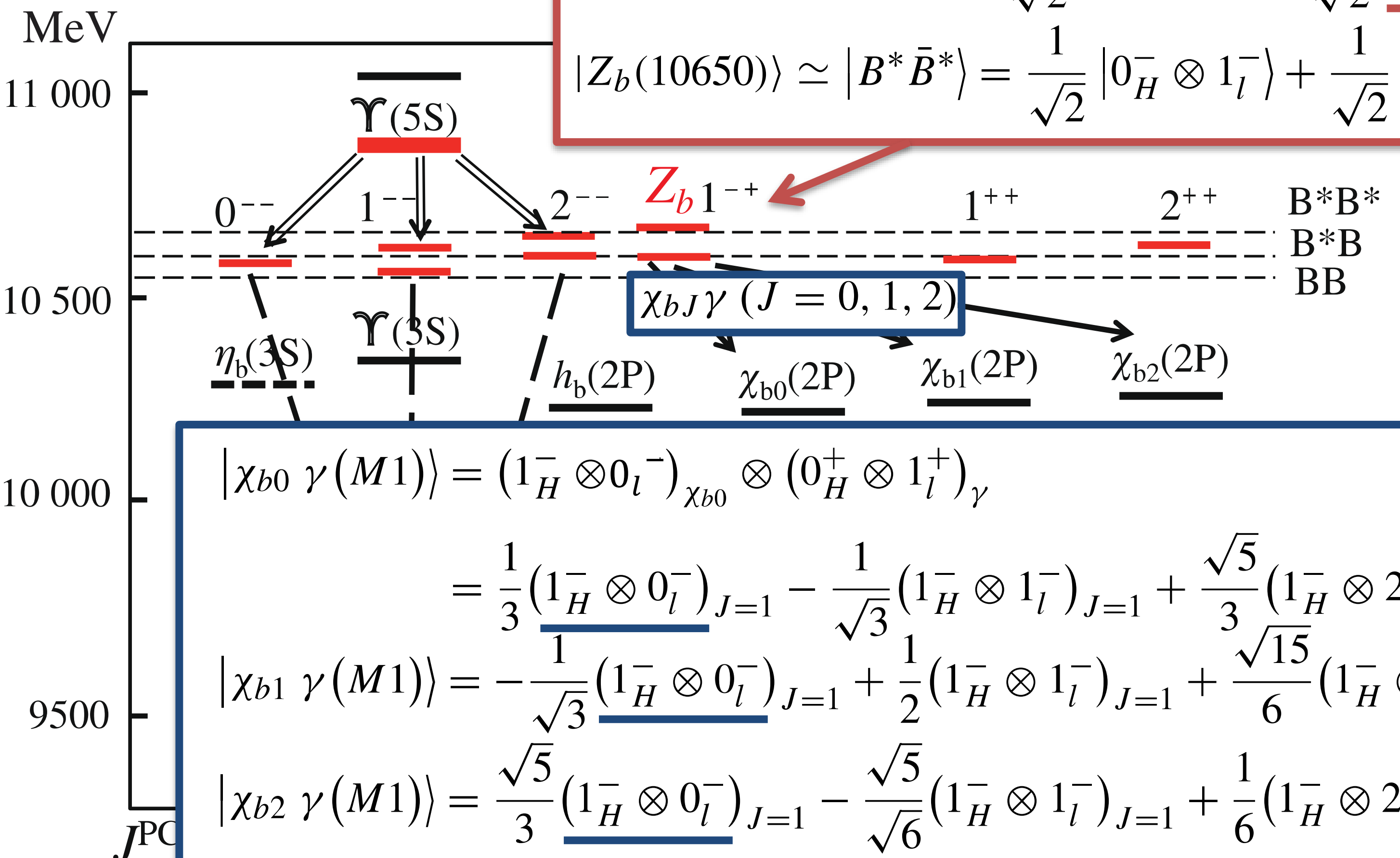
It seems consistent with experiments.

3. Heavy exotic hadrons -X, Y, Z hadrons-

$Z_b(10610)^+$ & $Z_b(10650)^+$

hadronic molecule interpretation

③ Radiative decay (I_Z=0)



$$|Z_b(10610)\rangle \simeq |B\bar{B}^*\rangle = \frac{1}{\sqrt{2}}|0_H^-\otimes 1_l^-\rangle - \frac{1}{\sqrt{2}}\underline{|1_H^-\otimes 0_l^-\rangle}$$

$$|Z_b(10650)\rangle \simeq |B^*\bar{B}^*\rangle = \frac{1}{\sqrt{2}}|0_H^-\otimes 1_l^-\rangle + \frac{1}{\sqrt{2}}\underline{|1_H^-\otimes 0_l^-\rangle}$$

$$|\chi_{b0}\,\gamma(M1)\rangle = (1_H^-\otimes 0_l^-)_{\chi_{b0}}\otimes(0_H^+\otimes 1_l^+)_\gamma$$

$$= \frac{1}{3}\underline{(1_H^-\otimes 0_l^-)}_{J=1} - \frac{1}{\sqrt{3}}(1_H^-\otimes 1_l^-)_{J=1} + \frac{\sqrt{5}}{3}(1_H^-\otimes 2_l^-)_{J=1}$$

$$|\chi_{b1}\,\gamma(M1)\rangle = -\frac{1}{\sqrt{3}}\underline{(1_H^-\otimes 0_l^-)}_{J=1} + \frac{1}{2}(1_H^-\otimes 1_l^-)_{J=1} + \frac{\sqrt{15}}{6}(1_H^-\otimes 2_l^-)_{J=1}$$

$$|\chi_{b2}\,\gamma(M1)\rangle = \frac{\sqrt{5}}{3}\underline{(1_H^-\otimes 0_l^-)}_{J=1} - \frac{\sqrt{5}}{\sqrt{6}}(1_H^-\otimes 1_l^-)_{J=1} + \frac{1}{6}(1_H^-\otimes 2_l^-)_{J=1}$$

$$\Gamma(Z_b\to\chi_{b0}\gamma):\Gamma(Z_b\to\chi_{b1}\gamma):\Gamma(Z_b\to\chi_{b2}\gamma) = 1:3:5$$

A. E. Bondar, A. Garmash, A. I. Milstein, R. Mizuk, and M. B. Voloshin, Phys. Rev. D84, 054010 (2011)
M. B. Voloshin, Phys. Rev. D 84, 031502 (2011)
S. Ohkoda, S. Yasui, and A. Hosaka, Phys. Rev. D 89, 074029 (2014)

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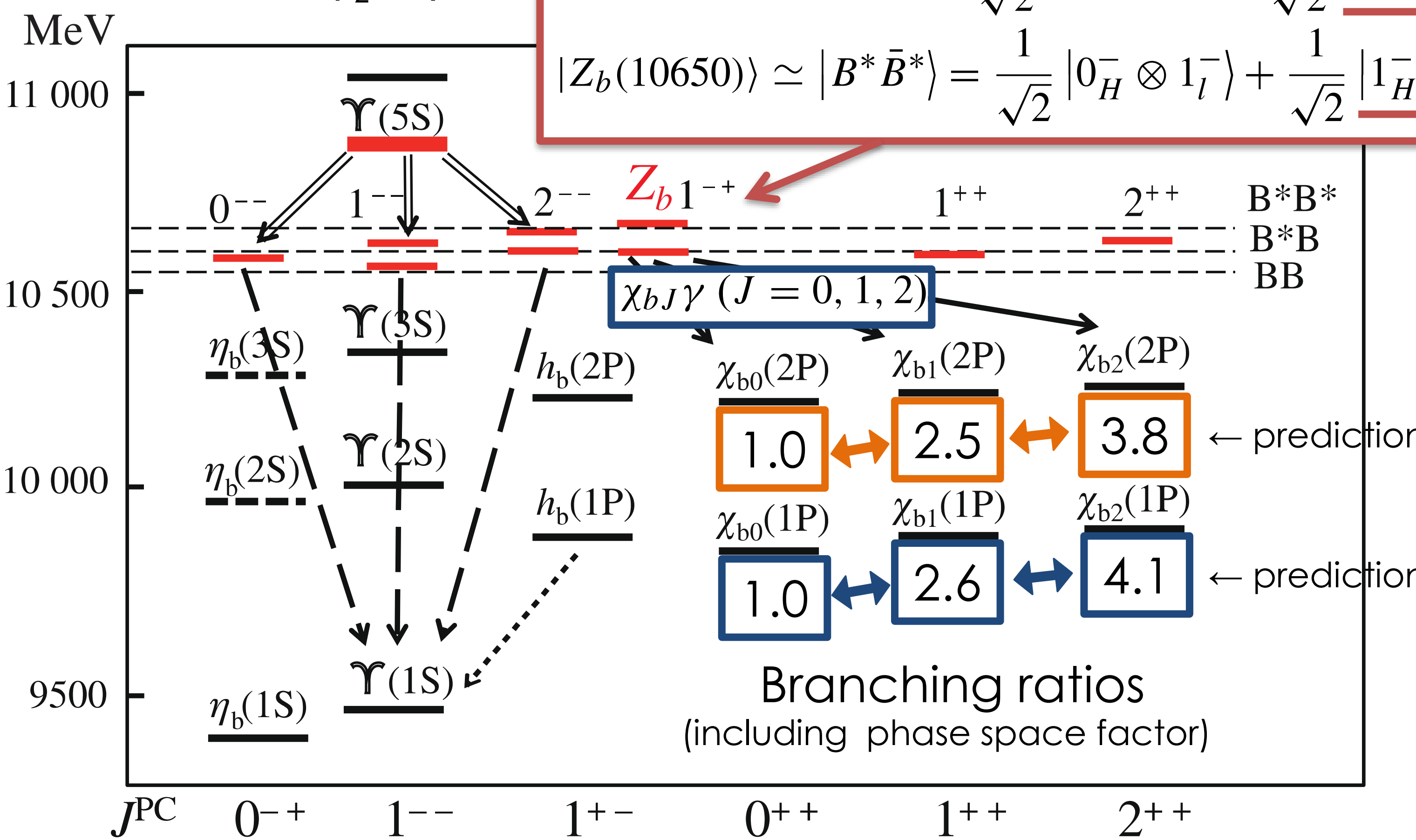
$$|Z_b(10650)\rangle \simeq |B^*\bar{B}^*\rangle = \frac{1}{\sqrt{2}}|0_H^- \otimes 1_l^-\rangle + \frac{1}{\sqrt{2}}|1_H^- \otimes 0_l^-\rangle$$



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3. Heavy exotic hadrons -X, Y, Z hadrons-

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Charm

Pentaquark

3. Heavy exotic hadrons -X, Y, Z hadrons-

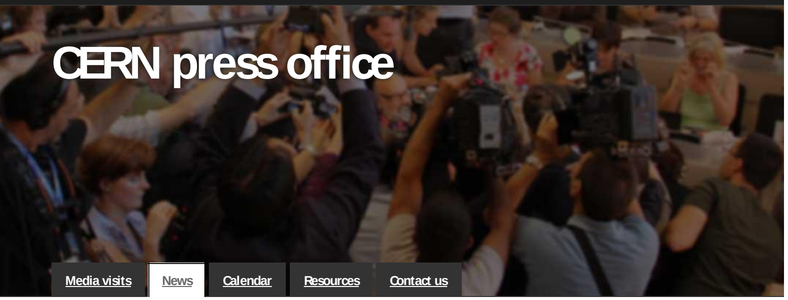
$P_c(4380)$

$P_c(4450)$

3. Heavy exotic hadrons -X, Y, Z hadrons-

$P_c(4380)$ & $P_c(4450)$

first charm pentaquark



PARTICLE PHYSICS

2015

Forsaken pentaquark particle spotted at CERN

Exotic subatomic species confirmed at Large Hadron Collider after earlier false sightings.

BY MATTHEW CHALMERS

An exotic particle made up of five quarks has been found a decade after experiments seemed to rule out its existence. The short-lived 'pentaquark' was spotted by researchers analysing data on the decay of unstable particles in the LHCb experiment at the Large Hadron Collider (LHC) at CERN, Europe's particle-physics laboratory near Geneva. The finding, says LHCb spokesperson Guy Wilkinson, opens a new era in physicists' understanding of the strong nuclear force that holds atomic nuclei together.

"The pentaquark is not just any new particle — it represents a way to aggregate quarks, namely the fundamental constituents of ordinary protons and neutrons, in a pattern that has never been observed before," he says. "Studying its properties may allow us to understand better how ordinary matter, the protons and neutrons from which we're all made, is constituted."

Protons and neutrons are made up of three kinds of quarks bound together, but theorists calculate that, in principle, particles could be made of up to five quarks. Such particles would be rich testing grounds for quantum chromodynamics (QCD) — the theory that describes the forces that hold quarks together.

In 2002, researchers at the SPring-8 synchrotron in Harima, Japan, caused a stir when they announced that they had discovered a pentaquark, roughly 1.5 times heavier than a proton, inferring its existence from the debris of collisions between high-energy photons and neutrons. Within a year, more than ten other labs had reported finding evidence for the particle by reanalysing data. But many others saw no evidence for such a state and, in 2005, ▶

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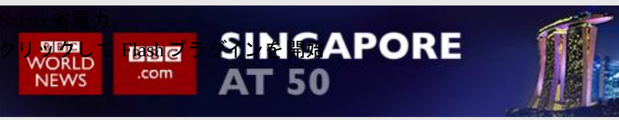
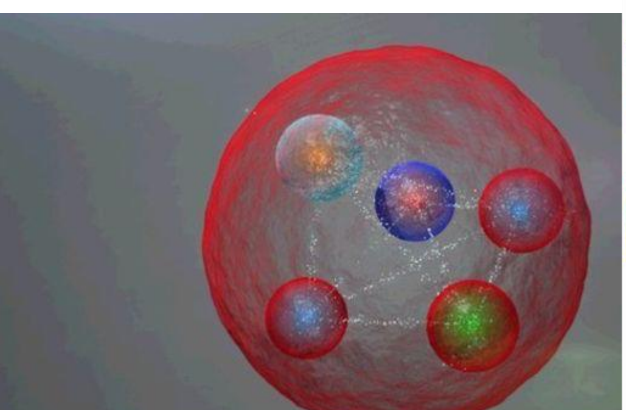


Science & Environment

Large Hadron Collider discovers pentaquark particle

By Paul Rincon
Science editor, BBC News website

14 July 2015 | Science & Environment



CERN Physicists Confirm Existence of Pentaquark Particles | Physics | Sci-News.com
2015/07/30 15:51

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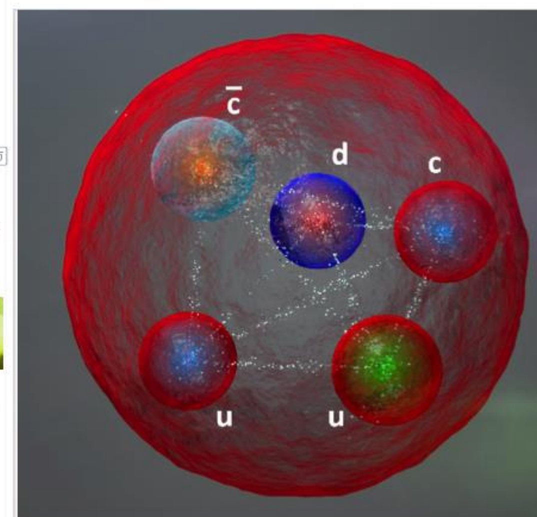
Weyl Fermion: Long-Sought Massless Particle Finally Observed

Physicists from CERN's LHCb experiment announced today the observati… of a pentaquark, a hadron consisting of five quarks.



SCIENTIFIC AMERICAN 日本版 日経サイエンス

アクセストップ5

編集部のピックアップ

Contents

Shopping

2015年7月15日
LHCでペンタクォークが発見されました！

ツイート 44

欧州合同原子核研究機構（CERN）は7月14日，世界最強加速器LHCを用いた国際共同実験でクォークと呼ぶ素粒子5個からなる「ペンタクォーク」という新タイプの粒子を発見したと発表しました。「物質」の存在形態に関する私たちの認識を新たにする発見で，宇宙初期の研究にもインパクトを与えることになりそうです。ペンタクォークについては現在販売中の日経サイエンス最新号の巻頭特集「クォークの世界」で詳しく紹介しています。

LHCでは4つの大きな国際共同実験が行われています。ヒッグス粒子を発見したのは，超高エネルギー領域で新たな素粒子を探索するATLAS実験とCMS実験。ペンタクォークを発見したのは，これらの実験ではなくLHCb実験です。LHCb実験はLHCによる陽子どうしの衝突で生み出されるB中間子の崩壊を高精度で測定し，「CP対称性の破れ」を詳しく調べるのが主な研究テーマですが，新タイプの複合粒子（何個かの素粒子が集まってできた粒子）の探索でも存在感を示しています。今回の研究成果もその1つになります。ちなみにCP対称性の破れについては別冊日経サイエンス165『素粒子論の一世紀』でたいへん詳しく紹介しています。

クォークは核子（陽子と中性子）を構成する素粒子です。核子はクォーク3個からなり，反クォーク… 粒子も知られています。核子どうしを結びつける力を媒介するパイ中間子（湯川秀樹博士が存在を予…ど中間子と総称されるタイプはクォーク1個と反クォーク1個からなります。長い間，知られていた…けで，クォーク4個からなるテトラクォークや今回のペンタクォークなどは理論的には存在が予測…の，実験では見つかっていませんでした。

arXiv:1507.03414v1 [hep-ex] 13 Jul 2015

CERN-PH-EP-2015-153
LHCb-PAPER-2015-029
July 13, 2015

Observation of $J/\psi p$ resonances consistent with pentaquark states in $\Lambda_b^0 \to J/\psi K^- p$ decays

1900 citations

The LHCb collaboration[1]

Abstract

Observations of exotic structures in the $J/\psi p$ channel, that we refer to as pentaquark-charmonium states, in $\Lambda_b^0 \to J/\psi K^- p$ decays are presented. The data sample corresponds to an integrated luminosity of 3 fb^{-1} acquired with the LHCb detector from 7 and 8 TeV pp collisions. An amplitude analysis is performed on the three-body final-state that reproduces the two-body mass and angular distributions. To obtain a satisfactory fit of the structures seen in the $J/\psi p$ mass spectrum, it is necessary to include two Breit-Wigner amplitudes that each describe a resonant state. The significance of each of these resonances is more than 9 standard deviations. One has a mass of $4380 \pm 8 \pm 29$ MeV and a width of $205 \pm 18 \pm 86$ MeV, while the second is narrower, with a mass of $4449.8 \pm 1.7 \pm 2.5$ MeV and a width of $39 \pm 5 \pm 19$ MeV. The preferred J^P assignments are of opposite parity, with one state having spin $3/2$ and the other $5/2$.

masses

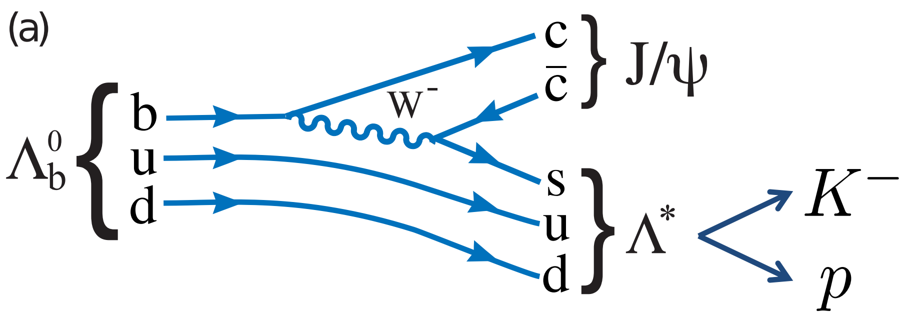
decay widths

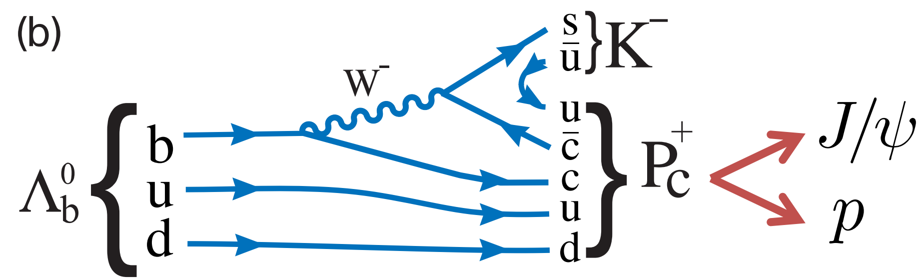
3. Heavy exotic hadrons -X, Y, Z hadrons-

$P_c(4380)$ & $P_c(4450)$

first charm pentaquark

$$\Lambda_b^0 \to J/\psi K^- p$$

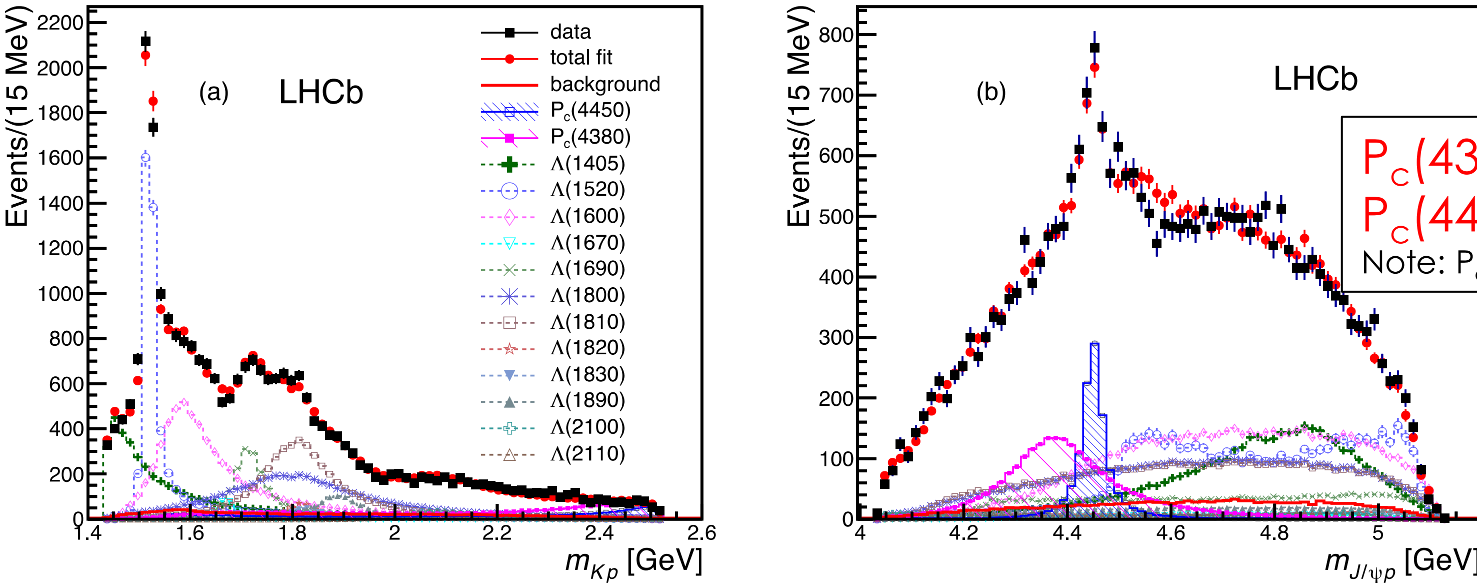




State	J^P	M_0 (MeV)	Γ_0 (MeV)
$\Lambda(1405)$	$1/2^-$	$1405.1^{+1.3}_{-1.0}$	50.5 ± 2.0
$\Lambda(1520)$	$3/2^-$	1519.5 ± 1.0	15.6 ± 1.0
$\Lambda(1600)$	$1/2^+$	1600	150
$\Lambda(1670)$	$1/2^-$	1670	35
$\Lambda(1690)$	$3/2^-$	1690	60
$\Lambda(1800)$	$1/2^-$	1800	300
$\Lambda(1810)$	$1/2^+$	1810	150
$\Lambda(1820)$	$5/2^+$	1820	80
$\Lambda(1830)$	$5/2^-$	1830	95
$\Lambda(1890)$	$3/2^+$	1890	100
$\Lambda(2100)$	$7/2^-$	2100	200
$\Lambda(2110)$	$5/2^+$	2110	200
$\Lambda(2350)$	$9/2^+$	2350	150
$\Lambda(2585)$	?	≈ 2585	200

3. Heavy exotic hadrons -X, Y, Z hadrons-

LHCb, Phys. Rev. Lett. 115, 072001 (2015)



$P_c(4380)$ Γ=205 MeV
$P_c(4450)$ Γ=39 MeV
Note: $P_c(4450)$ is now $P_c(4457)$.

Figure 3: Fit projecti
(see Table 1). The da
results of the fit. The
squares with the sha
topped with (purple)
shown. The error ba

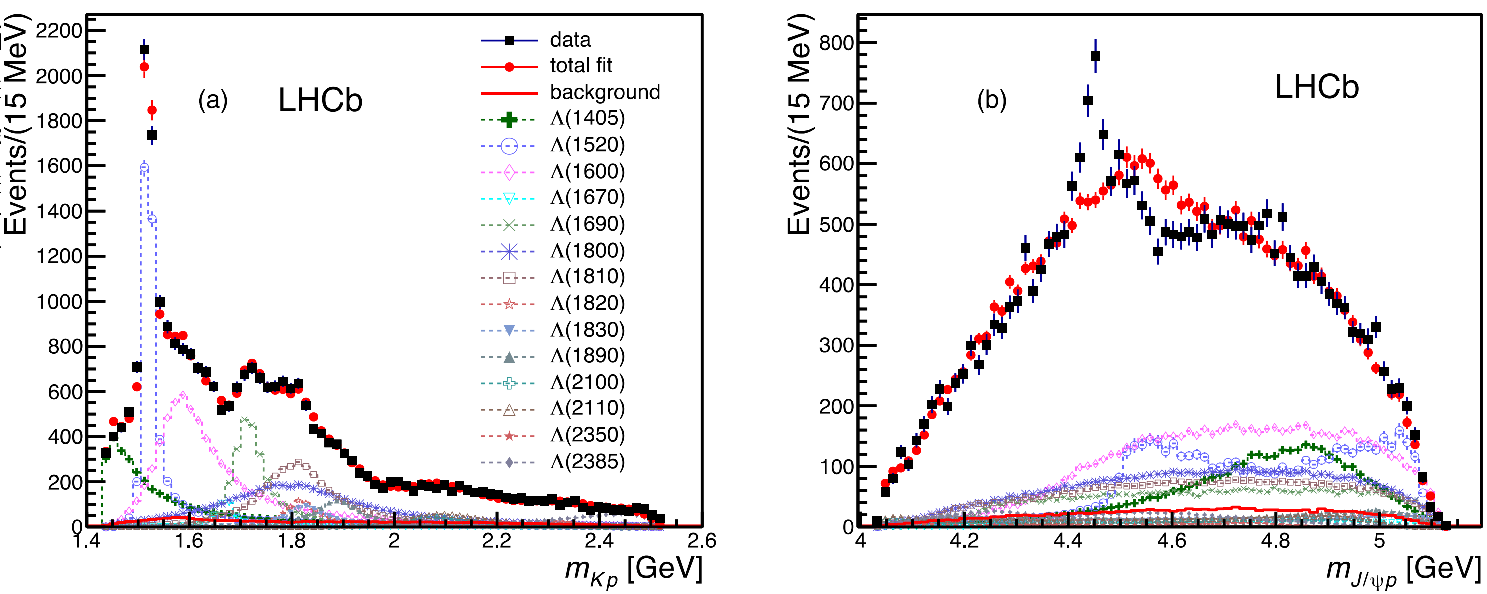


Figure 6: Results for (a) m_{Kp} and (b) $m_{J/\psi p}$ for the extended Λ^* model fit without P_c^+ states.

3. Heavy exotic hadrons -X, Y, Z hadrons-

LHCb, Phys. Rev. Lett. 115, 072001 (2015)

Argand Plot

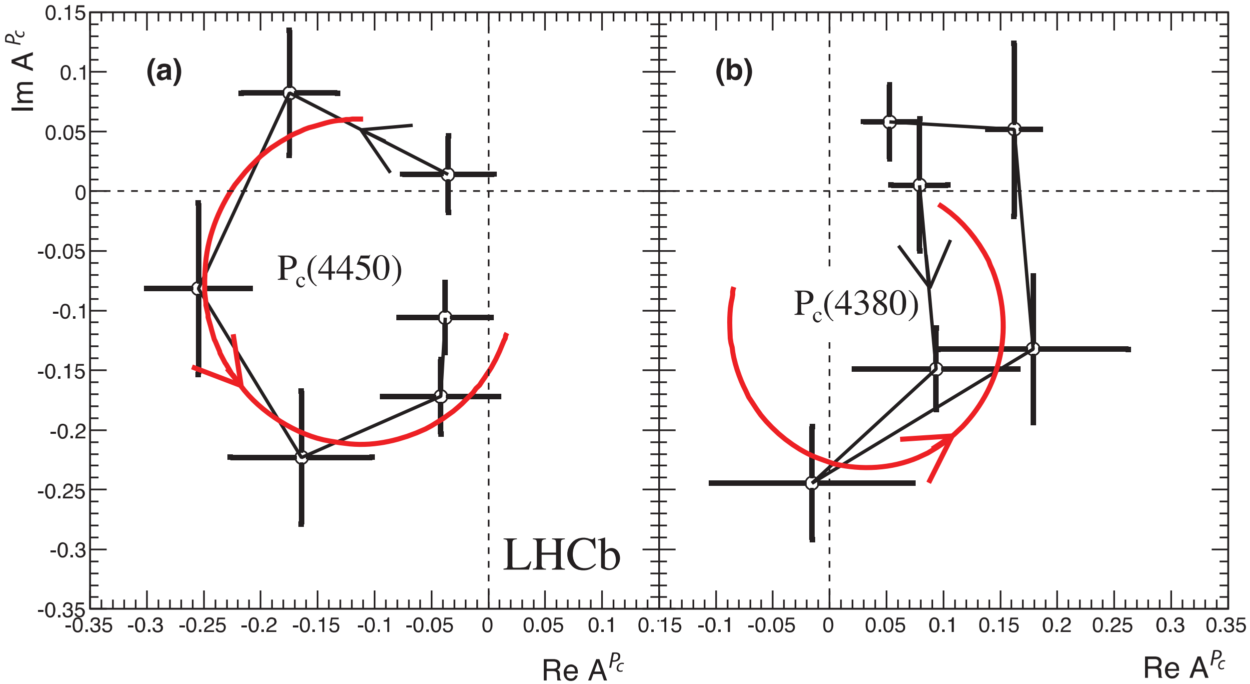
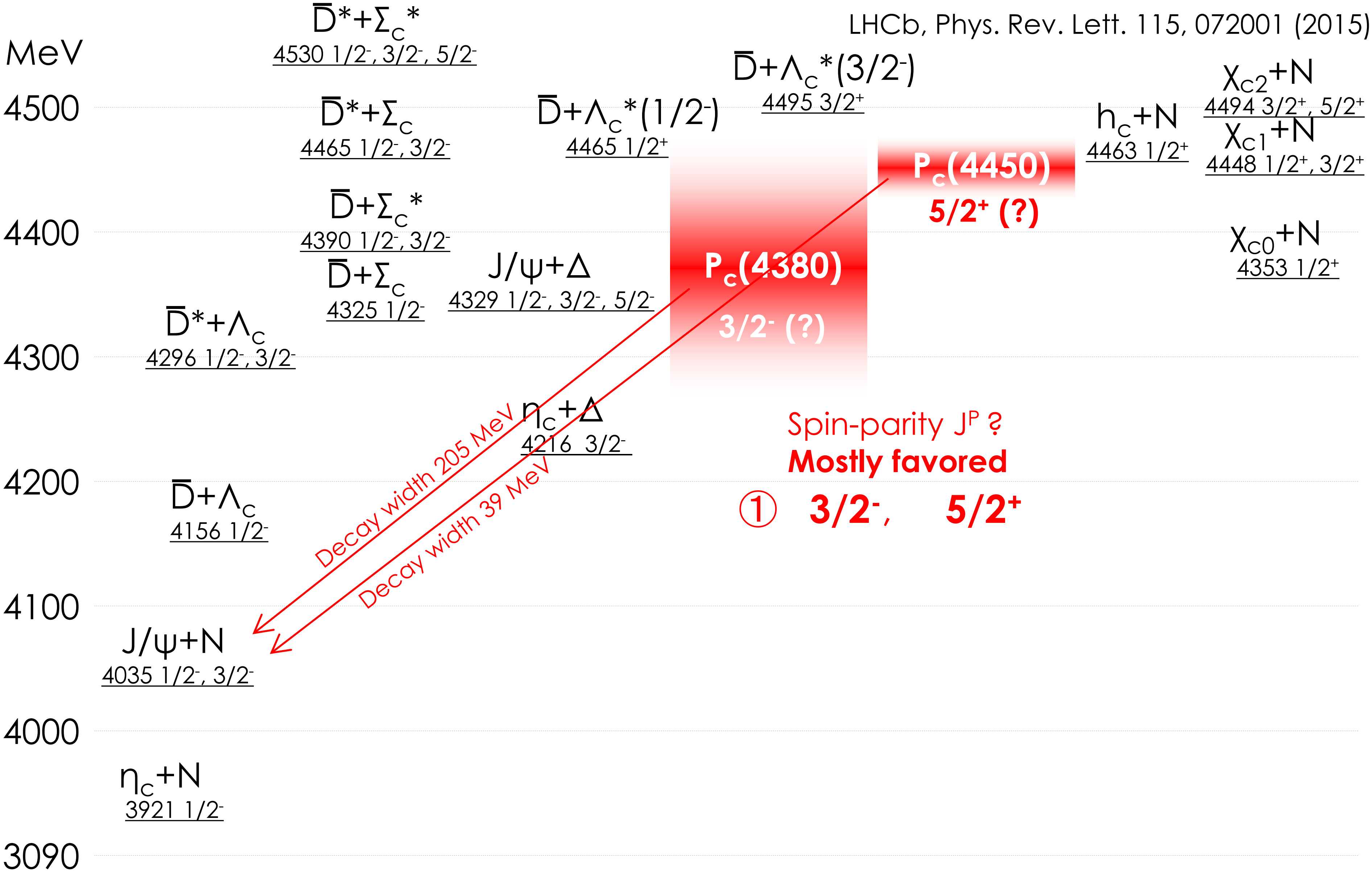
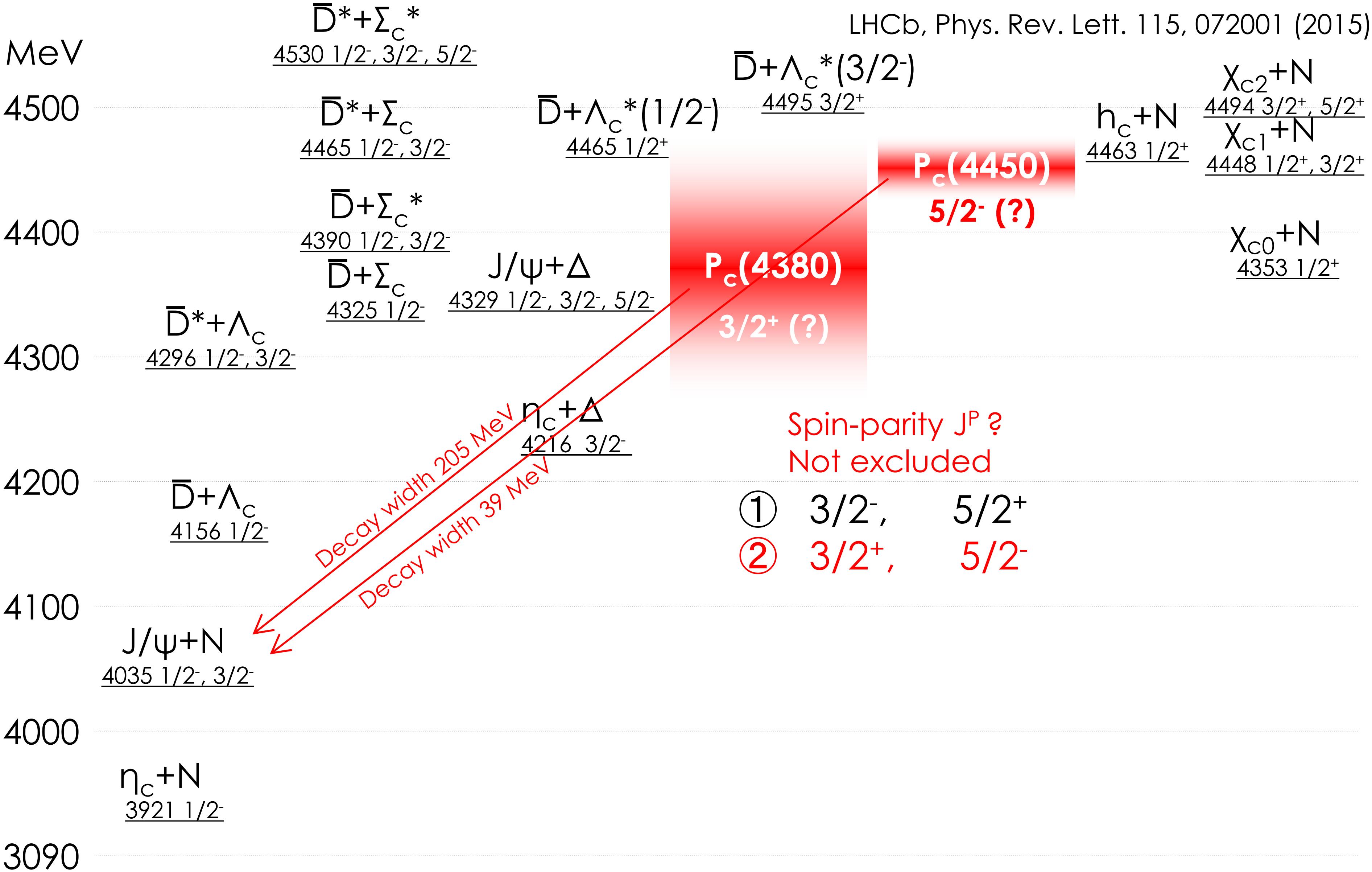


Figure 9: Fitted values of the real and imaginary parts of the amplitudes for the baseline ($3/2^-$, $5/2^+$) fit for a) the $P_c(4450)^+$ state and b) the $P_c(4380)^+$ state, each divided into six $m_{J/\psi p}$ bins of equal width between $-\Gamma_0$ and $+\Gamma_0$ shown in the Argand diagrams as connected points with error bars ($m_{J/\psi p}$ increases counterclockwise). The solid (red) curves are the predictions from the Breit-Wigner formula for the same mass ranges with M_0 (Γ_0) of 4450 (39) MeV and 4380 (205) MeV, respectively, with the phases and magnitudes at the resonance masses set to the average values between the two points around M_0. The phase convention sets $B_{0,\frac{1}{2}} = (1, 0)$ for $\Lambda(1520)$. Systematic uncertainties are not included.

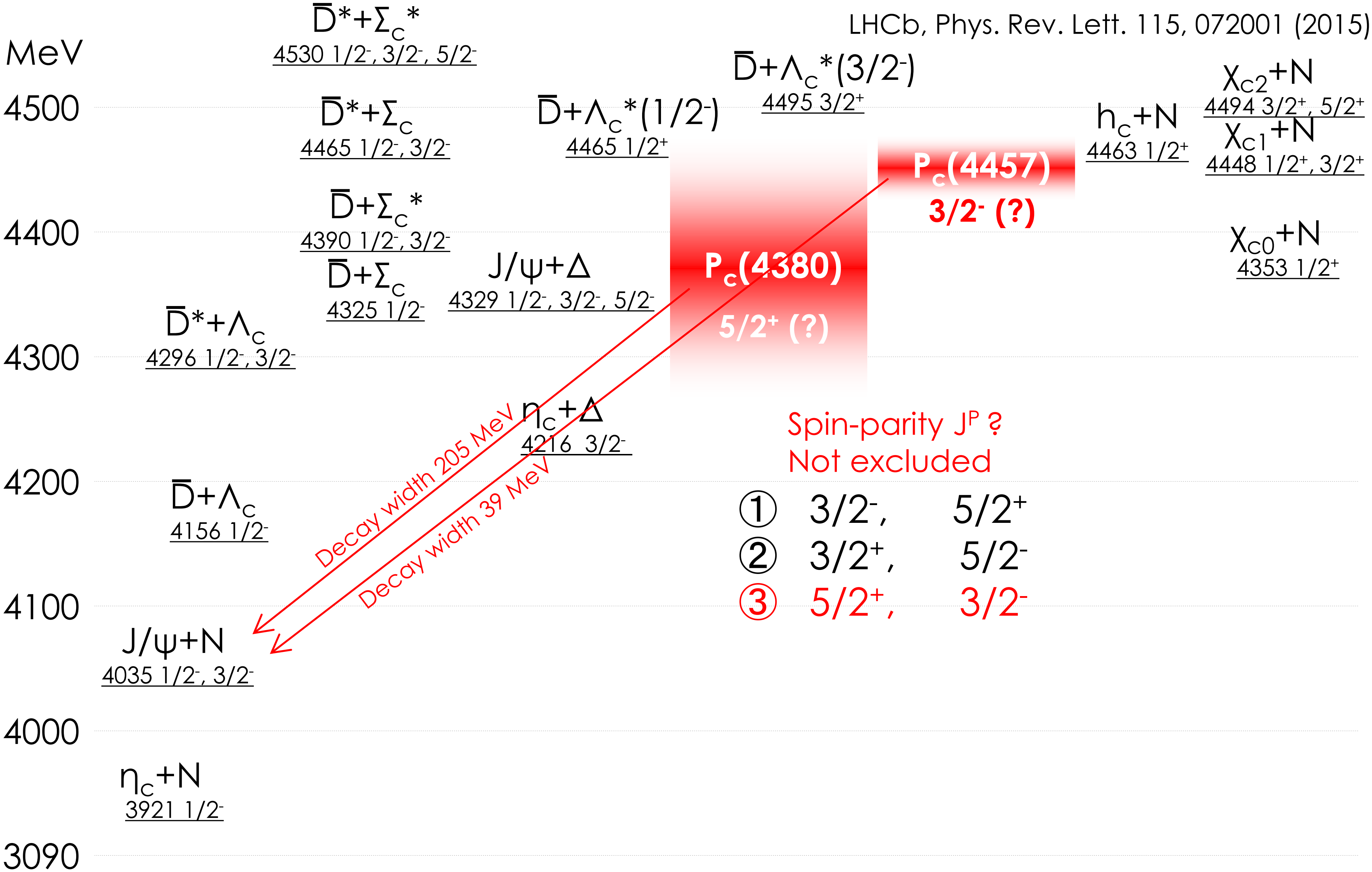
Thresholds of hadron states above 4 GeV



Thresholds of hadron states above 4 GeV



Thresholds of hadron states above 4 GeV



P_c should contain at least

$c\bar{c}uud$

Hidden charm

So, what is the structure?

1. Pentaquark

Quark spin/orbital excitations
Inter-*quark* correlations (diquarks)

2. Hadronic molecule

Inter-*hadron* correlations

3. Cusp effect

Kinematic anomaly

4. Other things?

Lattice QCD, AdS/QCD, …

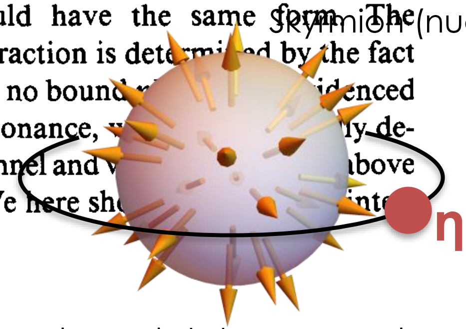
1. Pentaquark

Quark spin/orbital excitations
Inter-*quark* correlations (diquarks)

Skyrme model (1962)

der Waals interaction is sufficiently strong that heavy quarkonium states – $c\bar{c}$ and $b\bar{b}$ – form bound states with nuclei [1]. Although the approximations used to estimate the effective low energy QCD Van der Waals force in ref. [1] have been criticized as leading to a large overestimate of the strength of the interaction [2], the original suggestion has been substantiated by the observation that in the infinite mass limit for the heavy quark the actual binding energy of quarkonium in nuclear matter can be obtained exactly in QCD [3]. To get another perspective on this interesting possibility for a new type of baryonic matter we here investigate the question of bound states of quarkonium with nucleons using the bound state to the question at hand.

The virtue of the bound state version of the topological soliton model is that its form applies to all heavy flavour sectors, the only difference between the sectors being the different flavour quantum numbers and the masses and decay constants of the different heavy flavour mesons. Thus the interaction between the nucleons and the η, the $\eta_c(2980)$ and the η_b (unobserved) should have the same form. The strength of this interaction is determined by the fact that there should be no bound ... idenced by the $N(1535)$ resonance, ... ly decays into the ηN channel and ... above the ηN threshold. We here sh... inte...



Motivation

1. What is $c\bar{c}$-nucleon interaction?

   Color van der Waals force, Scale anomaly

   Kaidalov, Volkovitsky (1992)
   Luke, Manohar, Savage (1992)

2. Flavor extension: N(1535)~ηN ⇒ $N_{c\bar{c}}$(?)~η_cN

Gobbi, Riska, Nucl. Phys. A568, 779 (1994)

$$\mathscr{L} = -\tfrac{1}{4} f_\pi^2 \,\mathrm{Tr}(L_\mu L^\mu) + \frac{x}{32\varepsilon^2}\mathrm{Tr}[L_\mu, L_\nu]^2 + \frac{1-x}{16\varepsilon^2}\left\{(\mathrm{Tr}\, L_\mu L_\nu)^2 - (\mathrm{Tr}\, L_\mu L^\mu)^2\right\},$$

$$U = \sqrt{U_\mathrm{H}}\, U_\pi \sqrt{U_\mathrm{H}}$$

$$U_\mathrm{H} = \mathrm{e}^{i(\lambda_0\eta_0 + \lambda_8\eta_8)/f_\eta}$$

$N_{c\bar{c}}$ mass estimated to be 2800 MeV (too light?)

Binding energy ~ 1300 MeV (!)

What is the QCD mechanism to prevent the too-deeply bound state?

Skyrme model (1962)

der Waals interaction is sufficiently strong that heavy quarkonium states – $c\bar{c}$ and $b\bar{b}$ – form bound states with nuclei [1]. Although the approximations used to estimate the effective low energy QCD Van der Waals force in ref. [1] have been criticized as leading to a large overestimate of the strength of the interaction [2], the original suggestion has been substantiated by the observation that in the infinite mass limit for the heavy quark the actual binding energy of quarkonium in nuclear matter can be obtained exactly in QCD [3]. To get another perspective on this interesting possibility for a new type of baryonic matter we here investigate the question of bound states of quarkonium with nucleons using the bound state to the question at hand.

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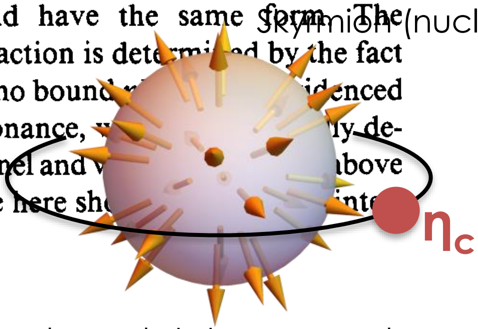
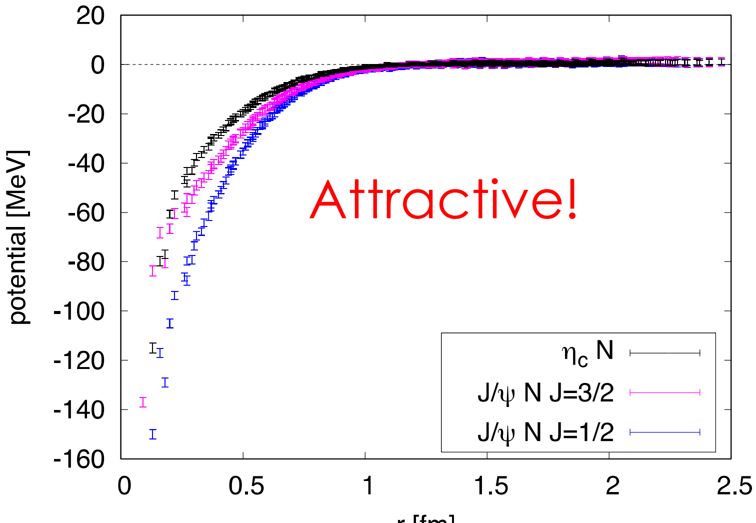
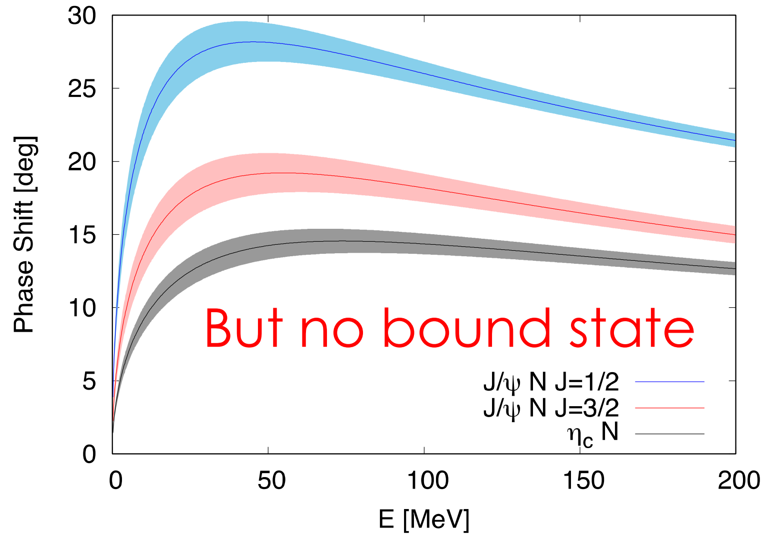


bound-state approach

J/ψ, η_c-nucleon potential from lattice QCD

Sugiura, Ikeda, Ishii, arXix:1905.02336 [hep-lat]





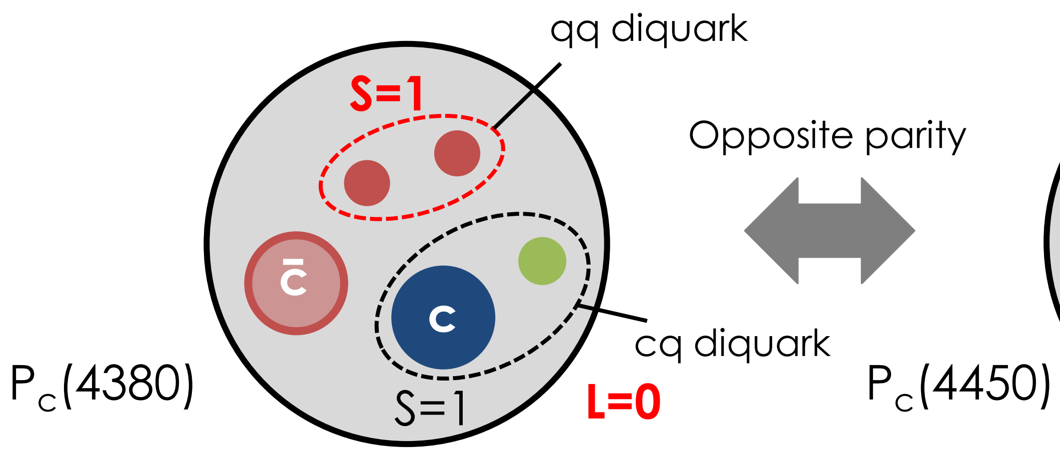
Cf. Two-pion exchange in φN, HAL QCD, PRD106, 074507 (2022)

Diquark model

$P_c(4380)$ Γ=205 MeV, $P_c(4450)$ Γ=39 MeV

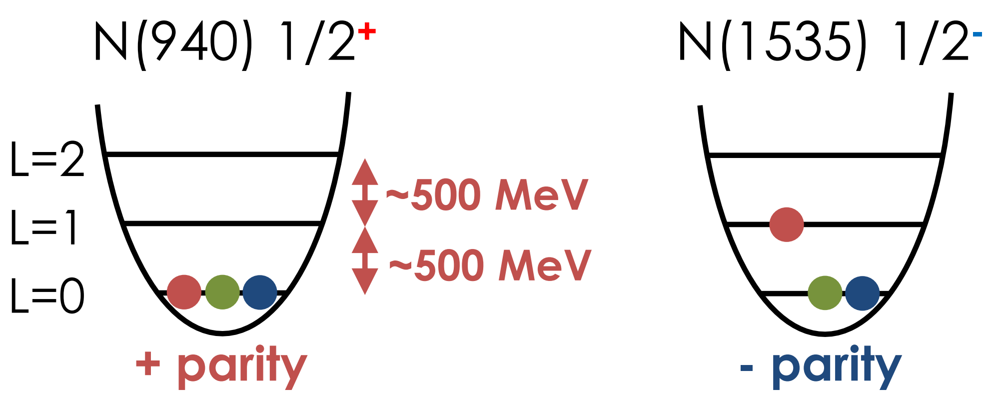
Maiani, Polosa, Riquer, Phys. Lett. B749 (2015) 289

Q. Why is the mass difference between $P_c(4380)$ and $P_c(4450)$ small (~70 MeV)?



$P(3/2^-) = \{\bar{c}[cq]_{s=1}[q'q'']_{s=1}, L = 0\}$ $P(5/2^+) = \{\bar{c}[cq]_{s=1}[q'q'']_{s=0}, L = 1\}$

Opposite parity → Angular-momentum excitation
~ 500 MeV mass difference



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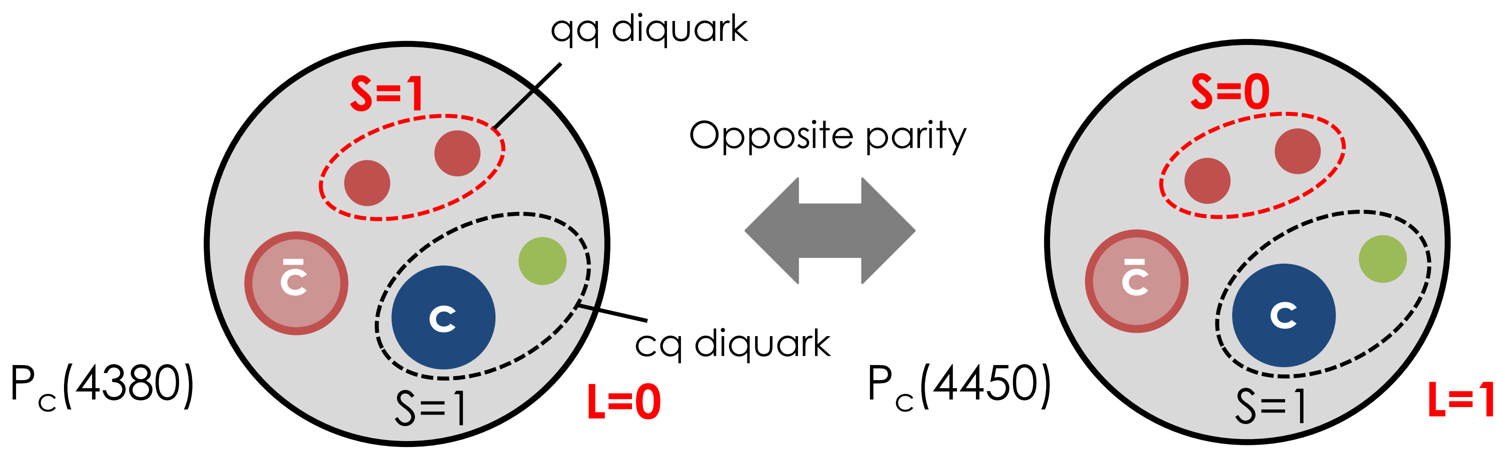

$P(3/2^-) = \{\bar{c}[cq]_{s=1}[q'q'']_{s=1}, L = 0\}$ $P(5/2^+) = \{\bar{c}[cq]_{s=1}[q'q'']_{s=0}, L = 1\}$

$$M(5/2^+)-M(3/2^-) = \Delta M_{spin} + \Delta M_{anglular\ momentum}$$

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Diquark model

Maiani, Polosa, Riquer, Phys. Lett. B749 (2015) 289

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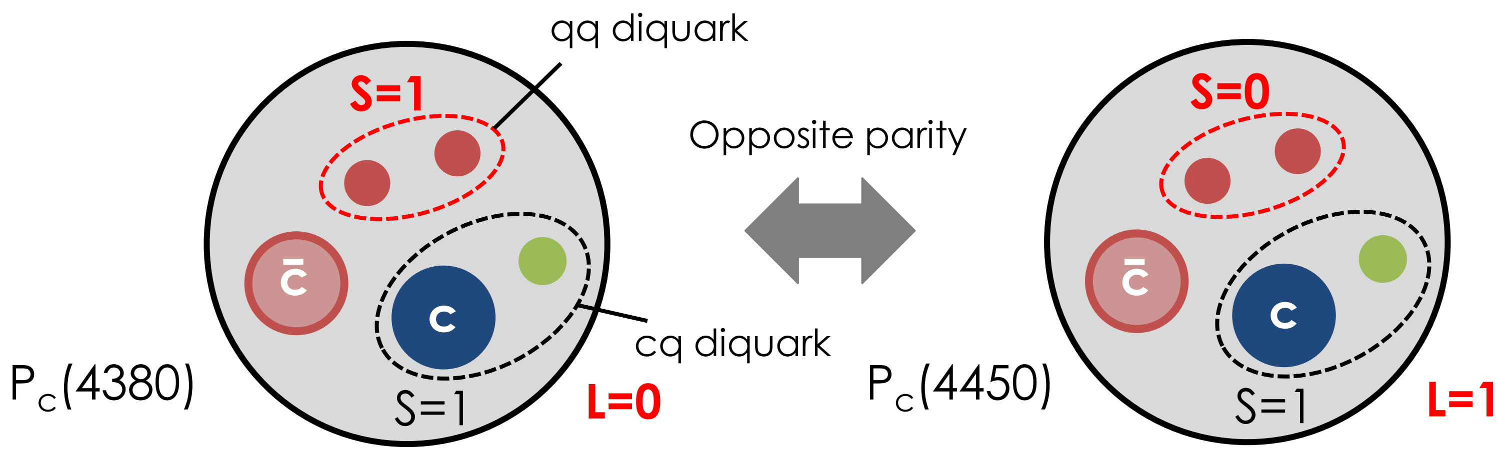

$P(3/2^-) = \{\bar{c}[cq]_{s=1}[q'q'']_{s=1}, L=0\}$ $\quad P(5/2^+) = \{\bar{c}[cq]_{s=1}[q'q'']_{s=0}, L=1\}$

$$M(5/2^+)-M(3/2^-) = \Delta M_{spin} + \Delta M_{anglular\ momentum}$$

$$= \underline{(-200\ \mathrm{MeV})} + \underline{300\ \mathrm{MeV}}$$

$\Sigma_c(2455) - \Lambda_c(2286) \simeq 170$ MeV $\quad \Lambda(1405) - \Lambda(1116) \sim 290$ MeV

$c(qq)_{S=1}$ $\quad c(qq)_{S=0}$ $\quad (uds)_{L=1}$ $\quad (uds)_{L=0}$

Diquark model

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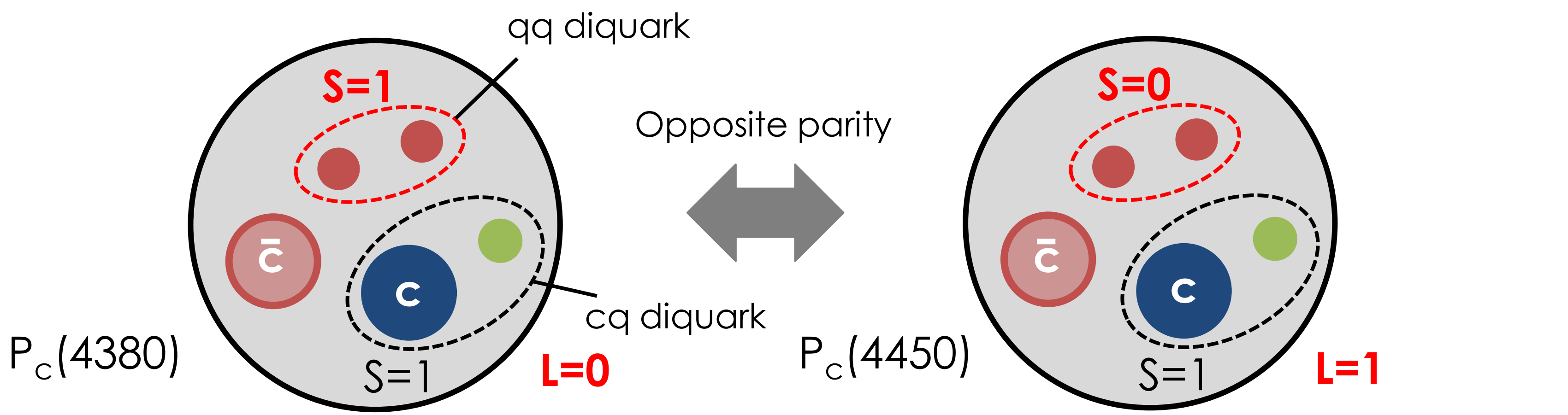

$P(3/2^-) = \{\bar{c}[cq]_{s=1}[q'q'']_{s=1}, L=0\}$ $\quad$ $P(5/2^+) = \{\bar{c}[cq]_{s=1}[q'q'']_{s=0}, L=1\}$

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$\Sigma_c(2455) - \Lambda_c(2286) \simeq 170$ MeV $\quad$ $\Lambda(1405) - \Lambda(1116) \sim 290$ MeV

$c(qq)_{S=1}$ $\quad$ $c(qq)_{S=0}$ $\quad$ $(uds)_{L=1}$ $\quad$ $(uds)_{L=0}$

$$= 100\ \mathrm{MeV}\ (\sim 70\ \mathrm{MeV})$$

exp.

$P_c(4380)$ Γ=205 MeV, $P_c(4450)$ Γ=39 MeV

Diquark model

Maiani, Polosa, Riquer, Phys. Lett. B749 (2015) 289

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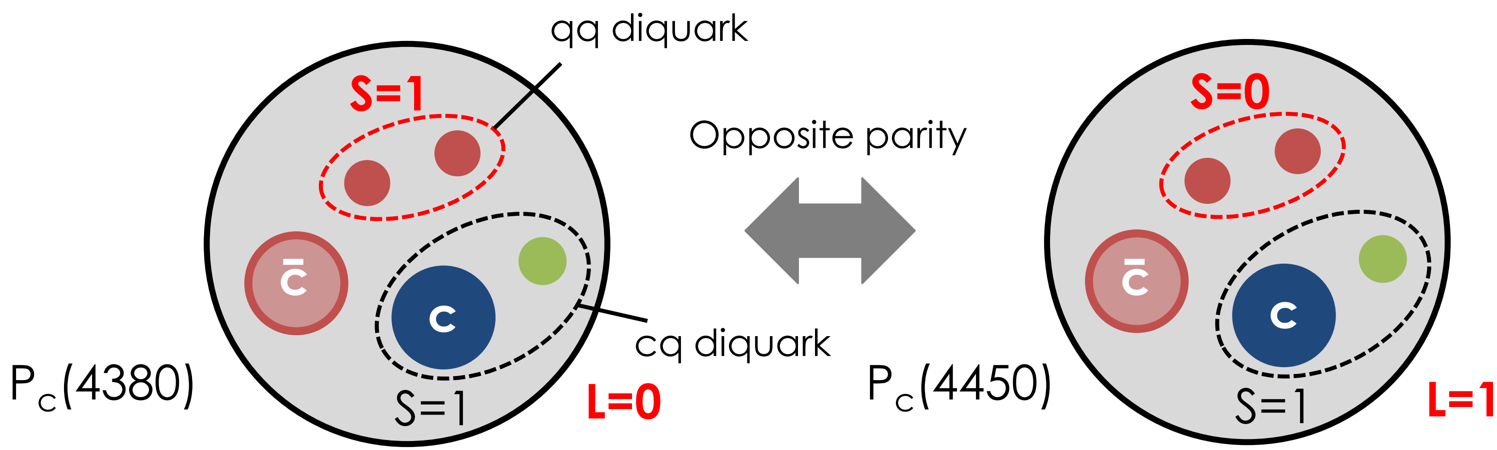

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If this is true, we can make an extension from SU(2) flavor symmetry to SU(3).

$$P_A = \epsilon^{\alpha\beta\gamma}\{\bar{c}_\alpha [cq]_{\beta, s=0,1} [q'q'']_{\gamma, s=0}, L\}$$
$$= 3 \otimes \bar{3} = 1 \oplus 8$$
$$P_S = \epsilon^{\alpha\beta\gamma}\{\bar{c}_\alpha [cq]_{\beta, s=0,1} [q'q'']_{\gamma, s=1}, L\}$$
$$= 3 \otimes 6 = 8 \oplus 10$$

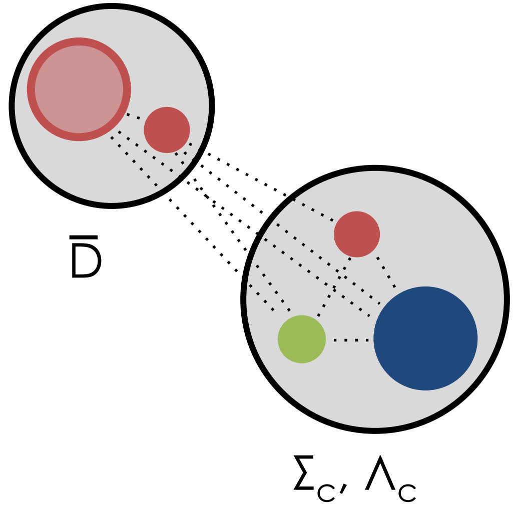
$P_c(4380)$ Γ=205 MeV, $P_c(4450)$ Γ=39 MeV

Quark model calculation

Wang, Huang, Zhang, Zou, Phys. Rev. C84 015203 (2011)

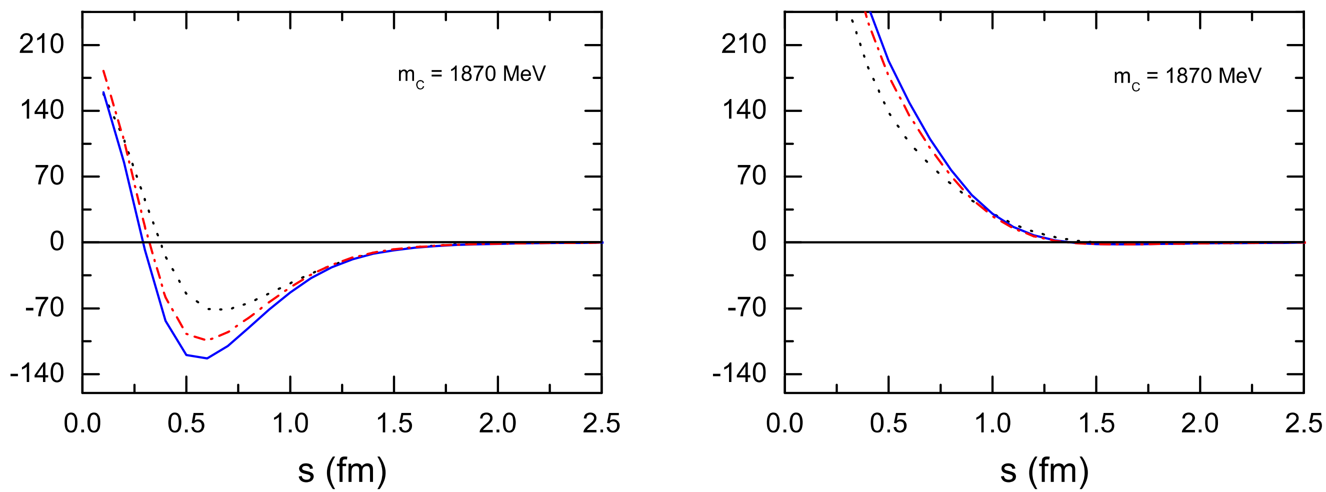
$$H = \sum_i T_i - T_G + \sum_{i,j} V_{ij}$$

$$V_{ij} = \begin{cases} V_{ij}^{OGE} + V_{ij}^{conf} + \sum_M V_{ij}^M, & (ij = qq) \\ V_{ij}^{OGE} + V_{ij}^{conf}, & (ij = qQ, q\bar{Q}, Q\bar{Q}) \end{cases}$$



$\bar{D}\Sigma_c$-$\bar{D}\Lambda_c$: resonating group method

$$\sum_{\beta'} \int [H_{\beta\beta'}(R, R') - E N_{\beta\beta'}(R, R')] \chi_{\beta'}(R') \, dR' = 0$$



mass: 4.279-4.316 GeV Very good agreement with LHCb!

2. Hadronic molecule

Inter-*hadron* correlations

Meson-baryon coupling model

$\eta' N$, $\eta_c N$, $D\bar{\Sigma}_c$, $D\bar{\Lambda}_c$

$P_c(4380)$ Γ=205 MeV, $P_c(4450)$ Γ=39 MeV

Hofmann, Lutz, Nucl. Phys. A763, 90 (2005)

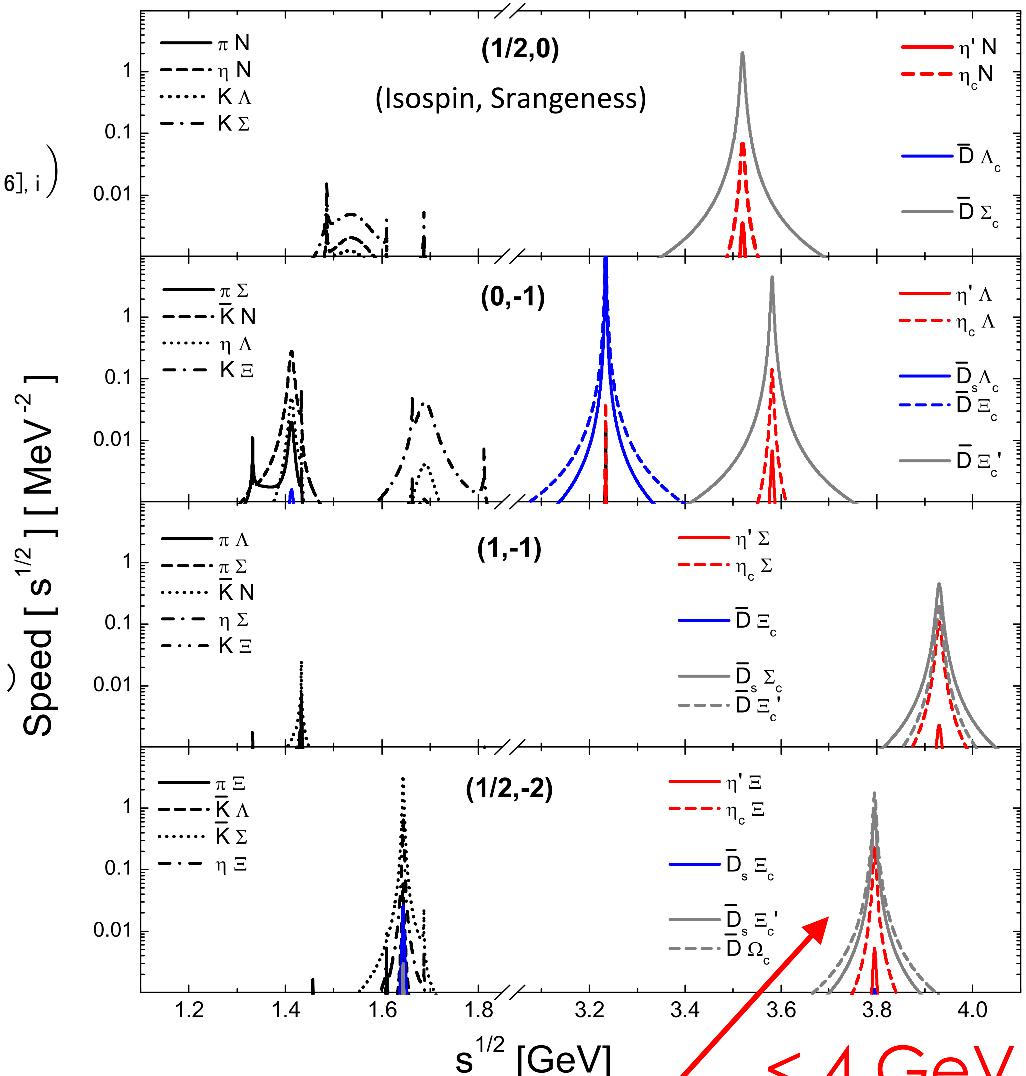
Lagrangian with Flavor SU(4) symmetry

$$L^{SU(4)}_{kin} = \frac{1}{4}\sum_{i,j=1}^{4}\left((\partial_\mu \Phi^{j}_{[16],j})(\partial^\mu \Phi^{j}_{[16],i}) - m^2_{[16]}\Phi^{j}_{[16],j}\Phi^{j}_{[16],i}\right) + \frac{1}{2}\sum_{i,j,k=1}^{4}\bar{B}^{[20]}_{ijk}\left(i\,\partial\!\!\!/ - M_{[20]}\right)B^{ijk}_{[20]}.$$

$$L^{SU(4)}_{int} = \frac{i}{4}\, g\, \mathrm{tr}\left(\left[(\partial_\mu \Phi_{[16]}),\, \Phi_{[16]}\right]_{-} V^{\mu}_{[16]}\right)$$

Coupled-channel equation

$$M^{(I,S,C)}(\sqrt{s}) = \left[1 - V^{(I,S,C)}(\sqrt{s})\,J^{(I,S,C)}(\sqrt{s})\right]^{-1} V^{(I,S,C)}(\sqrt{s})$$



< 4 GeV

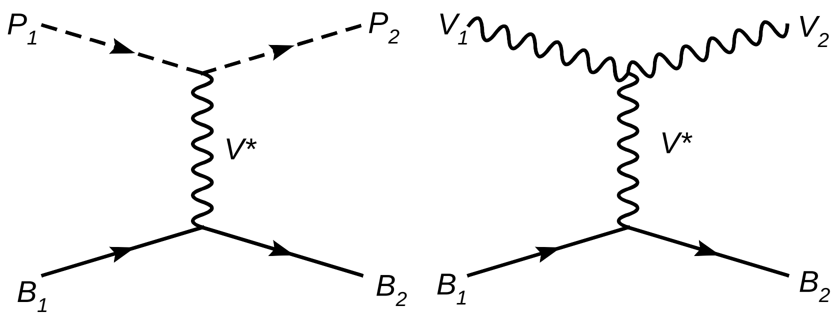
Deeply bound-state

$P_c(4380)$ Γ=205 MeV, $P_c(4450)$ Γ=39 MeV

Meson-baryon coupling model

$\bar{D}\Sigma_c$, $D\bar{\Sigma}_c^*$, $D\bar{\Lambda}_c$

Wu, Molina, Oset, Zou, Phys. Rev. Lett. 105 232001 (2010), Phys. Rev. C84 015202 (2011)



(a) (b)

$$L_{VVV} = ig\langle V^\mu [V^\nu, \partial_\mu V_\nu]\rangle$$
$$L_{PPV} = -ig\langle V^\mu [P, \partial_\mu P]\rangle$$
$$L_{BBV} = g(\langle \bar{B}\gamma_\mu [V^\mu, B]\rangle + \langle \bar{B}\gamma_\mu B\rangle\langle V^\mu\rangle)$$

$$T = [1 - VG]^{-1} V$$

Very good agreement with LHCb!

(I,S)	z_R (MeV)	g_a		
$(1/2, 0)$		$\bar{D}\Sigma_c$	$\bar{D}\Lambda_c^+$	
	4269	2.85	0	
$(0, -1)$		$\bar{D}_s\Lambda_c^+$	$\bar{D}\Xi_c$	$\bar{D}\Xi_c'$
	4213	1.37	3.25	0
	4403	0	0	2.64

TABLE III: Pole positions z_R and coupling constants g_a for the states from $PB \to PB$.

(I,S)	z_R (MeV)	g_a		
$(1/2, 0)$		$\bar{D}^*\Sigma_c$	$\bar{D}^*\Lambda_c^+$	
	4418	2.75	0	
$(0, -1)$		$\bar{D}_s^*\Lambda_c^+$	$\bar{D}^*\Xi_c$	$\bar{D}^*\Xi_c'$
	4370	1.23	3.14	0
	4550	0	0	2.53

TABLE IV: Pole position and coupling constants for the bound states from $VB \to VB$.

(I,S)	M	Γ	Γ_i					
$(1/2, 0)$			πN	ηN	$\eta' N$	$K\Sigma$		$\eta_c N$
	4261	56.9	3.8	8.1	3.9	17.0		23.4
$(0, -1)$			$\bar{K}N$	$\pi\Sigma$	$\eta\Lambda$	$\eta'\Lambda$	$K\Xi$	$\eta_c\Lambda$
	4209	32.4	15.8	2.9	3.2	1.7	2.4	5.8
	4394	43.3	0	10.6	7.1	3.3	5.8	16.3

TABLE V: Mass (M), total width (Γ), and the partial decay width (Γ_i) for the states from $PB \to PB$, with units in MeV.

(I,S)	M	Γ	Γ_i					
$(1/2, 0)$			ρN	ωN	$K^*\Sigma$			$J/\psi N$
	4412	47.3	3.2	10.4	13.7			19.2
$(0, -1)$			$\bar{K}^*N$	$\rho\Sigma$	$\omega\Lambda$	$\phi\Lambda$	$K^*\Xi$	$J/\psi\Lambda$
	4368	28.0	13.9	3.1	0.3	4.0	1.8	5.4
	4544	36.6	0	8.8	9.1	0	5.0	13.8

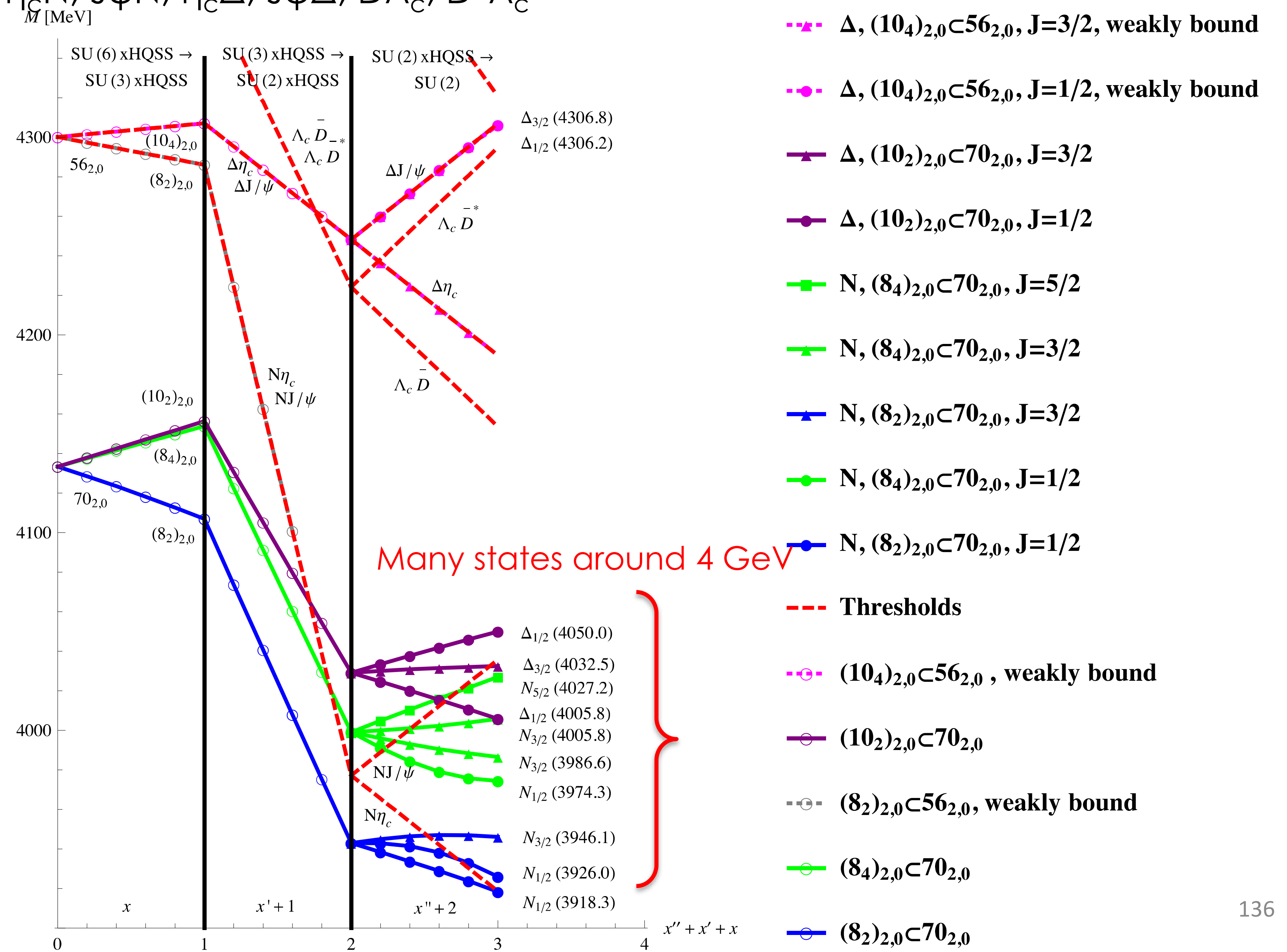
TABLE VI: Mass (M), total width (Γ), and the partial decay width (Γ_i) for the states from $VB \to VB$ with units in MeV.

P$_c$(4380) Γ=205 MeV, P$_c$(4450) Γ=39 MeV

Meson-baryon coupling model

$\eta_c N$, $J\psi N$, $\eta_c\Delta$, $J\psi\Delta$, $D\bar{\Lambda}_c$, $D^*\bar{\Lambda}_c$

Garcia-Recio, Nieves, Romanetz, Salcedo, Tolos, Phys. Rev. D87, 074034 (2013)

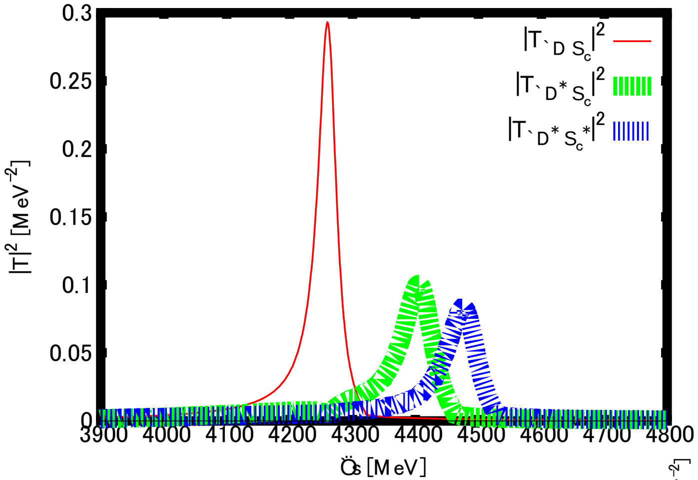


$P_c(4380)\ \Gamma=205$ MeV, $P_c(4450)\ \Gamma=39$ MeV

Meson-baryon coupling model

Xiao, Nieves, Oset, Phys. Rev. D88, 056012 (2013)

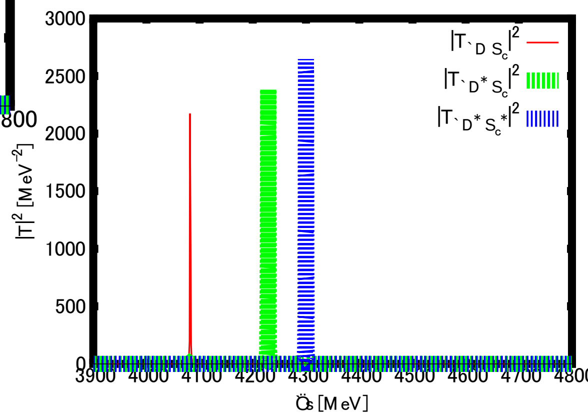
$\bar{D}\Sigma_c, D^*\bar{\Sigma}_c, D\bar{\Sigma}_c^*, D^*\bar{\Sigma}_c^*$



I=1/2, $J^P=1/2^-$

$$V_{ij} = -C_{ij}\,\frac{1}{4f^2}\,(k^0 + k'^0)\,\vec{\epsilon}\,\vec{\epsilon}\,'$$

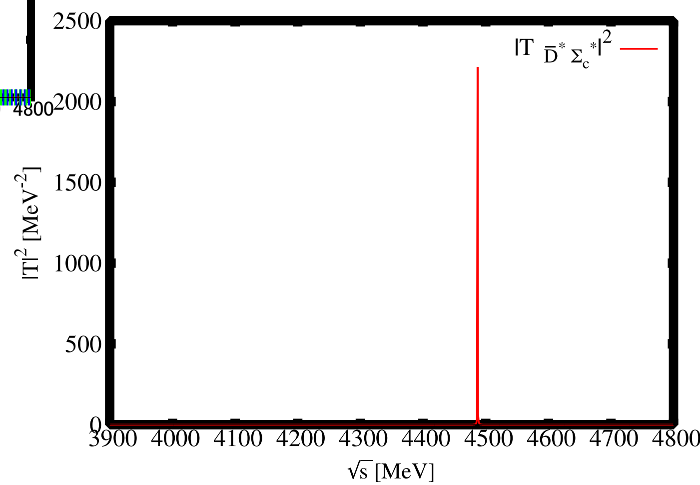
$$T = [1 - V G]^{-1} V$$



I=1/2, $J^P=3/2^-$

$$V_\mu = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & D_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & J/\psi \end{pmatrix}_\mu$$

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\tilde{\eta}_c}{\sqrt{12}} + \frac{\tilde{\eta}_c'}{\sqrt{4}} & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\tilde{\eta}_c}{\sqrt{12}} + \frac{\tilde{\eta}_c'}{\sqrt{4}} & K^0 & D^- \\ K^- & \bar{K}^0 & \frac{-2\eta_8}{\sqrt{6}} + \frac{\tilde{\eta}_c}{\sqrt{12}} + \frac{\tilde{\eta}_c'}{\sqrt{4}} & D_s^- \\ D^0 & D^+ & D_s^+ & -\frac{3\tilde{\eta}_c}{\sqrt{12}} + \frac{\tilde{\eta}_c'}{\sqrt{4}} \end{pmatrix}$$



I=1/2, $J^P=5/2^-$

$P_c(4380)$ Γ=205 MeV, $P_c(4450)$ Γ=39 MeV

QCD sum rule

Chen, Chen, Liu, Steele, Zhu, Phys. Rev. Lett. 115 (2015) 17

$$
\begin{aligned}
J^{\bar{D}^*\Sigma_c^*}_{\{\mu\nu\}} &= [\bar{c}_d\gamma_\mu d_d][\epsilon_{abc}(u_a^T C\gamma_\nu u_b)\gamma_5 c_c] + \{\mu \leftrightarrow \nu\}\,, \\
J^{\bar{D}\Sigma_c^*}_{\{\mu\nu\}} &= [\bar{c}_d\gamma_\mu\gamma_5 d_d][\epsilon_{abc}(u_a^T C\gamma_\nu u_b) c_c] + \{\mu \leftrightarrow \nu\}\,, \\
J^{\bar{D}^*\Lambda_c}_{\{\mu\nu\}} &= [\bar{c}_d\gamma_\mu u_d][\epsilon_{abc}(u_a^T C\gamma_\nu\gamma_5 d_b) c_c] + \{\mu \leftrightarrow \nu\}\,,
\end{aligned}
$$

3/2⁻

5/2⁺

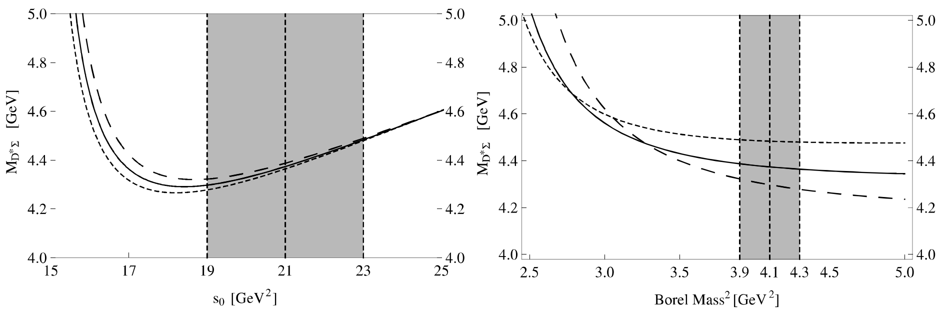


FIG. 1: The variation of $M_{[\bar{D}^*\Sigma_c],3/2^-}$ with respect to the threshold value s_0 (left) and the Borel mass M_B (right). In the left figure, the long-dashed, solid and short-dashed curves are obtained by fixing $M_B^2 = 3.9$, 4.1 and 4.3 GeV^2, respectively. In the right figure, the long-dashed, solid and short-dashed curves are obtained for $s_0 = 19$, 21 and 23 GeV^2, respectively.

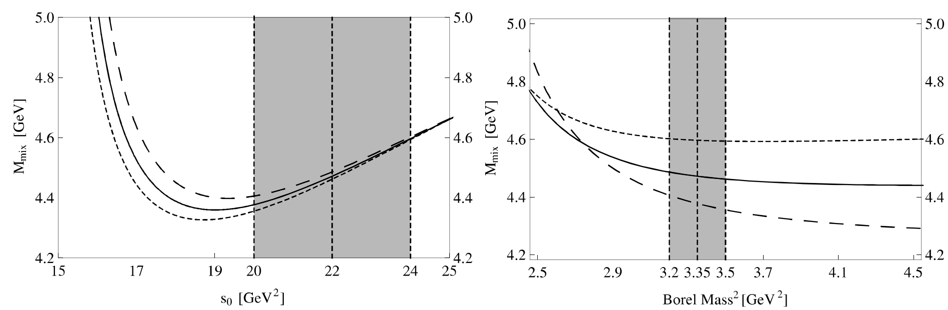


FIG. 2: The variation of $M_{[\bar{D}\Sigma_c^*\&\bar{D}^*\Lambda_c],5/2^+}$ with respect to the threshold value s_0 (left) and the Borel mass M_B (right).

Very good agreement with LHCb!

$$M_{[\bar{D}^*\Sigma_c],3/2^-} = 4.37^{+0.18}_{-0.12}\ \text{GeV}$$

$$M_{[\bar{D}\Sigma_c^*\&\bar{D}^*\Lambda_c],5/2^+} = 4.47^{+0.19}_{-0.12}\ \text{GeV}$$

Prediction for bottom analog:

$$
\begin{aligned}
M_{[\bar{B}^*\Sigma_b],3/2^-} &= 11.55^{+0.23}_{-0.14}\ \text{GeV} \\
M_{[\bar{B}\Sigma_b^*\&\bar{B}^*\Lambda_b],5/2^+} &= 11.66^{+0.28}_{-0.27}\ \text{GeV}
\end{aligned}
$$

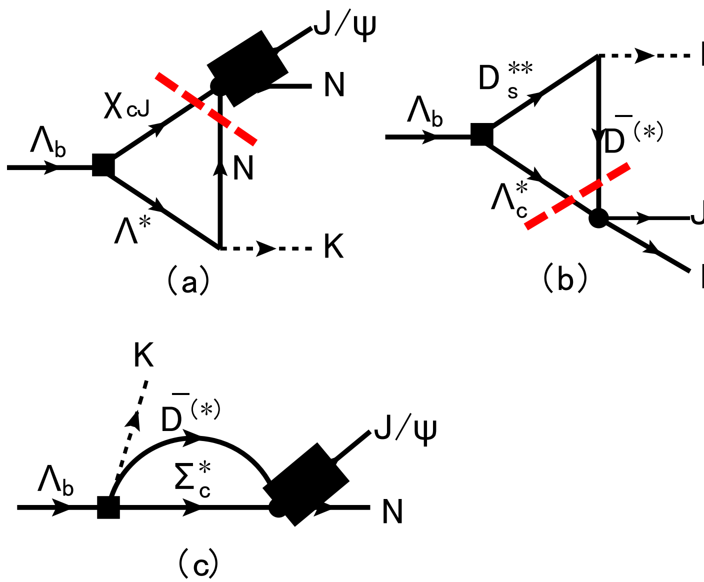
3. Cusp effect

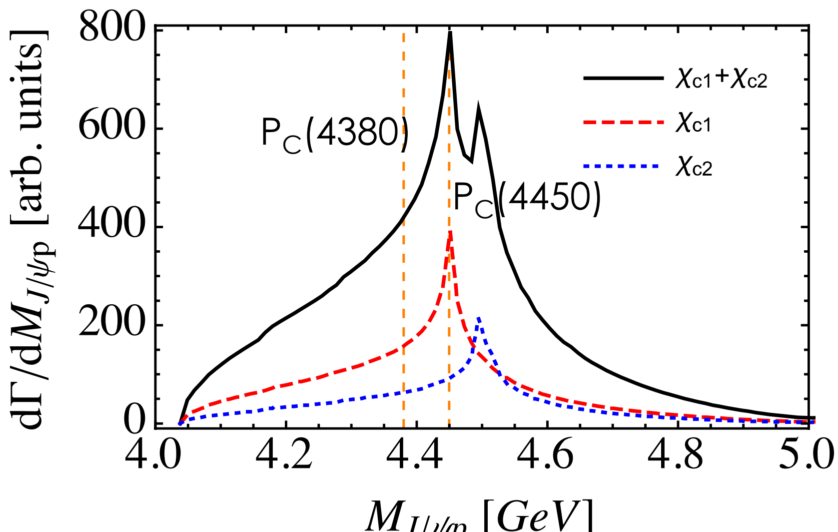
Kinematic anomaly

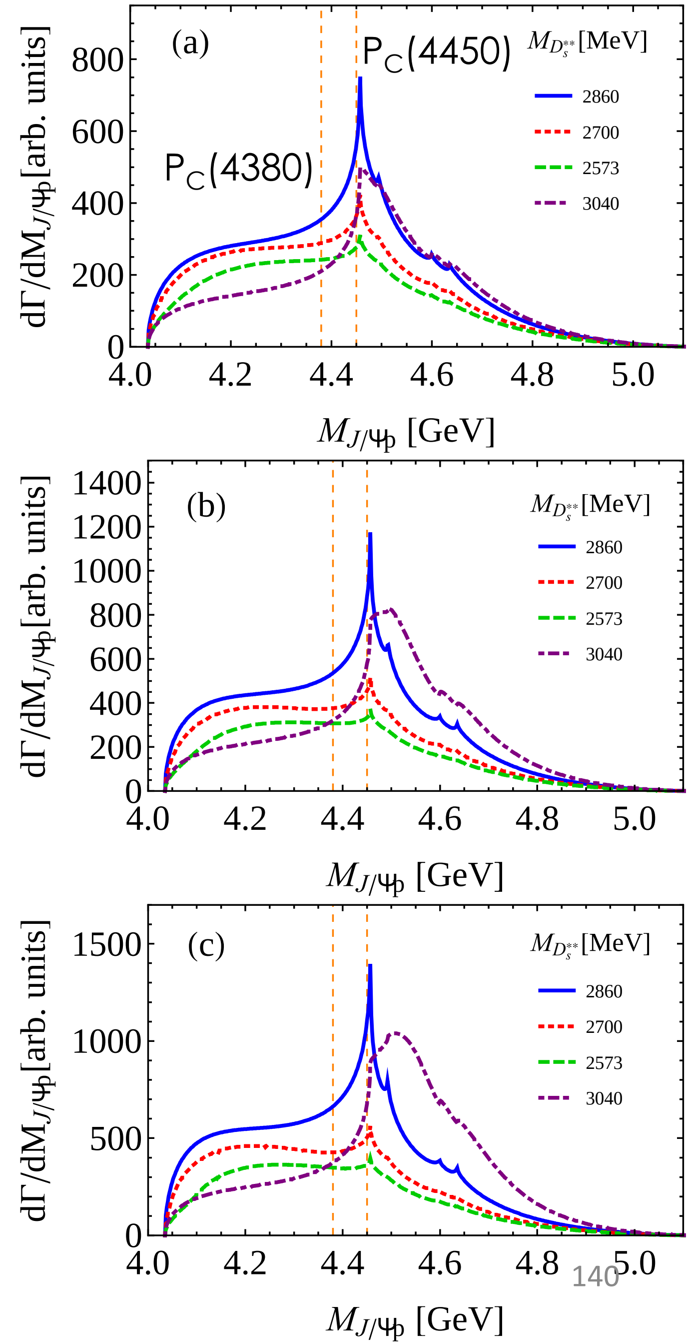
$P_c(4380)$ Γ=205 MeV, $P_c(4450)$ Γ=39 MeV

Anomalous triangle singularity

Liu, Wang, Zhao, Phys. Lett. B757 (2016) 231







4. Other things?

Lattice QCD, AdS/QCD, …

3. Heavy exotic hadrons -X, Y, Z hadrons-

Recent experiments

LHCb, Phys.Rev.Lett.115, 072001 (2015)

LHCb, Phys.Rev.Lett.122, 222001 (2019)

$P_c(4380)$? → $P_c(4312)$

same or different?

$P_c(4450)$ → $P_c(4440)$

→ $P_c(4457)$

two peaks!

Three peak states

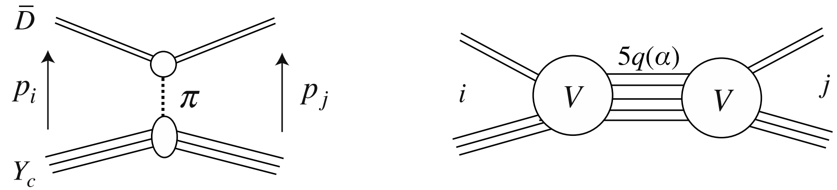
$P_c(4380)$ Γ~200 MeV (very large)

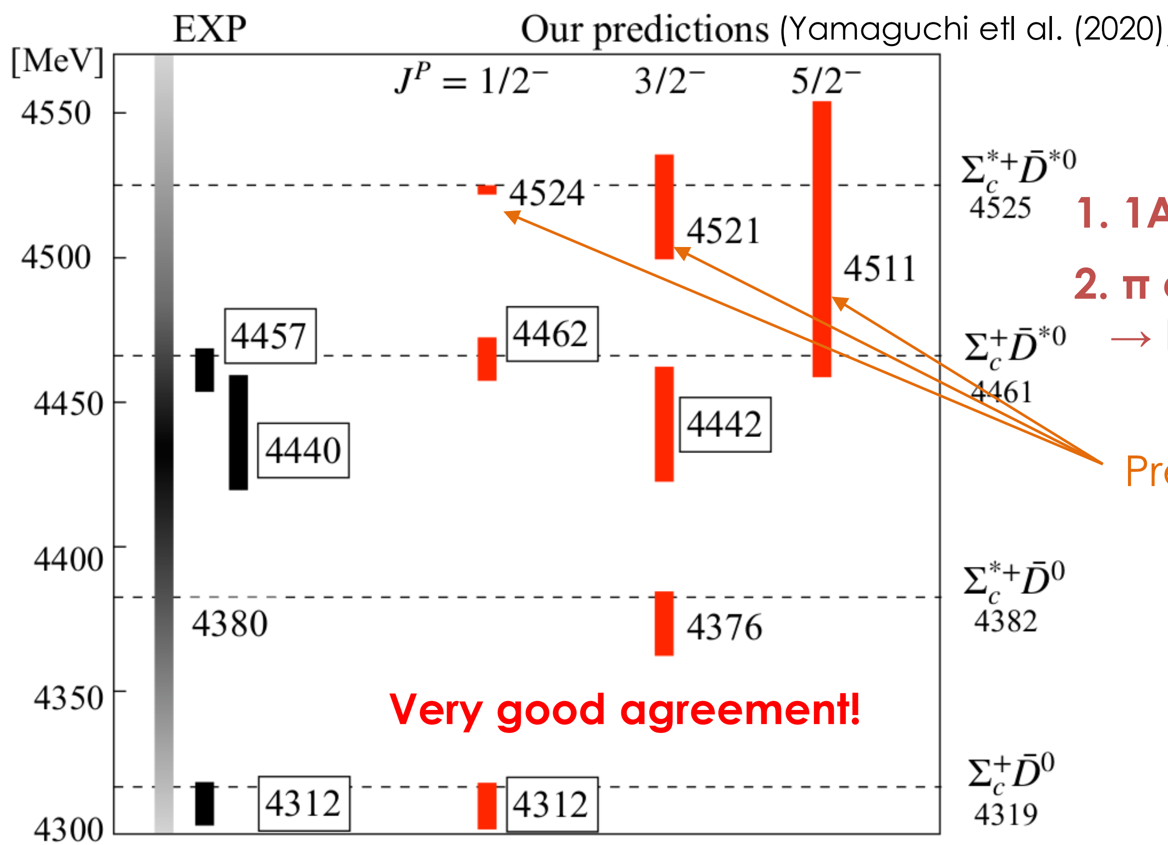
$$
\begin{array}{ll}
P_c(4312)^+: & M = (4311.9 \pm 0.7^{+6.8}_{-0.6})\ \text{MeV}\,,\ \Gamma = (9.8 \pm 2.7^{+3.7}_{-4.5})\ \text{MeV}\,,\\
P_c(4440)^+: & M = (4440.3 \pm 1.3^{+4.1}_{-4.7})\ \text{MeV}\,,\ \Gamma = (20.6 \pm 4.9^{+8.7}_{-10.1})\ \text{MeV}\,,\\
P_c(4457)^+: & M = (4457.3 \pm 0.6^{+4.1}_{-1.7})\ \text{MeV}\,,\ \Gamma = (6.4 \pm 2.0^{+5.7}_{-1.9})\ \text{MeV}\,.
\end{array}
$$

Brambilla et al. Phys. Rep. 873 (2020) 1

3. Heavy exotic hadrons -X, Y, Z hadrons-

π exchange pot. + short-range int.





1. 1Attraction by 5q

2. π exchange pot.
→ Decay widths

Y. Yamaguchi et al., Phys. Rev. D96, 114031 (2017)
Y. Yaamguchi et al. Phys. Rev. D101 (2020) 091502
Cf. Y. Shmizu, Y. Yamaguchi, M. Harada, Phys. Rev. D98, 014021 (2018)
Y. Shimizu, Y. Yamaguchi, M. Harada, PTEP2019 (2019) 123D01
Y. Shimizu, Y. Yamaguchi, M. Harada, arXiv:1904.00587 [hep-ph]

3. Heavy exotic hadrons -X, Y, Z hadrons-

Aren't there more charm pentaquarks?

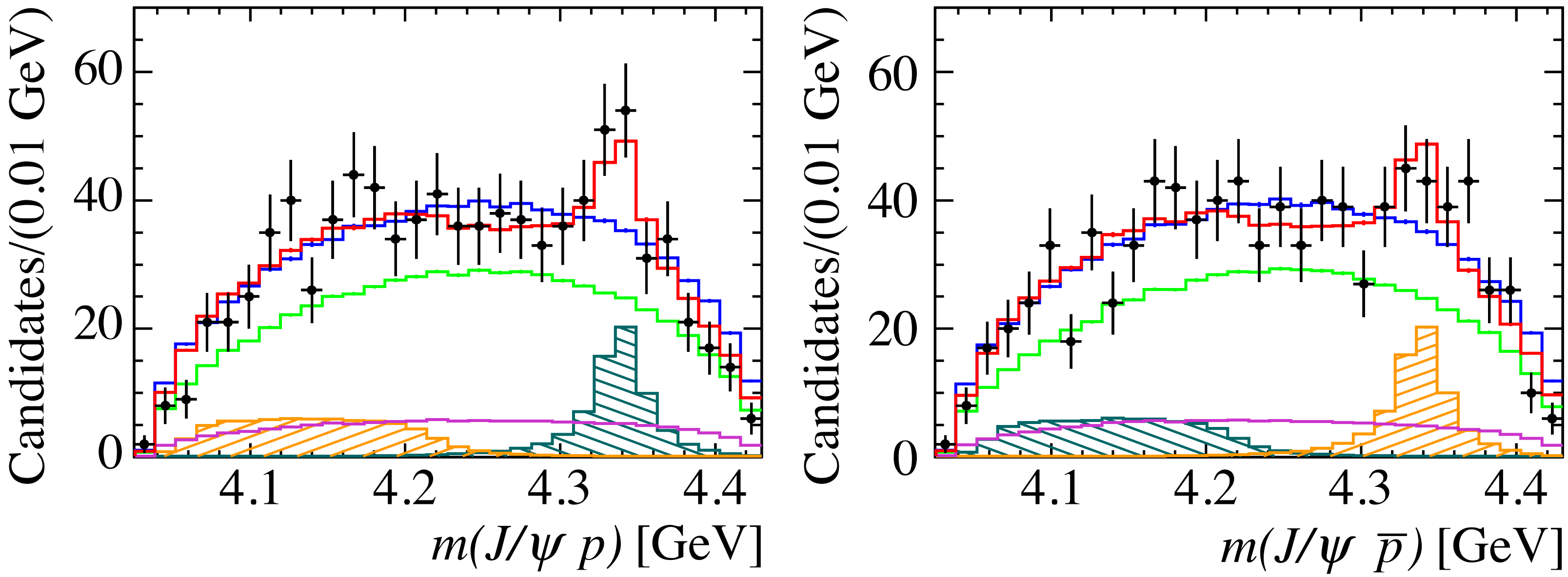
Yes, there are more!

3. Heavy exotic hadrons -X, Y, Z hadrons-

LHCb, Phys. Rev. Lett. 128 (2022) 062001

$P_c(4338)$ $J^P=1/2^+$

positive parity (new)



$$M_{P_c} = 4337^{+7}_{-4}{}^{+2}_{-2}\ \mathrm{MeV}$$

a few MeV below $\Sigma_c{}^*D^{bar}$ threshold

$$\Gamma_{P_c} = 29^{+26}_{-12}{}^{+14}_{-14}\ \mathrm{MeV}$$

$\Sigma_c{}^*D^{bar}$ dynamics (p-wave) may be relevant…

3. Heavy exotic hadrons -X, Y, Z hadrons-

Charm-strange pentaquark？

$$\boxed{P_{cs}(uds c\bar{c})}$$

P_c

$P_c(4380)$, $P_c(4450)$

u u d c $\bar{c}$

Flavor Octet

(I=1/2)

P_{cs}

u d s c $\bar{c}$

color octet

Flavor Singlet

(I=0)

Brief summary of quark model

Three quarks in SU(6)$_{\text{flavor+spin}}$ symmetry

$$\mathbf{6}\times\mathbf{6}\times\mathbf{6}=\mathbf{20}_{\mathrm{A}}+\mathbf{70}_{\mathrm{MA}}+\mathbf{70}_{\mathrm{MS}}+\mathbf{56}_{\mathrm{S}}$$

SU(6)$_{\text{flavor+spin}}$ representation

$$\begin{cases}\mathbf{20}=(\mathbf{8},2)+(\mathbf{1},4),\\ \mathbf{70}=(\mathbf{8},4)+(\mathbf{10},2)+(\mathbf{8},2)+(\mathbf{1},2),\\ \mathbf{56}=(\mathbf{10},4)+(\mathbf{8},2),\end{cases}$$

Δ N

Three quarks in SU(3)$_{\text{color}}$ symmetry

$$\mathbf{3}\times\mathbf{3}\times\mathbf{3}=\mathbf{1}_{\mathrm{A}}+\mathbf{8}_{\mathrm{MA}}+\mathbf{8}_{\mathrm{MS}}+\mathbf{10}_{\mathrm{S}}$$

Totally antisymmetric representation (fermion systems)
A: antisymmetric
S: symmetric
MA: mixed-antisymmetric
MS: mixed-symmetric

Brief summary of quark model

Three quarks in **SU(6)$_{\text{flavor+spin}}$** symmetry

$$\mathbf{6} \times \mathbf{6} \times \mathbf{6} = \mathbf{20}_{\mathrm{A}} + \mathbf{70}_{\mathrm{MA}} + \mathbf{70}_{\mathrm{MS}} + \mathbf{56}_{\mathrm{S}}$$

SU(6)$_{\text{flavor+spin}}$ representation

$$\begin{cases} \mathbf{20} = (\mathbf{8}, 2) + (\mathbf{1}, 4), \\ \mathbf{70} = (\mathbf{8}, 4) + (\mathbf{10}, 2) + (\mathbf{8}, 2) + (\mathbf{1}, 2), \\ \mathbf{56} = (\mathbf{10}, 4) + (\mathbf{8}, 2), \end{cases}$$

Colored qqq

Lowest-dimension: Mostly attractive in color-spin int.

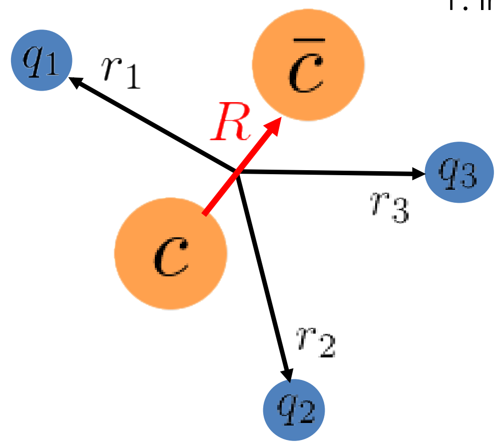
$\lambda \cdot \lambda \quad \sigma \cdot \sigma$

Three quarks in **SU(3)$_{\text{color}}$** symmetry

$$\mathbf{3} \times \mathbf{3} \times \mathbf{3} = \mathbf{1}_{\mathrm{A}} + \mathbf{8}_{\mathrm{MA}} + \mathbf{8}_{\mathrm{MS}} + \mathbf{10}_{\mathrm{S}}$$

Totally antisymmetric representation (fermion systems)
A: antisymmetric
S: symmetric
MA: mixed-antisymmetric
MS: mixed-symmetric

Y. Irie, M. Oka, S.Y., Phys. Rev. D97, 034006 (2018)



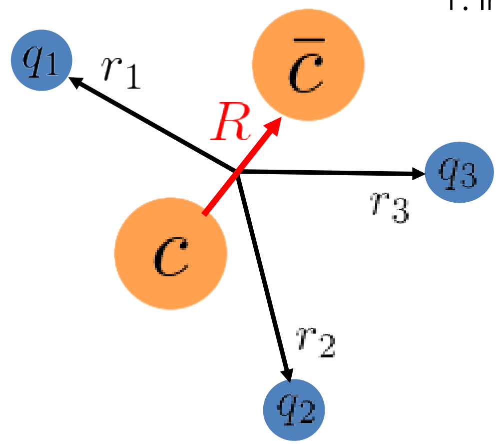
$$\psi = \phi(\boldsymbol{R}, \boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3)\psi^{s,c}_{c\bar{c}}\psi^{s,c,f}_{uds}$$

$$\phi(R, r_1, r_2, r_3) = \frac{1}{(2\pi a^2)^{\frac{3}{4}}}\frac{1}{(\pi b^2)^{\frac{9}{4}}}\exp\left(-\frac{|R|^2}{4a^2} - \frac{|r_1|^2 + |r_2|^2 + |r_3|^2}{2b^2}\right)$$

(I, J^P)	octet type ($\mathbf{8}$)						singlet type ($\mathbf{1}$)					
		component	color	spin	flavor	isospin		component	color	spin	flavor	isospin
$(0, 1/2^-)$	$P_{cs\mathbf{8}}$	$c\bar{c}$	$\mathbf{8}$	0	—	—	$P_{cs\mathbf{1}}$	$c\bar{c}$	$\mathbf{1}$	0	—	—
		uds	$\mathbf{8}$	$1/2$	$\mathbf{1}$	0		uds	$\mathbf{1}$	$1/2$	$\mathbf{8}$	0
$(0, 1/2^-)$	$P'_{cs\mathbf{8}}$	$c\bar{c}$	$\mathbf{8}$	1	—	—	$P'_{cs\mathbf{1}}$	$c\bar{c}$	$\mathbf{1}$	1	—	—
		uds	$\mathbf{8}$	$1/2$	$\mathbf{1}$	0		uds	$\mathbf{1}$	$1/2$	$\mathbf{8}$	0
$(0, 3/2^-)$	$P^*_{cs\mathbf{8}}$	$c\bar{c}$	$\mathbf{8}$	1	—	—	$P^*_{cs\mathbf{1}}$	$c\bar{c}$	$\mathbf{1}$	1	—	—
		uds	$\mathbf{8}$	$1/2$	$\mathbf{1}$	0		uds	$\mathbf{1}$	$1/2$	$\mathbf{8}$	0

Y. Irie, M. Oka, S.Y., Phys. Rev. D97, 034006 (2018)

Quark model
(5-body system)



baryon	model A	model B	experiments [24]
$N(1/2^+)$	1048	1019	939
$\Delta(3/2^+)$	1247	1220	1232
$\Lambda(1/2^+)$	1116	1116	1116
$\Sigma(1/2^+)$	1193	1193	1193
$\Sigma^*(3/2^+)$	1330	1327	1385

$$K = -\frac{\nabla_R^2}{2\mu_{c\bar{c}}} - \frac{\nabla_1^2}{2m_1} - \frac{\nabla_3^2}{2m_2} - \frac{\nabla_3^2}{2m_3},$$

+

model A
(conventional int.)

$$V_{\mathrm{Coulomb}} = \sum_{i<j} \frac{\alpha_s}{4r_{ij}} \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j,$$

$$V_{\mathrm{CMI}} = -\frac{\alpha_s}{4} \sum_{i<j} \frac{\pi}{m_i m_j} \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j \left(1 + \frac{2}{3} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\right) \delta^{(3)}(r_{ij}),$$

$$V_{\mathrm{conf}} = -\sigma \sum_{i<j} \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j r_{ij},$$

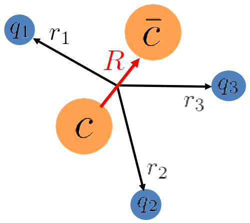
model B
(+instanton-induced int.)

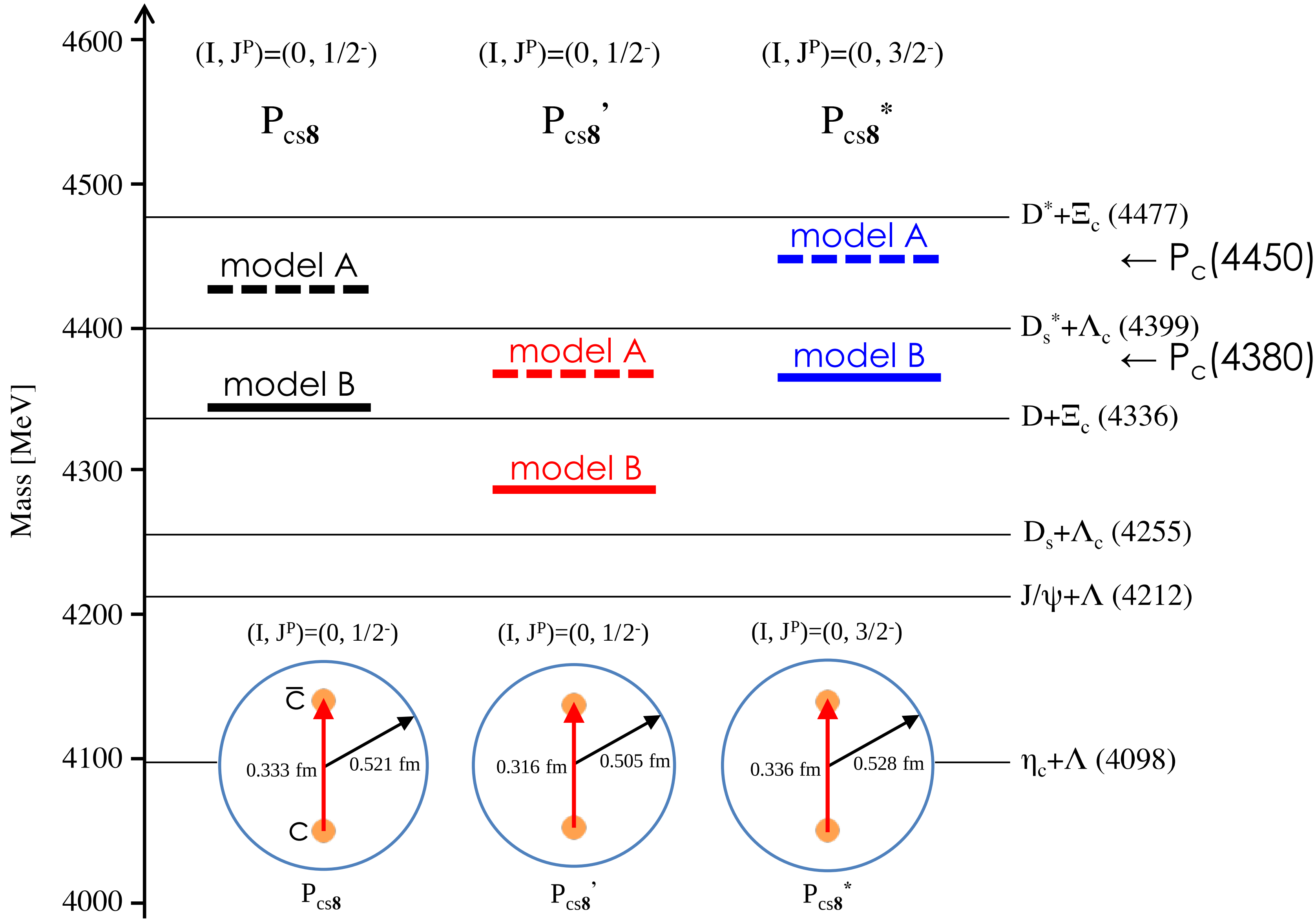
$$V_{\mathrm{III2}} = U_0^{(2)} \frac{15}{8} \sum_{i<j} \mathcal{A}_2^f \frac{1}{m_i m_j} \left(1 - \frac{1}{5} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\right) \delta^{(3)}(r_{ij}),$$

$$V_{\mathrm{III3}} = V_0 \frac{189}{40} \sum_{(ijk)} \mathcal{A}_3^f \left(1 - \frac{1}{7} (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + \boldsymbol{\sigma}_j \cdot \boldsymbol{\sigma}_k + \boldsymbol{\sigma}_k \cdot \boldsymbol{\sigma}_i)\right) \delta^{(3)}(r_{ij}) \delta^{(3)}(r_{jk}),$$

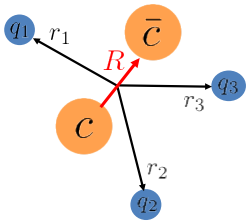
	model A	model B
m_u [MeV]	313	313
m_s [MeV]	521.7	521.7
m_c [MeV]	1497.4	1497.4
α_{s1}	0.769	0.715
α_{s2}	0.5461	0.5461
σ_1 [MeV/fm]	178	178
σ_2 [MeV/fm]	135.63	135.63
C_Λ [MeV]	-1130	-1470
C_{η_c} [MeV]	-61	-61
$U_0^{(2)}$	—	-1.331
V_0 [MeV^{-5}]	—	5.271×10^{-13}
p (L-L)	—	0.4
p (H-H,H-L)	—	0

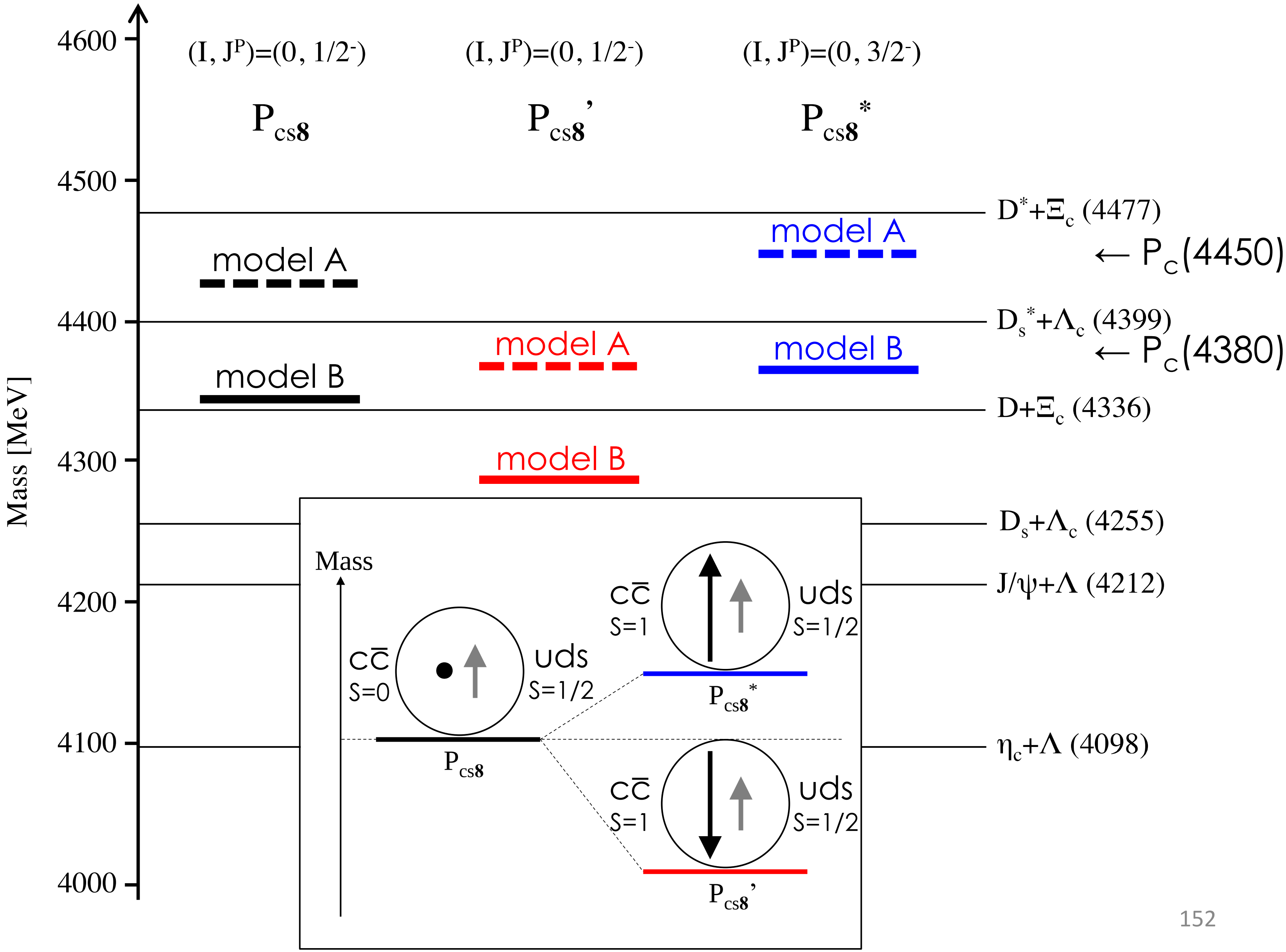
Y. Irie, M. Oka, S.Y., Phys. Rev. D97, 034006 (2018)



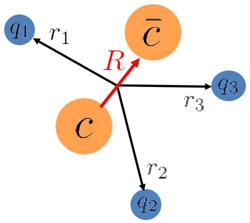


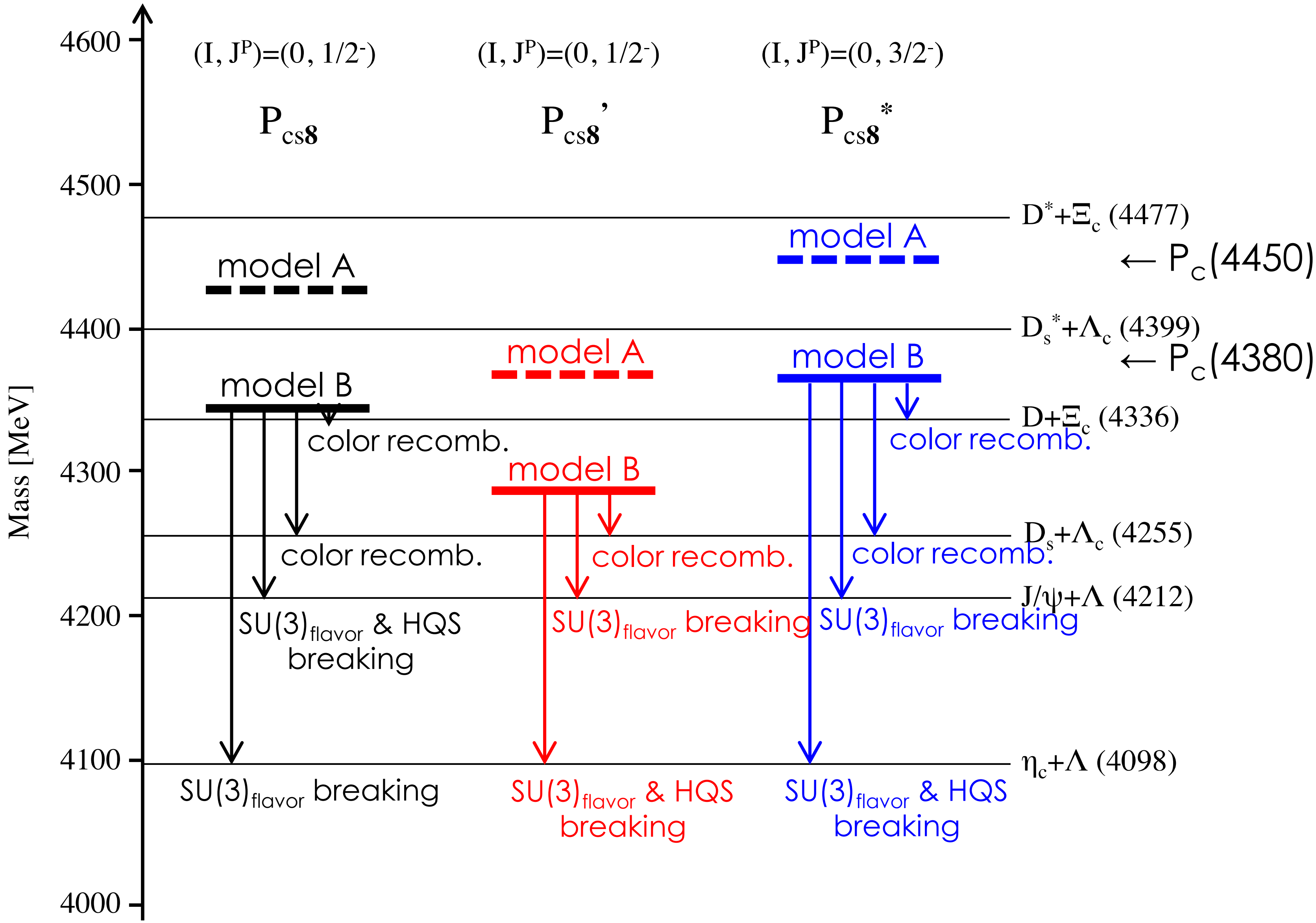
Y. Irie, M. Oka, S.Y., Phys. Rev. D97, 034006 (2018)



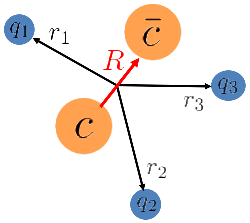


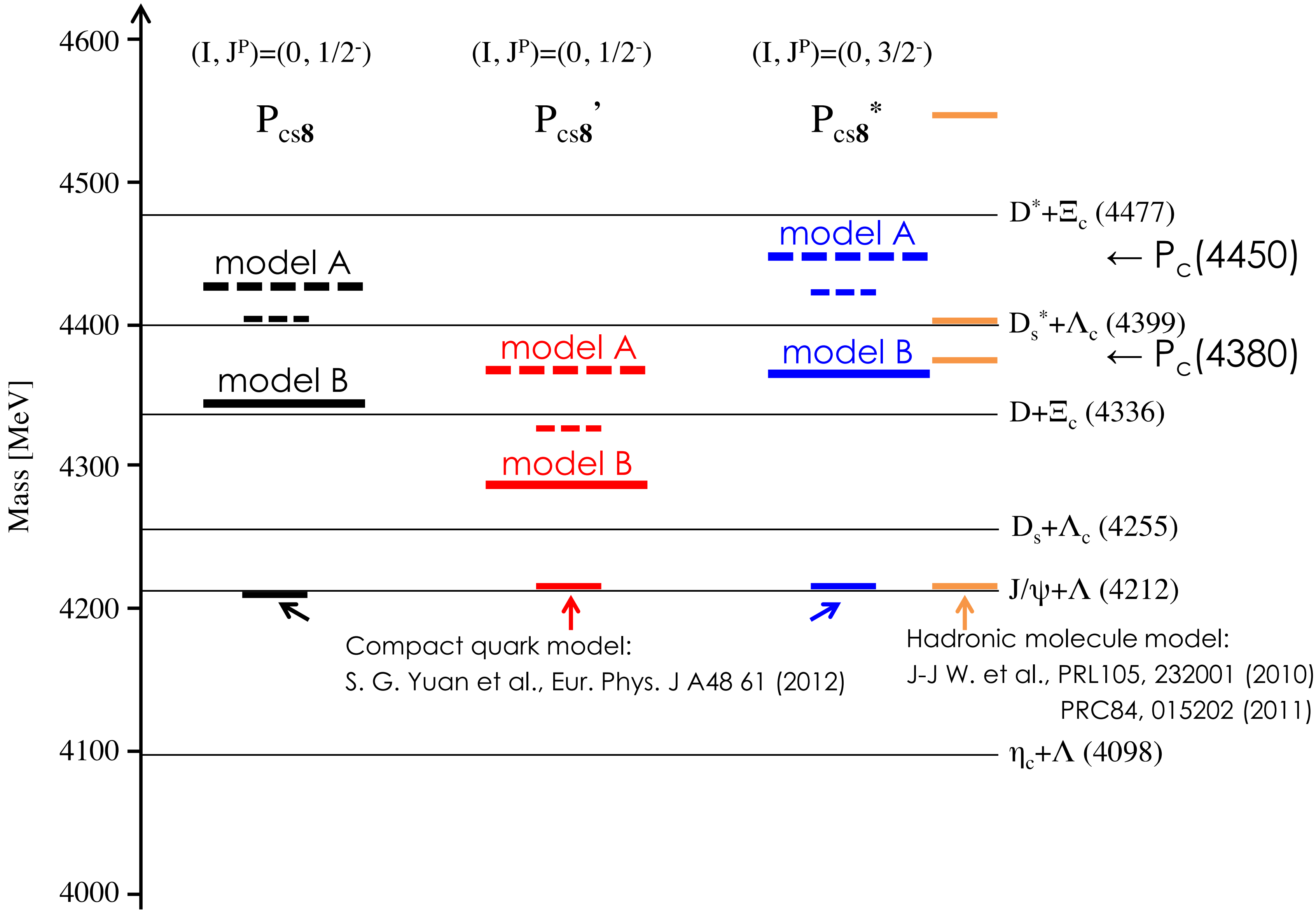
Y. Irie, M. Oka, S.Y., Phys. Rev. D97, 034006 (2018)





Y. Irie, M. Oka, S.Y., Phys. Rev. D97, 034006 (2018)

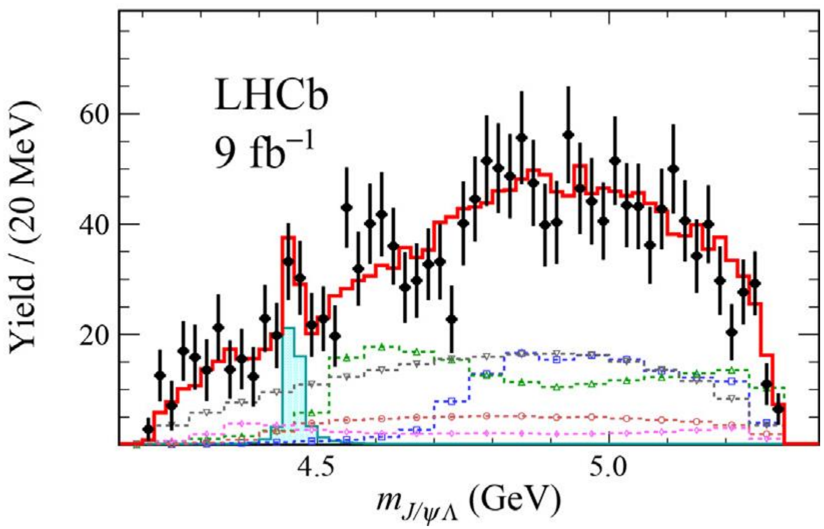


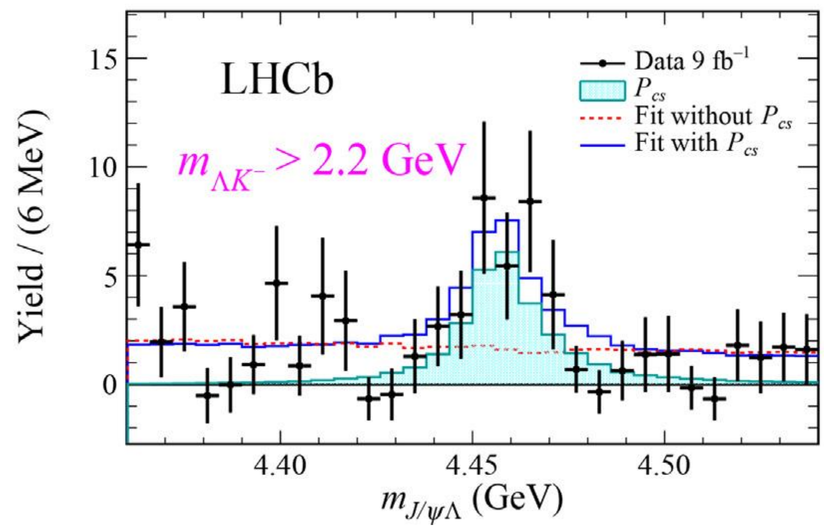


3. Heavy exotic hadrons -X, Y, Z hadrons-

LHCb, Sci. Bull. 66 (2021) 1278

$$B^- \to J/\psi\Lambda\bar{p}$$





State	M_0 (MeV)	Γ_0 (MeV)	FF (%)
$P_{cs}(4459)^0$	$4458.8 \pm 2.9^{+4.7}_{-1.1}$	$17.3 \pm 6.5^{+8.0}_{-5.7}$	$2.7^{+1.9+0.7}_{-0.6-1.3}$
$\Xi(1690)^-$	$1692.0 \pm 1.3^{+1.2}_{-0.4}$	$25.9 \pm 9.5^{+14.0}_{-13.5}$	$22.1^{+6.2+6.7}_{-2.6-8.9}$
$\Xi(1820)^-$	$1822.7 \pm 1.5^{+1.0}_{-0.6}$	$36.0 \pm 4.4^{+7.8}_{-8.2}$	$32.9^{+3.2+6.9}_{-6.2-4.1}$
$\Xi(1950)^-$	1910.6 ± 18.4	105.7 ± 23.2	$11.5^{+5.8+49.9}_{-3.5-9.4}$
$\Xi(2030)^-$	2022.8 ± 4.7	68.2 ± 8.5	$7.3^{+1.8+3.8}_{-1.8-4.1}$
NR	–	–	$35.8^{+4.6+10.3}_{-6.4-11.2}$

$P_{cs}(4459)$

two peaks ?

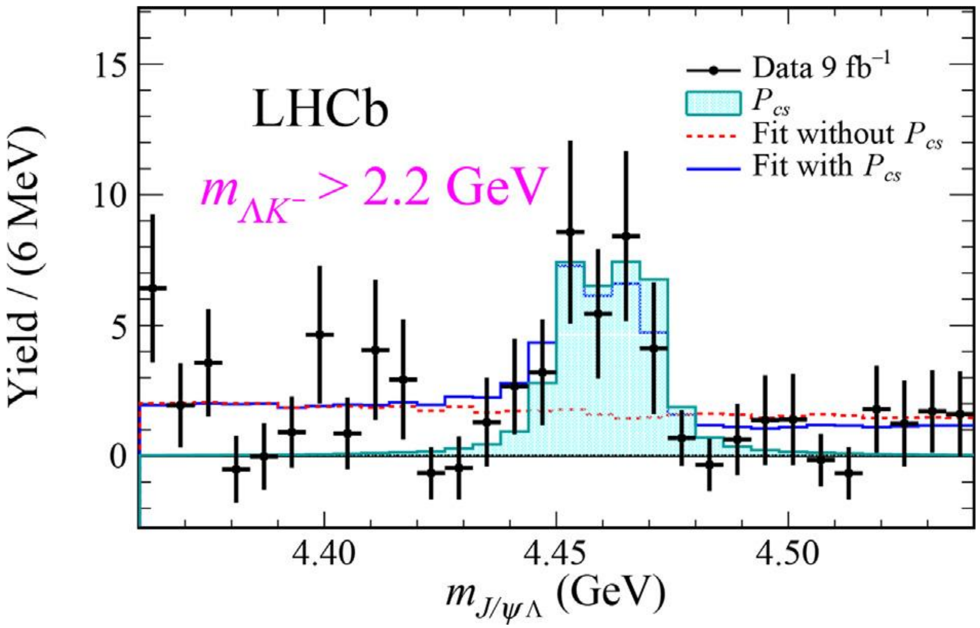
(→next page)

3. Heavy exotic hadrons -X, Y, Z hadrons-

LHCb, Sci. Bull. 66 (2021) 1278

$$B^- \to J/\psi\Lambda\bar{p}$$

two peaks？



$P_{cs}(4459)$

$M = 4458.8 \pm 2.9$ MeV
$\Gamma = 17.3 \pm 6.5$ MeV

$P_{cs}(4455)$

$M = 4454.9 \pm 2.7$ MeV
$\Gamma = 7.5 \pm 9.7$ MeV

$P_{cs}(4468)$

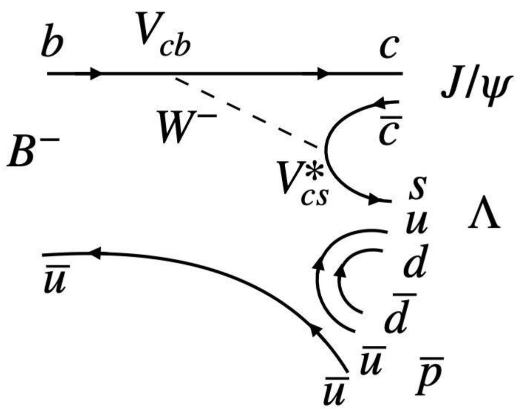
$M = 4467.8 \pm 3.7$ MeV
$\Gamma = 5.2 \pm 5.3$ MeV

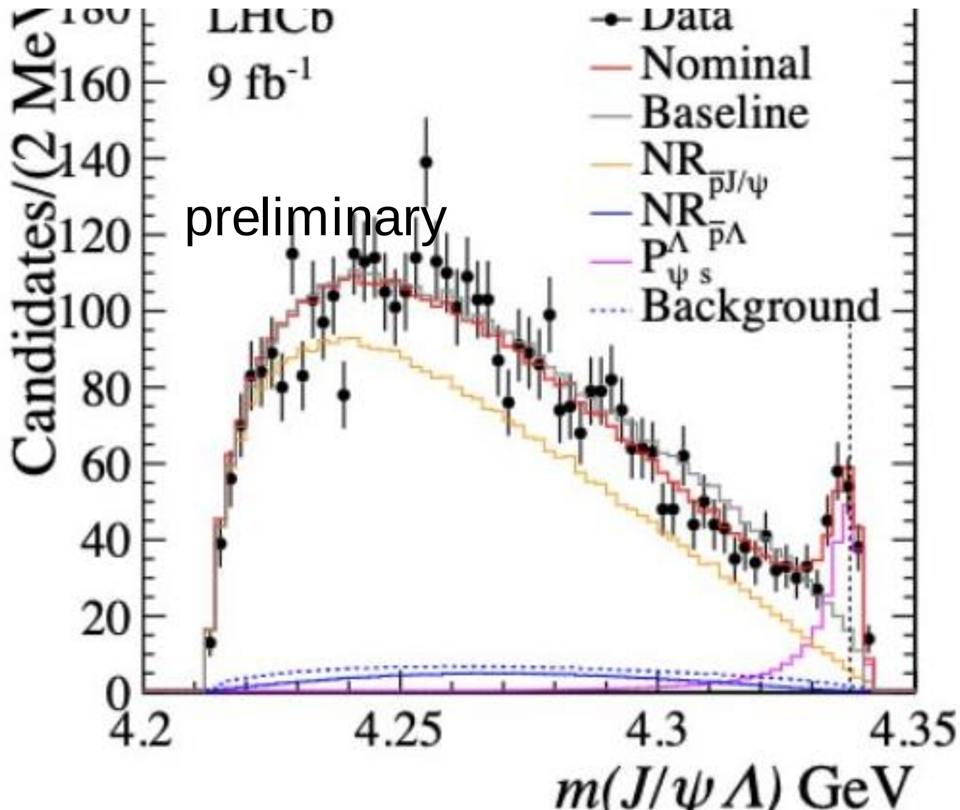
cannot be distinguished statistically…

3. Heavy exotic hadrons -X, Y, Z hadrons-

LHCb-PAPER-2022-031

$$B^- \to J/\psi \Lambda \bar{p}$$



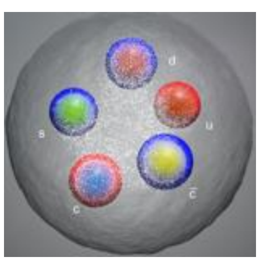


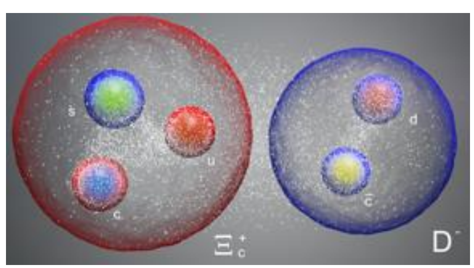
$m(P^{\Lambda}_{\psi s})$ $4338.2 \pm 0.7\ MeV$

$\Gamma(P^{\Lambda}_{\psi s})$ $7.0 \pm 1.2\ MeV$

spin: $J = 1/2$

parity: $\mathrm{P} = -1$ favoreed

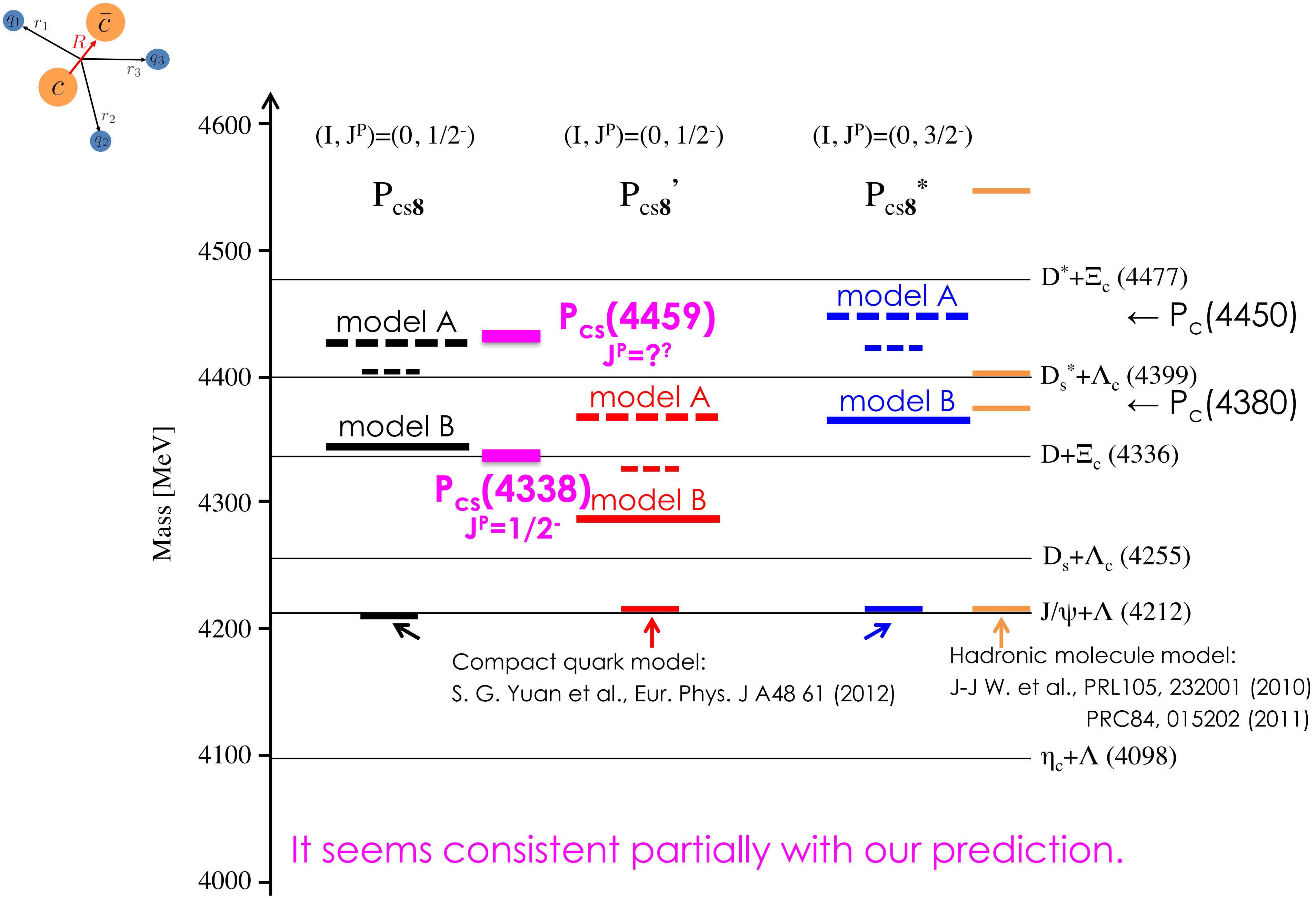




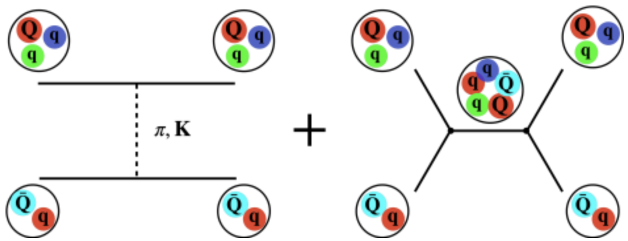
$P_{cs}(4338)\ J^P=1/2^-$

https://indico.cern.ch/event/1176505/attachments/2475130/4248283/CERN%20seminar_LHCb.pdf

Y. Irie, M. Oka, S.Y., Phys. Rev. D97, 034006 (2018)



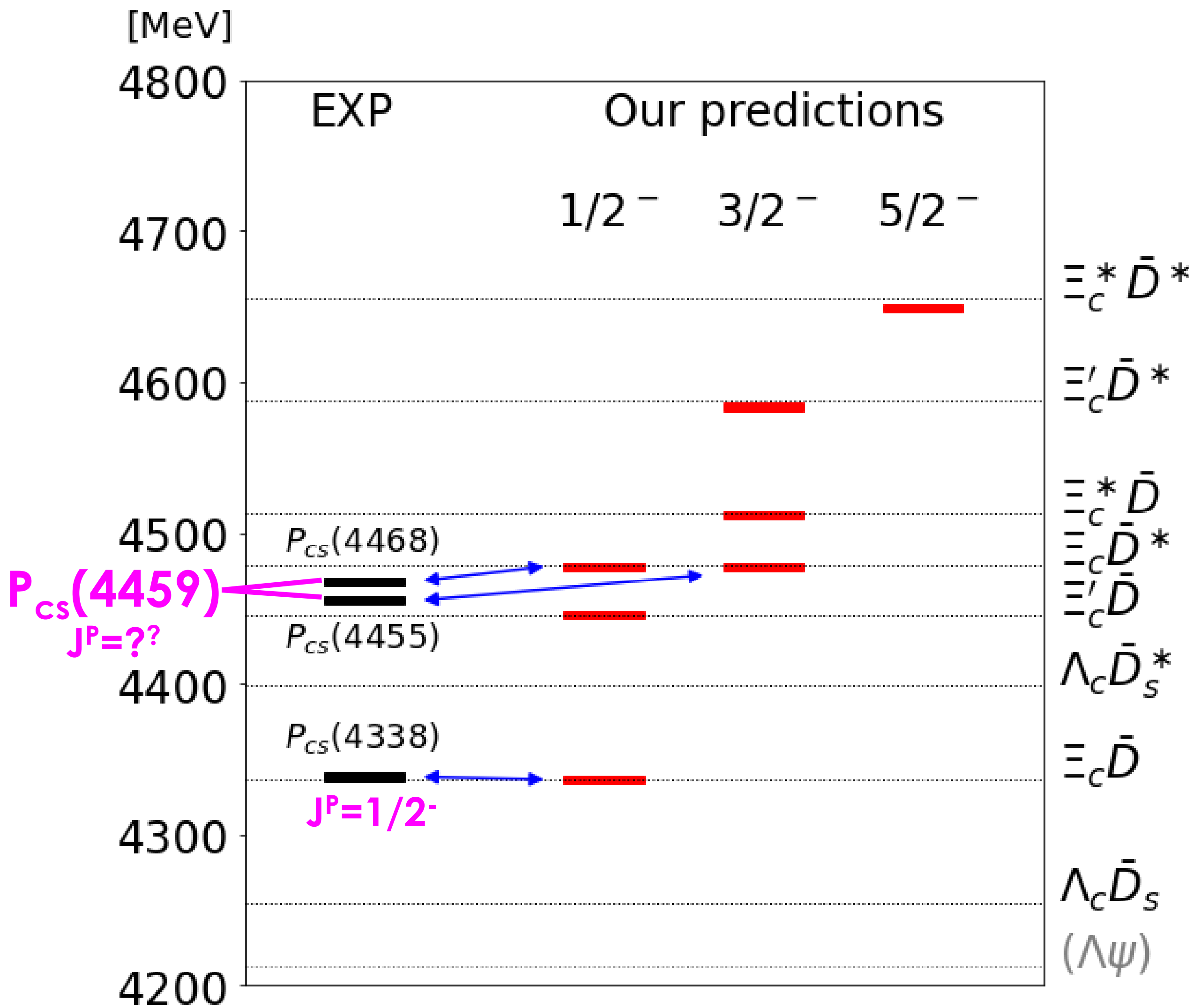
It seems consistent partially with our prediction.

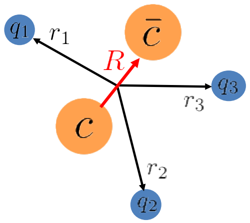


Giachino, Hosaka, Santopinto, Takeuchi, Takizawa, Yamaguchi, 2209.10413 [hpe-ph]

Hadronic molecule + quark core model

It seems consistent with experiments.





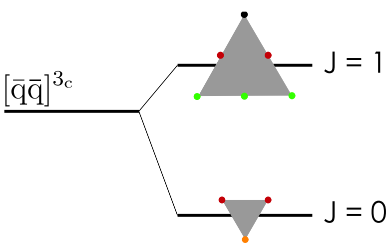
Through researches of heavy hadrons...

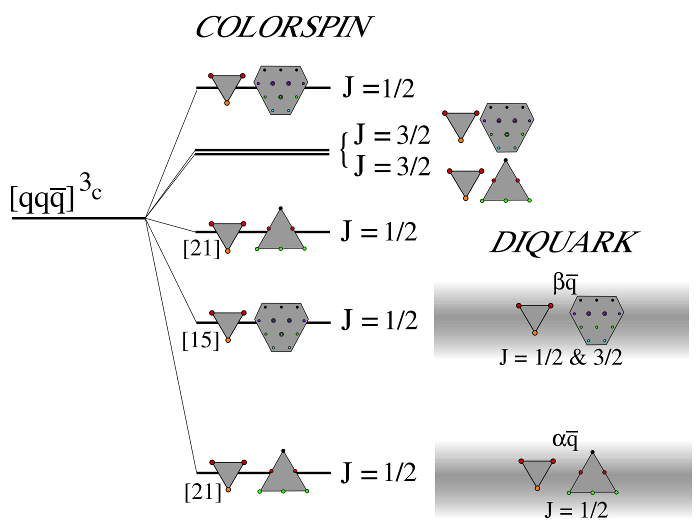
we can access the colorful world!

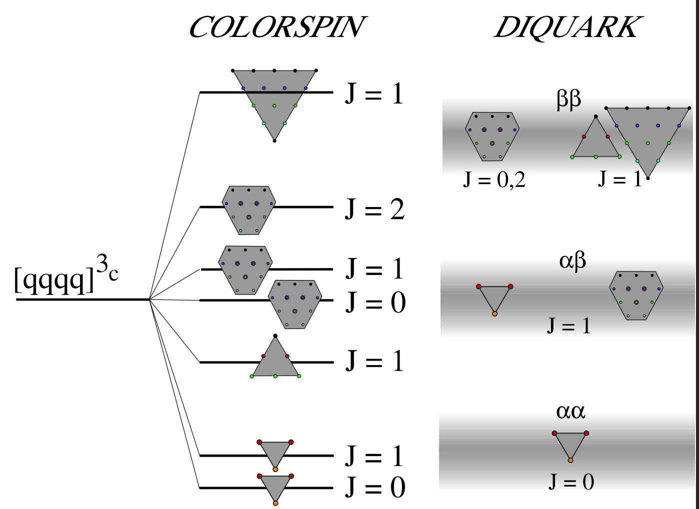
Exotic hadrons: *mass spectrum of colorful states*

R.L. Jaffe, Phys. Rev. D72, 074508 (2005)

$[\bar{q}\bar{q}]^{3_c}\bar{Q}$ $\quad$ $[qq\bar{q}]^{3_c}\bar{Q}$ $\quad$ $[qqqq]^{3_c}\bar{Q}$







Ex. $\Lambda_c = [qq]_{J=0}c$

$\Sigma_c^{(*)} = [qq]_{J=1}c$

Ex. $X(5568) = [s u \bar{d}]_{J=?}\bar{b}$

D0, PRL117, 022003 (2016)

Ex. $\bar{D}N = qqqq\bar{c}$

No experiment

3. Heavy exotic hadrons -X, Y, Z hadrons-

3. Heavy exotic hadrons -X, Y, Z hadrons-

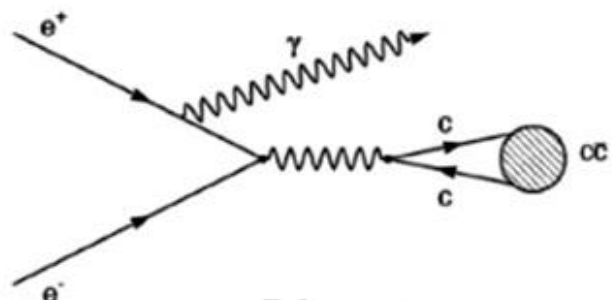
More X, Y, Z, ...?

3. Heavy exotic hadrons -X, Y, Z hadrons-

Y(4360) & Y(4660)

Cf. Y(4260)

too many ψ'?



$\rightarrow \psi(2S)\pi^+\pi^-$

$J^{PC} = 1^{--}$

Some confusions...

Single peak?

mass: $4324\ \mathrm{MeV}/c^2$

width: $172\ \mathrm{MeV}/c^2$ (relatively broad)

B. Aubert et al. [BaBar], Phys. Rev. Lett. 98, 212001 (2007)

less statistics...

Double peaks?

mass: $4361\ \mathrm{MeV}/c^2$ & $4664\ \mathrm{MeV}/c^2$

X. L. Wang et al. [Belle], Phys. Rev. Lett. 99, 142002 (2007)

J. P. Lees et al. [BaBar], Phys. Rev. D D89, 111103 (2014)

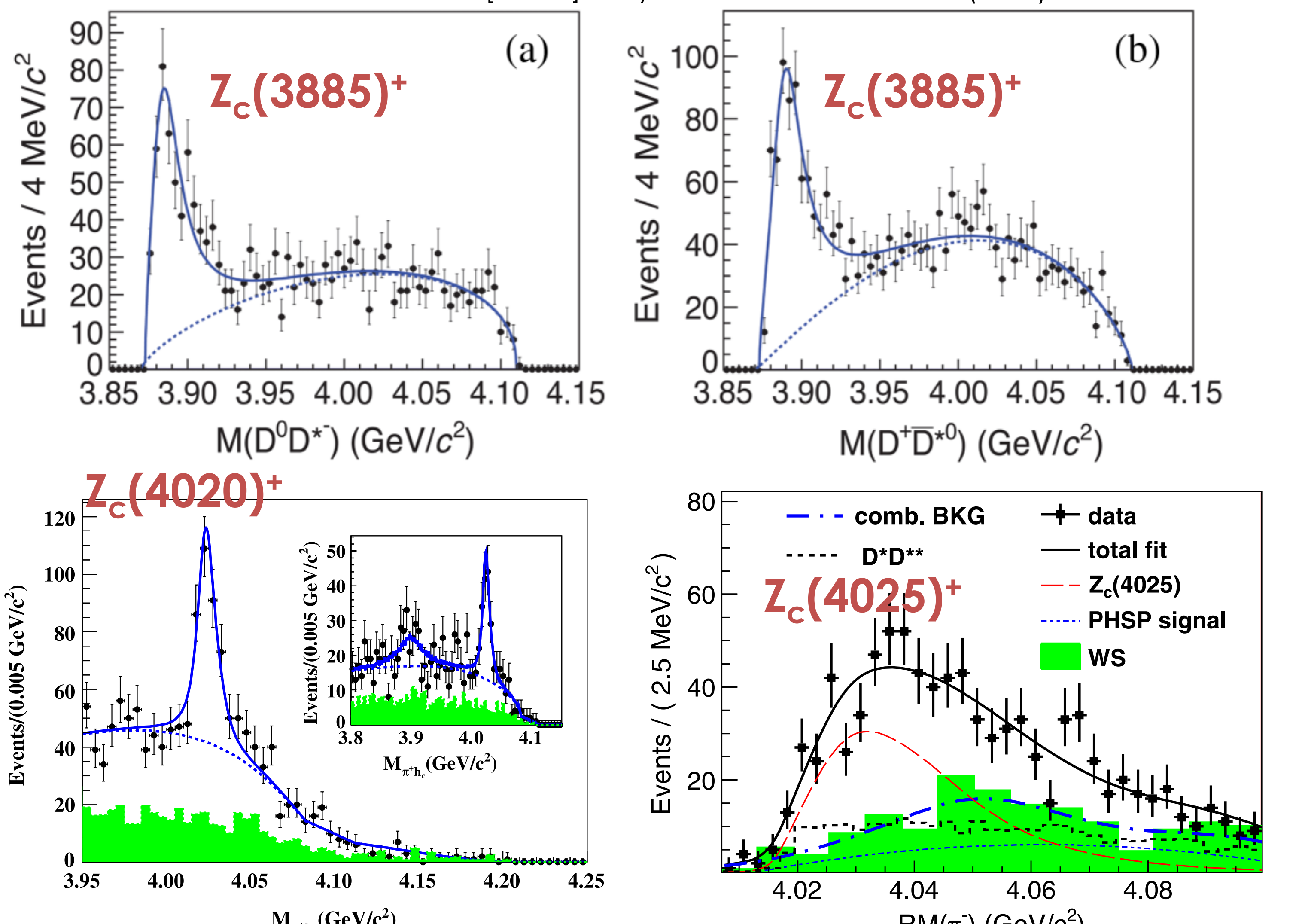
confirmed double peaks.

3. Heavy exotic hadrons -X, Y, Z hadrons-

$Z_c(3885)^+$, $Z_c(4020)^+$, $Z_c(4025)^+$ Cf. $Z_c(3900)^+$

Other charged charmonium?

M. Ablikim et al. [BESIII], Phys. Rev. Lett. 110, 252001 (2013)



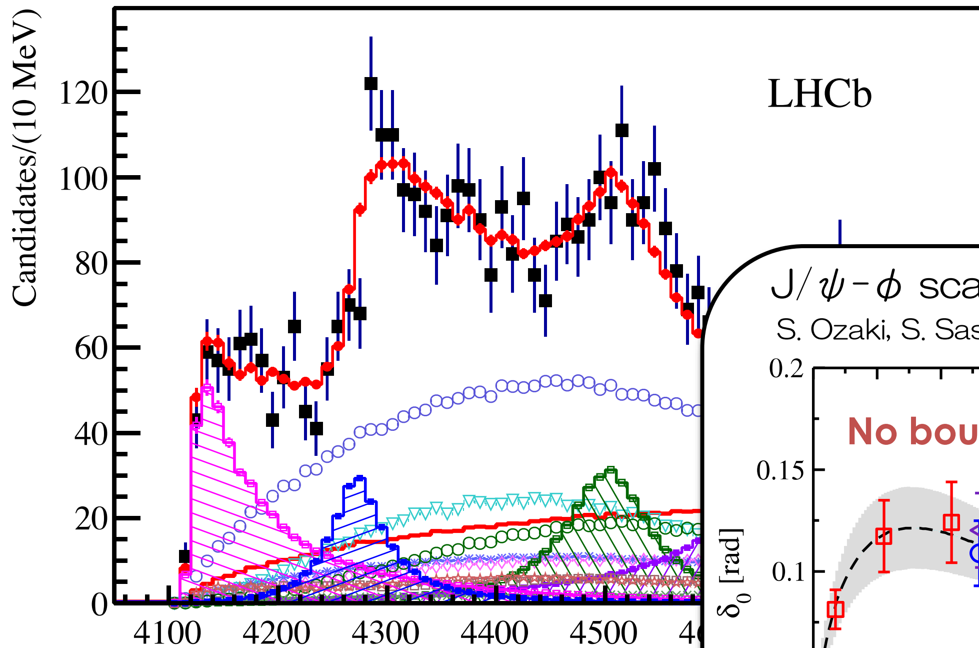
M. Ablikim et al. [BESIII], Phys. Rev. Lett. 111, 242001 (2013)

M. Ablikim et al. [BESIII], Phys. Rev. Lett. 112, 132001 (2014)

3. Heavy exotic hadrons -X, Y, Z hadrons-

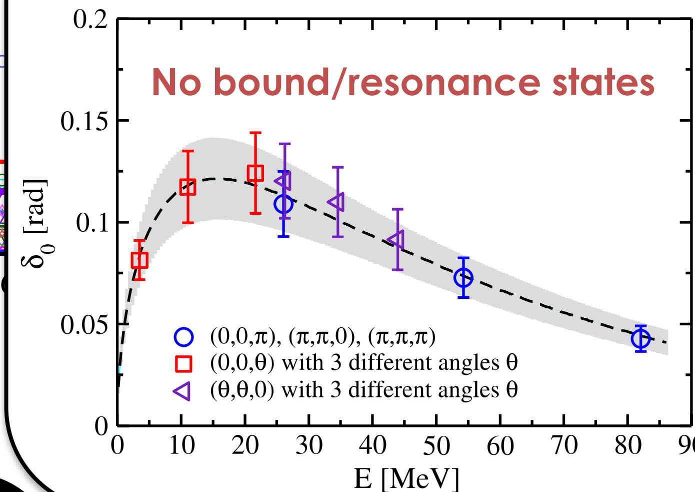
LHCb, Phys. Rev. Lett. 118, 022003 (2017)

X(4140), X(4274), X(4500), X(4700)



$J/\psi-\phi$ scattering in lattice QCD

S. Ozaki, S. Sasaki, PRD87, 014506 (2013)



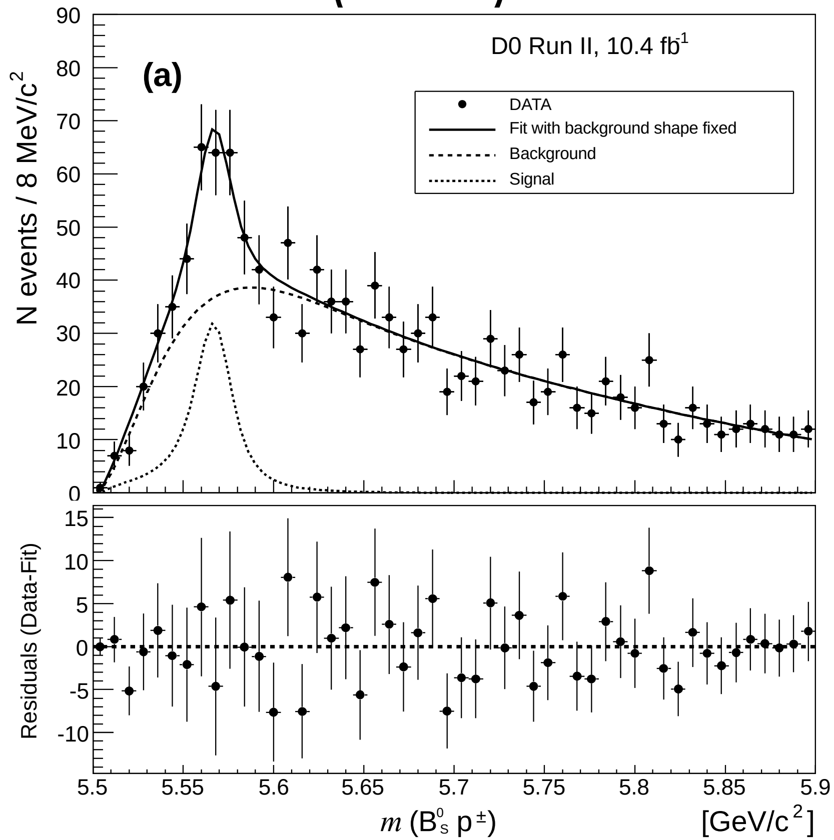
$c\bar{c}ss$

Quartet state?

3. Heavy exotic hadrons -X, Y, Z hadrons-

X(5568)

D0, PRL117, 022003 (2016)



$s\bar{b}u\bar{d}$

All different flavors!

3. Heavy exotic hadrons -X, Y, Z hadrons-

Talk file by E. S. Norella and C. Chen, CERN Seminar 5th July, 2022

New tetraquark candidates in several $B \to D\bar{D}h$ decays

$T^{a}_{c\bar{s}0}(2900)^{++} \to D_s^+\pi^+$ in $B^+ \to \bar{D}^- D_s^+\pi^+$
$T^{a}_{c\bar{s}0}(2900)^{0} \to D_s^+\pi^-$ in $B^0 \to \bar{D}^0 D_s^+\pi^-$

- Quark contents: $[c\bar{s}u\bar{d}]$, $[c\bar{s}\bar{u}d]$

LHCb-PAPER-2022-026
LHCb-PAPER-2022-027
In preparation

$X(3960) \to D_s^+ D_s^-$ in $B^+ \to D_s^+ D_s^- K^+$

- Quark content: $[c\bar{c}s\bar{s}]$?

LHCb-PAPER-2022-018
LHCb-PAPER-2022-019
In preparation

These two analyses are natural extensions of the $B^+ \to D^+D^-K^+$ study

Phys.Rev.D102(2020) 112003
Phys. Rev. Lett. 125 (2020) 242001

https://indico.cern.ch/event/1176505/attachments/2475130/4248283/CERN%20seminar_LHCb.pdf

3. Heavy exotic hadrons -X, Y, Z hadrons-

Double

Charm

Tetraquark

3. Heavy exotic hadrons -X, Y, Z hadrons-

Are exotic hadrons *unstable* in strong interaction？

Not necessarily! Some can be stable!

3. Heavy exotic hadrons -X, Y, Z hadrons-

T_{cc}

Double charm tetraquark

Z_c $\quad\quad$ T_{cc}

$c\bar{c}u\bar{d} \Leftrightarrow cc\bar{u}\bar{d}$

Hidden charm

$C = 0$

Double charm

$|C| = 2$

3. Heavy exotic hadrons -X, Y, Z hadrons-

T_{cc}

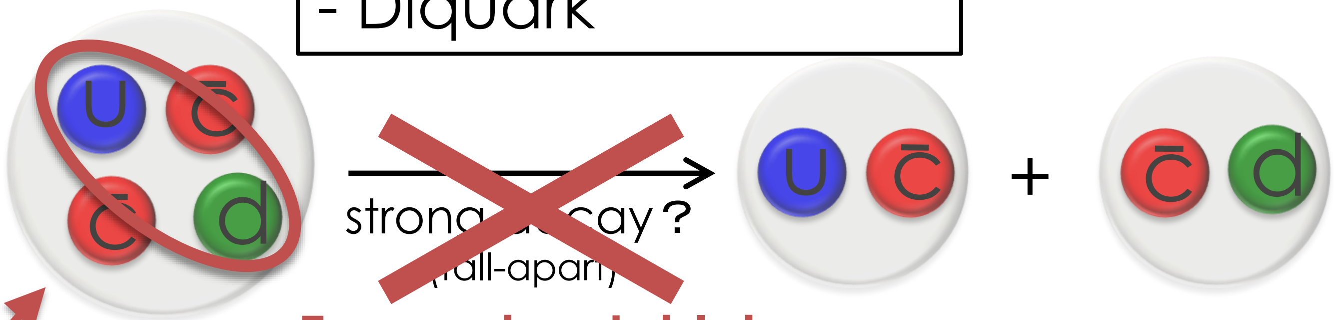
Double charm tetraquark

J.P. Ader, J.M. Richard and P. Taxil, Phys. Rev. D25, 2370 (1982)

- Color confinement
- Diquark

$I(J^P)=0(1^+)$

strong **ud** attraction



T_{cc} can be stable!

Gluon exchange force induces color-spin interaction

$$H_{int} = \sum_{i>j} \frac{C_H}{m_i m_j} \vec{s}_i \cdot \vec{s}_j \qquad C_H = v_0 \vec{\lambda}_i \cdot \vec{\lambda}_j \langle \delta(r_{ij}) \rangle$$

ud pair $1/m_c^0$ dominant attraction ($\bar{3}_c$, I=0, $^1S_0$)

$\bar{c}u$ pair $1/m_c$ suppressed

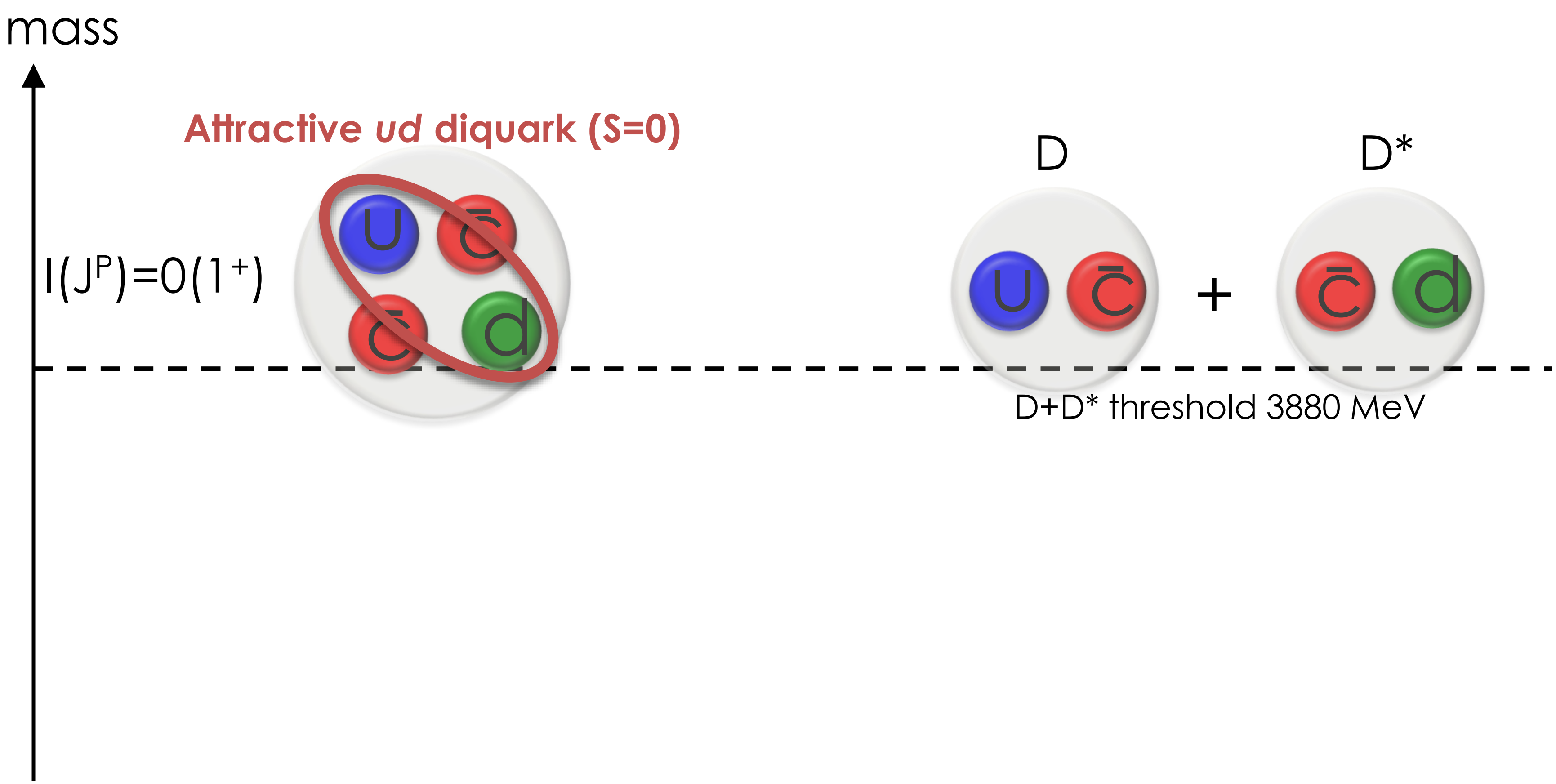
$\bar{c}\bar{c}$ pair $1/m_c^2$ more suppressed ($\bar{3}_c$, $^3S_1$)

3. Heavy exotic hadrons -X, Y, Z hadrons-

T_{cc}

Double charm tetraquark

J.P. Ader, J.M. Richard and P. Taxil, Phys. Rev. D25, 2370 (1982)

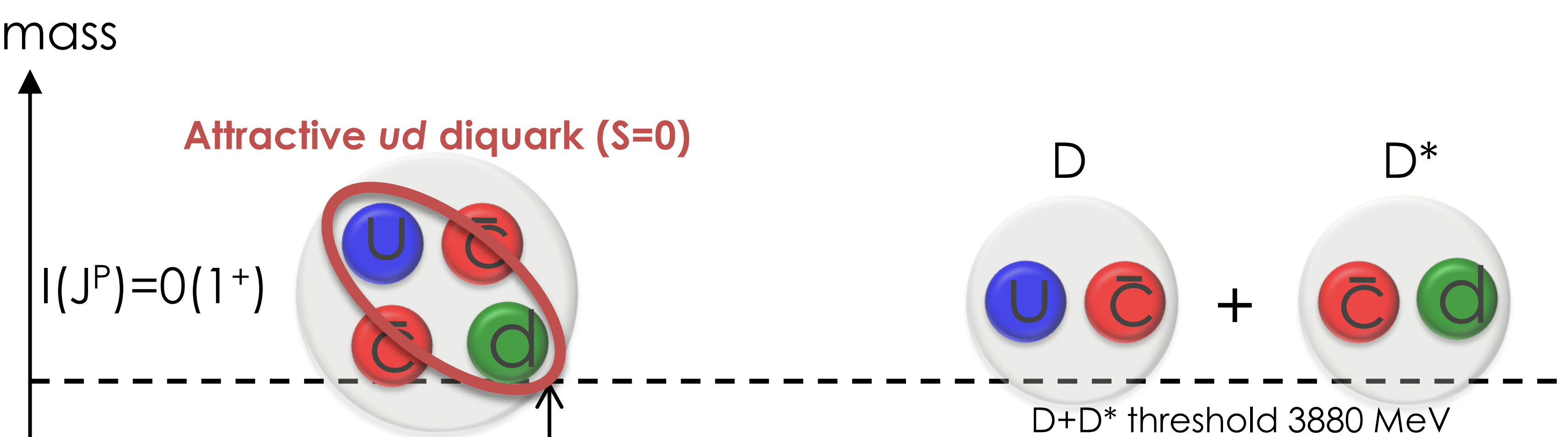


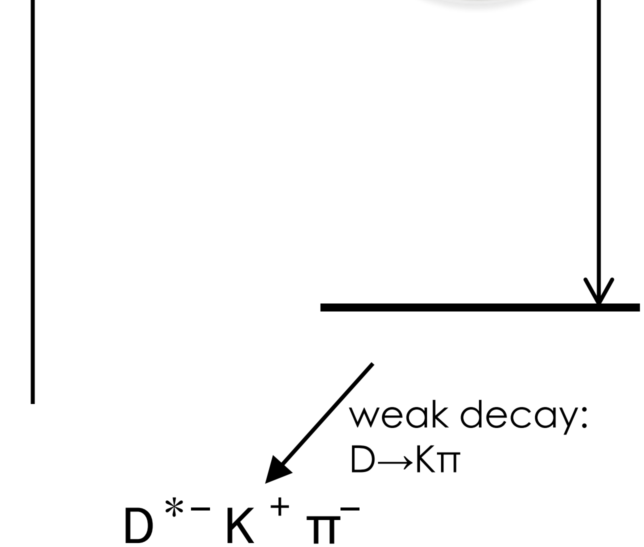
3. Heavy exotic hadrons -X, Y, Z hadrons-

T_{cc}

Double charm tetraquark

J.P. Ader, J.M. Richard and P. Taxil, Phys. Rev. D25, 2370 (1982)





	$ud\bar{c}\bar{c}$	$us\bar{c}\bar{c}$	$ds\bar{c}\bar{c}$
T^1_{cc}	-79.3	-8.7	-8.7
	$D^0 + D^{*-}$, $\bar{D}^{*0} + D^-$	$\bar{D}^0 + D_s^{*-}$	$D^- + D_s^{*-}$
	$ud\bar{b}\bar{b}$	$us\bar{b}\bar{b}$	$ds\bar{b}\bar{b}$
T^1_{bb}	-124.3	-62.3	-62.3
	$B^+ + B^{*0}$, $B^{*+} + B^0$	$B^+ + B_s^{*0}$	$B^0 + B_s^{*0}$
	$ud\bar{c}\bar{b}$	$us\bar{c}\bar{b}$	$ds\bar{c}\bar{b}$
T^1_{cb}	-59.0	$+2.9$	$+2.9$
	$B^{*+} + D^-$, $B^{*0} + \bar{D}^0$	$B_s^{*0} + D^0$	$B_s^{*0} + D^-$

S.-H. Lee, S. Yasui, Eur. Phys. J. C64,283 (2009)
S.-H. Lee, S. Yasui, W. Liu, C.-M. Ko, Eur. Phys. J. C54, 259 (2008)

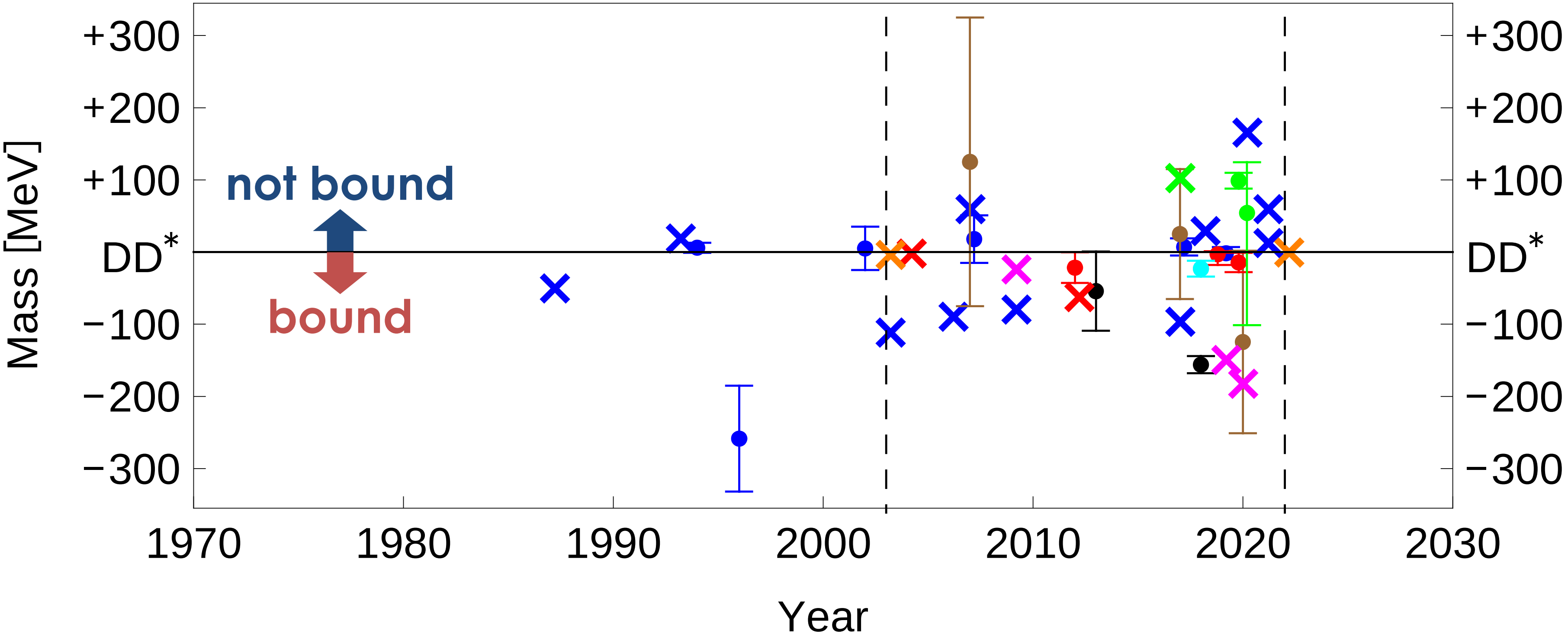
3. Heavy exotic hadrons -X, Y, Z hadrons-

Recent review:
H.-X. Chen, W. Chen, X. Liu, Y.-R. Liu, S.-L. Zhu, 2204.02649 [hep-ph]

T_{cc}

Double charm tetraquark

Summary of theoretical studies (chronological table!)



bound or **not bound** ?

3. Heavy exotic hadrons -X, Y, Z hadrons- T_{cc}

Double charm tetraquark

PRL **119,** 202002 (2017) PHYSICAL REVIEW LETTERS week ending 17 NOVEMBER 2017

Heavy-Quark Symmetry Implies Stable Heavy Tetraquark Mesons $Q_iQ_j\bar{q}_k\bar{q}_l$

Estia J. Eichten* and Chris Quigg†
Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510, USA

(Received 8 August 2017; published 15 November 2017)

~100 MeV **above** DD*bar threshold

State	J^P	j_ℓ	$m(Q_iQ_jq_m)$ (c.g.)	HQS relation	$m(Q_iQ_j\bar{q}_k\bar{q}_l)$	Decay channel	$\mathcal{Q}$ (MeV)
$\{cc\}[\bar{u}\,\bar{d}]$	1^+	0	3663[b]	$m(\{cc\}u)+315$	3978	D^+D^{*0} 3876	102
$\{cc\}[\bar{q}_k\bar{s}]$	1^+	0	3764[c]	$m(\{cc\}s)+392$	4156	$D^+D_s^{*-}$ 3977	179
$\{cc\}\{\bar{q}_k\bar{q}_l\}$	$0^+,1^+,2^+$	1	3663	$m(\{cc\}u)+526$	4146,4167,4210	D^+D^0, D^+D^{*0} 3734,3876	412,292,476
$[bc][\bar{u}\,\bar{d}]$	0^+	0	6914	$m([bc]u)+315$	7229	B^-D^+/B^0D^0 7146	83
$[bc][\bar{q}_k\bar{s}]$	0^+	0	7010[d]	$m([bc]s)+392$	7406	B_sD 7236	170
$[bc]\{\bar{q}_k\bar{q}_l\}$	1^+	1	6914	$m([bc]u)+526$	7439	B^*D/BD^* 7190/7290	249
$\{bc\}[\bar{u}\,\bar{d}]$	1^+	0	6957	$m(\{bc\}u)+315$	7272	B^*D/BD^* 7190/7290	82
$\{bc\}[\bar{q}_k\bar{s}]$	1^+	0	7053[d]	$m(\{bc\}s)+392$	7445	DB_s^* 7282	163
$\{bc\}\{\bar{q}_k\bar{q}_l\}$	$0^+,1^+,2^+$	1	6957	$m(\{bc\}u)+526$	7461,7472,7493	BD/B^*D 7146/7190	317,282,349
$\{bb\}[\bar{u}\,\bar{d}]$	1^+	0	10 176	$m(\{bb\}u)+306$	10 482	$B^-\bar{B}^{*0}$ 10 603	$\boxed{-121}$
$\{bb\}[\bar{q}_k\bar{s}]$	1^+	0	10 252[c]	$m(\{bb\}s)+391$	10 643	$\bar{B}\bar{B}_s^*/\bar{B}_s\bar{B}^*$ 10 695/10 691	$\boxed{-48}$
$\{bb\}\{\bar{q}_k\bar{q}_l\}$	$0^+,1^+,2^+$	1	10 176	$m(\{bb\}u)+512$	10 674,10 681,10 695	B^-B^0, B^-B^{*0} 10 559,10 603	115,78,136

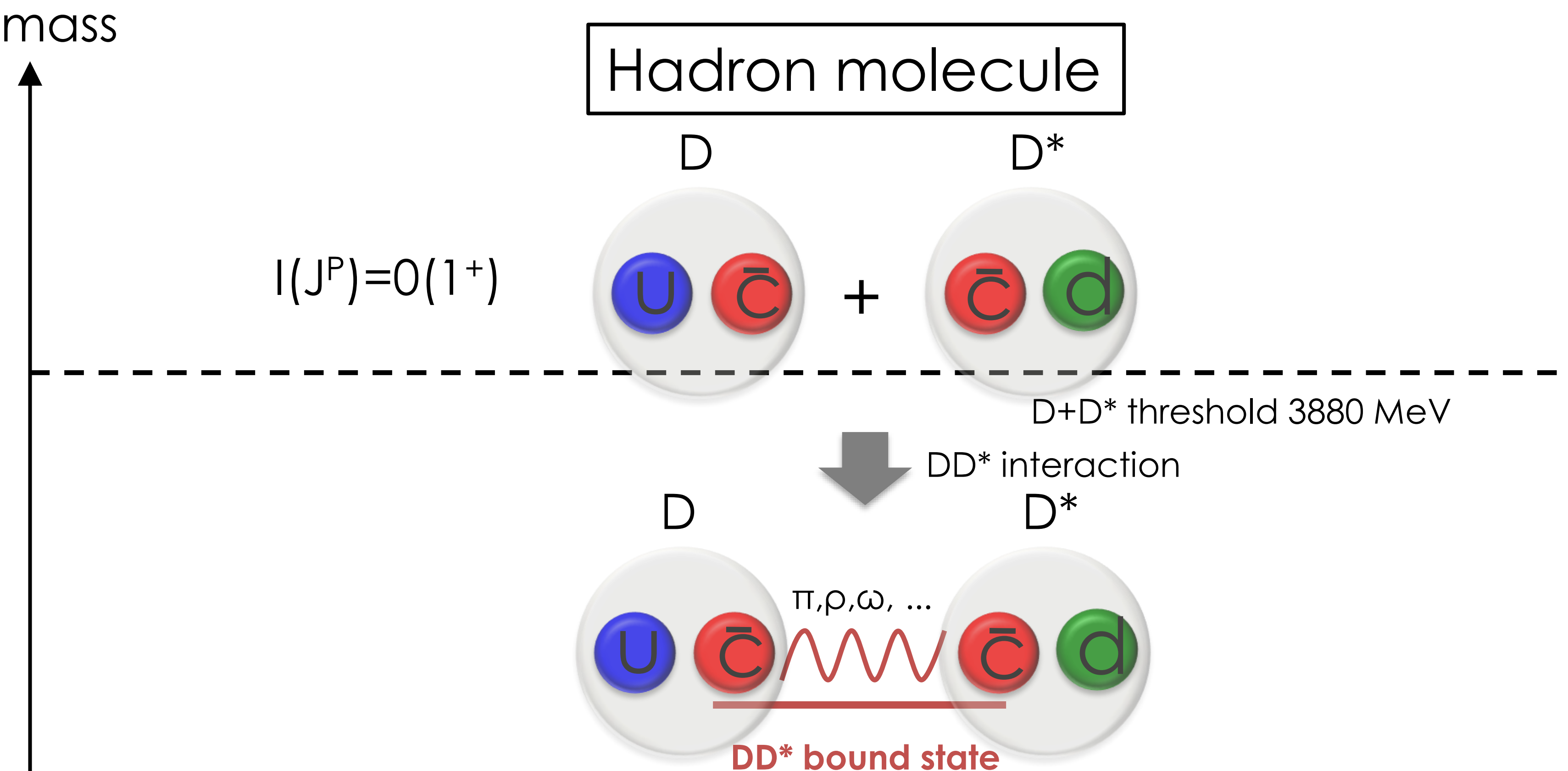
No bound state for T_{cc}?

3. Heavy exotic hadrons -X, Y, Z hadrons-

T_{cc}

Double charm tetraquark

A.V. Manohar, M. B. Wise,
Nucl. Phys. B399, 17 (1993)



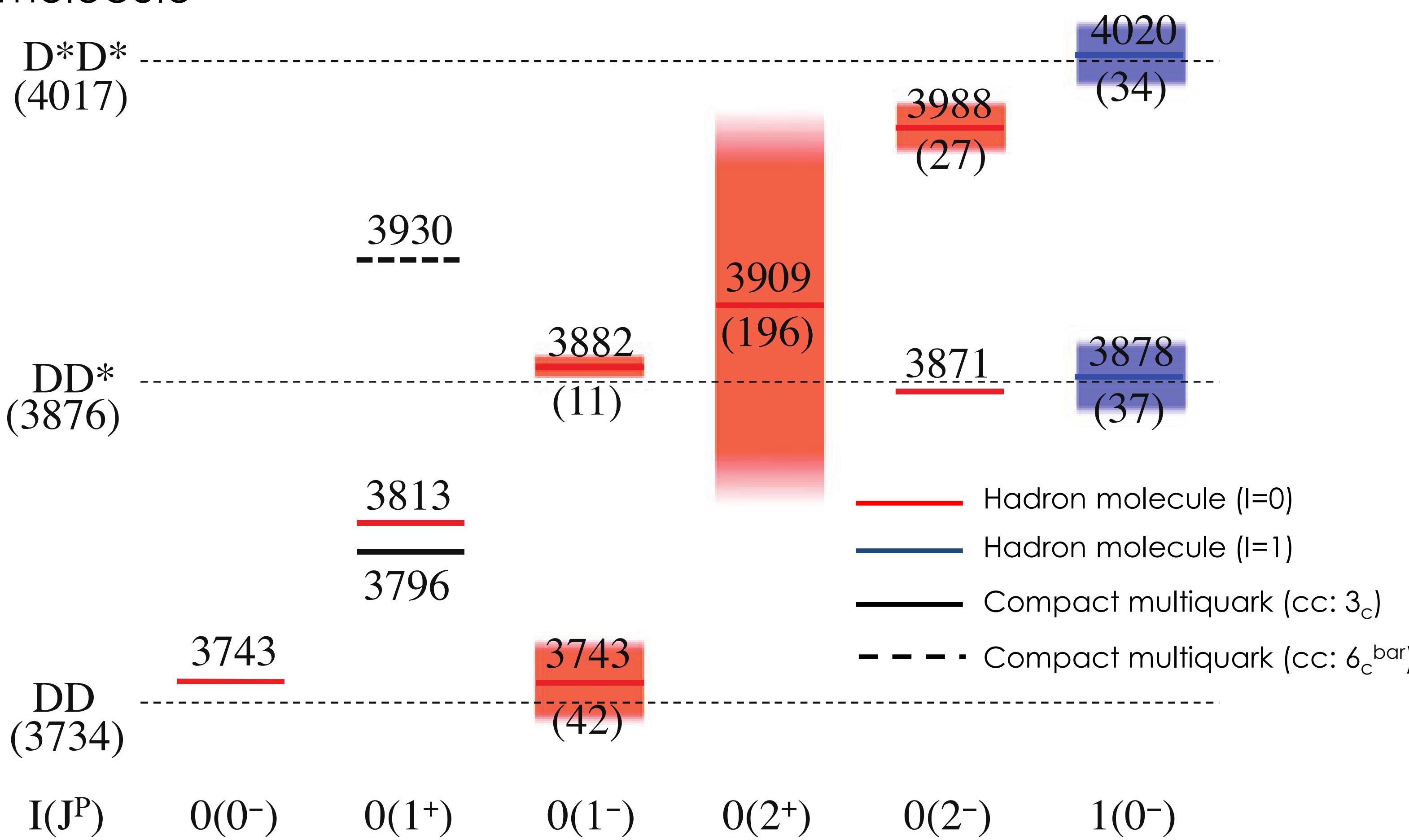
Basis: (HQS) $\frac{1}{\sqrt{2}}(DD^* - D^*D)(^3\mathrm{S}_1)$ $\frac{1}{\sqrt{2}}(DD^* - D^*D)(^3\mathrm{D}_1)$ $D^*D^*(^3\mathrm{S}_1)$ $D^*D^*(^3\mathrm{D}_1)$

HQS & Tensor are important, à la $Z_b \approx B\bar{B}, B^*\bar{B}^*, B^*\bar{B}^*$.

3. Heavy exotic hadrons -X, Y, Z hadrons- T_{cc}

Double charm tetraquark

Hadronic molecule



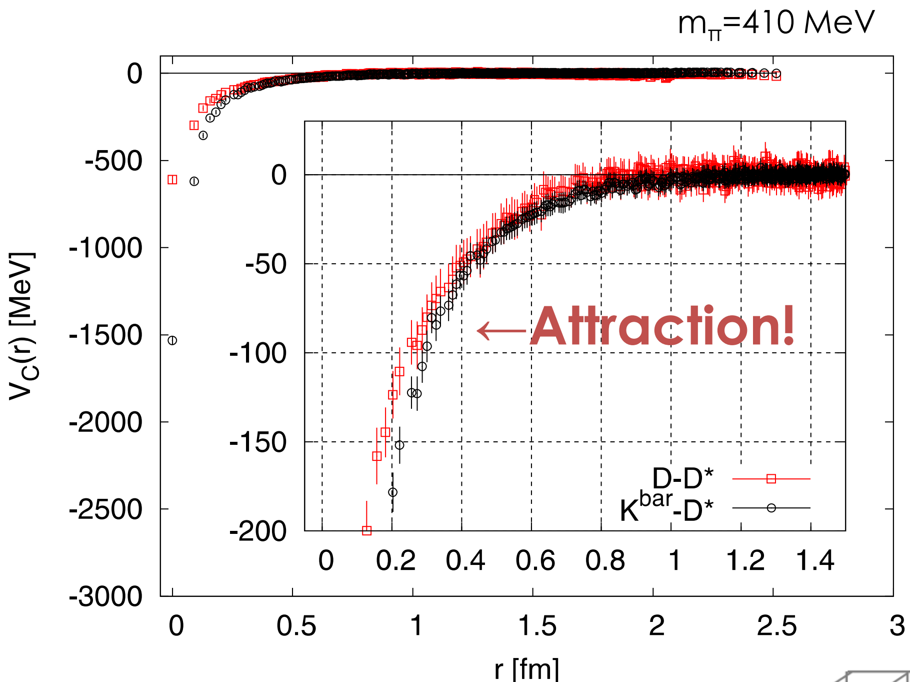
S. Ohkoda, Y. Yamaguchi, S.Y., K. Sudoh, A. Hosaka, Phys. Rev. D86, 034019 (2012)

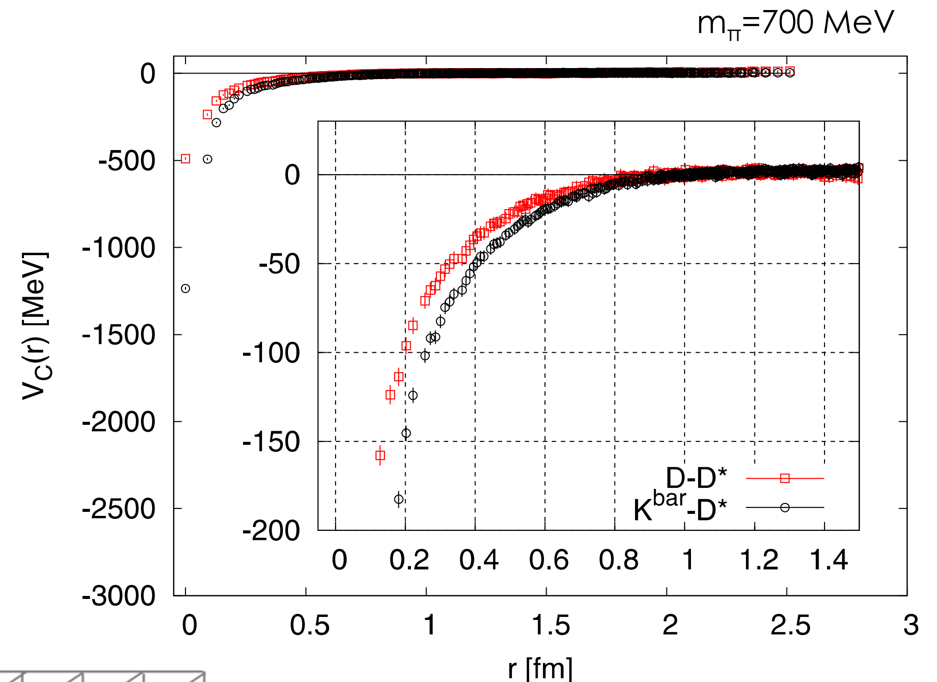
3. Heavy exotic hadrons -X, Y, Z hadrons- T_{cc}

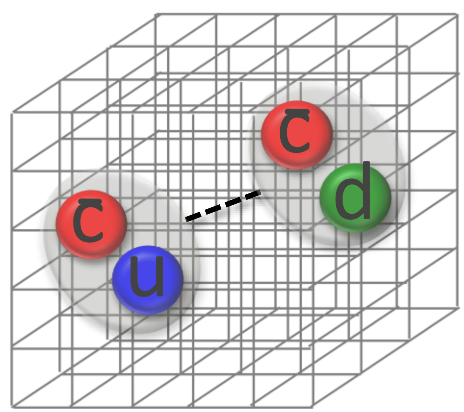
Double charm tetraquark

Lattice QCD simulation

Y. Ikeda (HAL Collaboration), PLB729, 85 (2014)





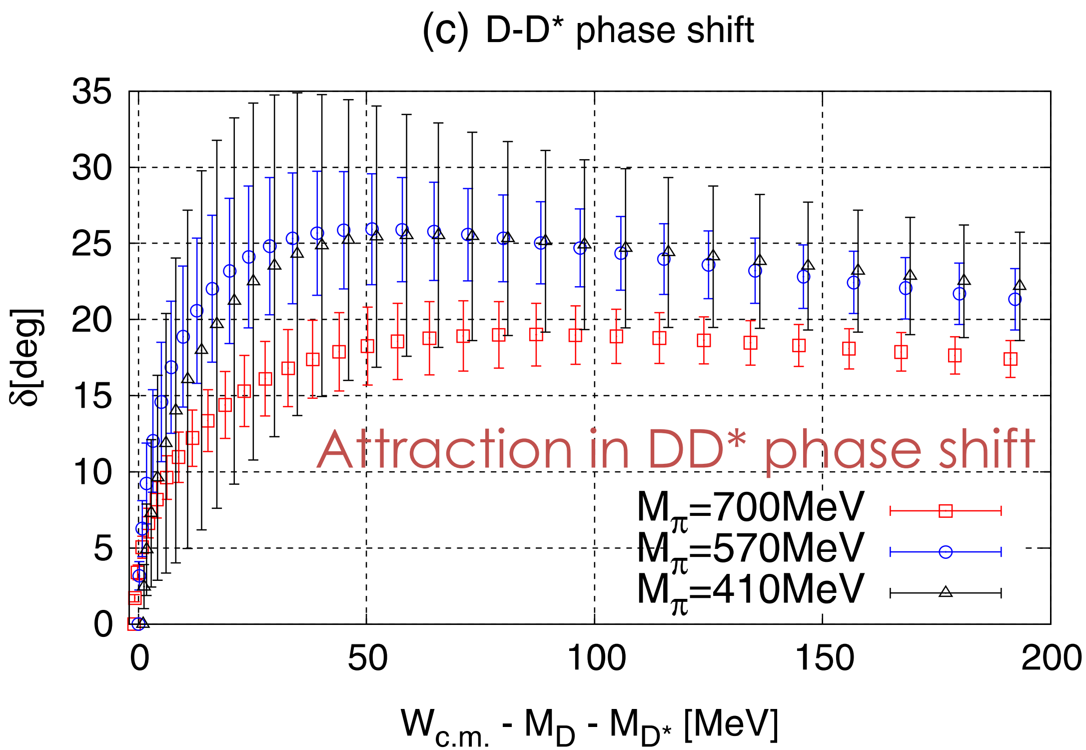


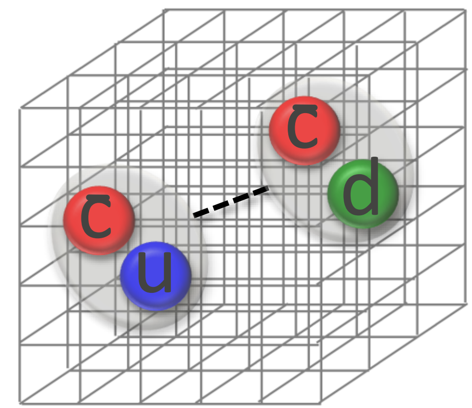
3. Heavy exotic hadrons -X, Y, Z hadrons- T_{cc}

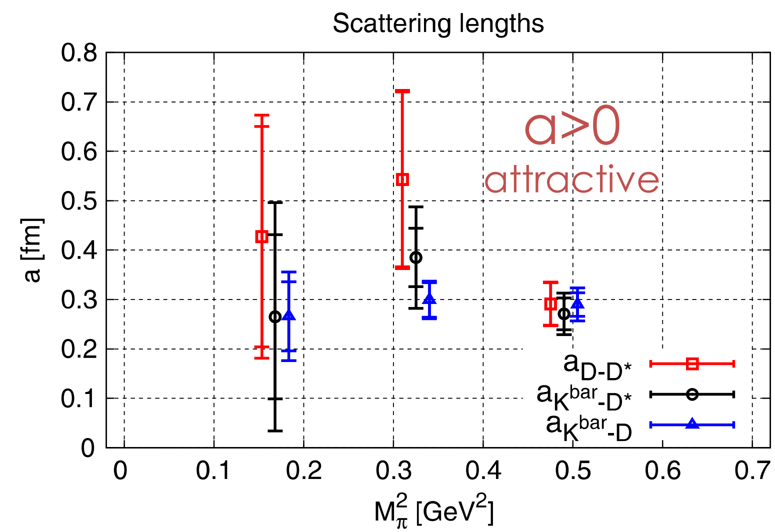
Double charm tetraquark

Lattice QCD simulation

Y. Ikeda (HAL Collaboration), PLB729, 85 (2014)







→ However, the attraction is *not* sufficiently strong to make a bound state....
(Due to the large pion mass?)

3. Heavy exotic hadrons -X, Y, Z hadrons- T_{cc}

Double charm tetraquark

T_{cc} really exists in our world!

2022

nature physics

LETTERS

https://doi.org/10.1038/s41567-022-01614-y

Check for updates

OPEN

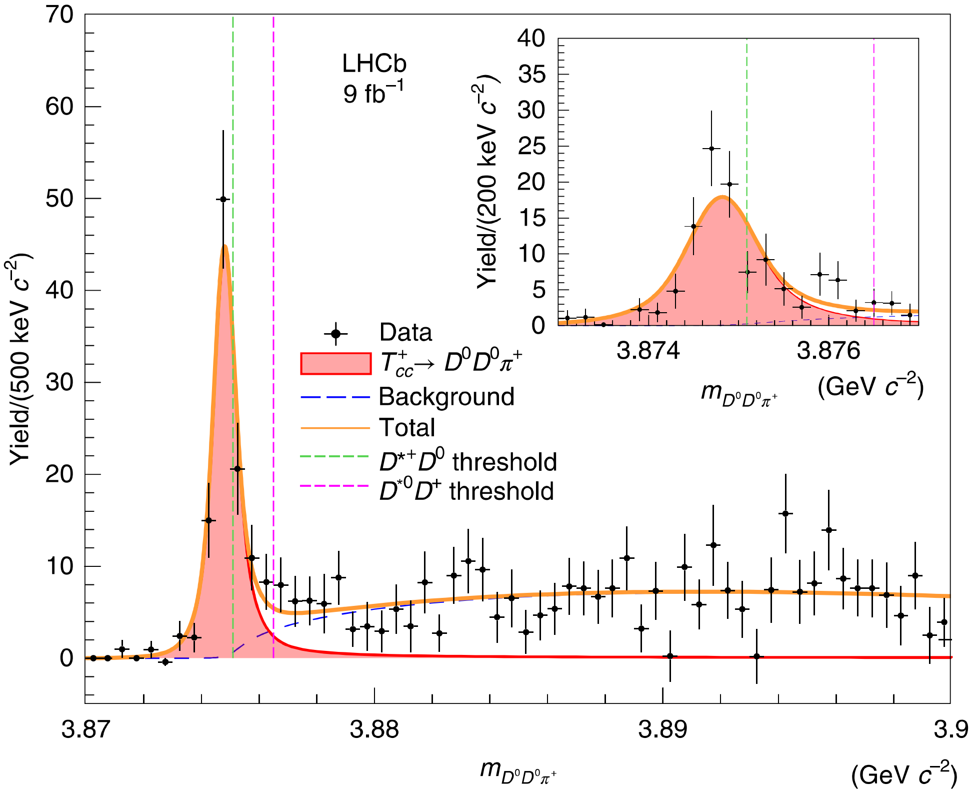
Observation of an exotic narrow doubly charmed tetraquark

LHCb Collaboration*

3. Heavy exotic hadrons -X, Y, Z hadrons- T_{cc}

Double charm tetraquark

LHCb, Nature Phys. 18 (2022) 751, Nature Commun. 13 (2022) 3351



Bound state below $D^{*+}D^0$ threshold,

$$\delta m_{\mathrm{BW}} = -273 \pm 61 \pm 5\,^{+11}_{-14}\ \underline{\mathrm{keV}\,c^{-2}},$$

$$\Gamma_{\mathrm{BW}} = 410 \pm 165 \pm 43\,^{+18}_{-38}\ \underline{\mathrm{keV}},$$

Very very shallow (keV)!

Anyway, we should explore more on T_{cc} and related states.

Very hot topic ongoing.

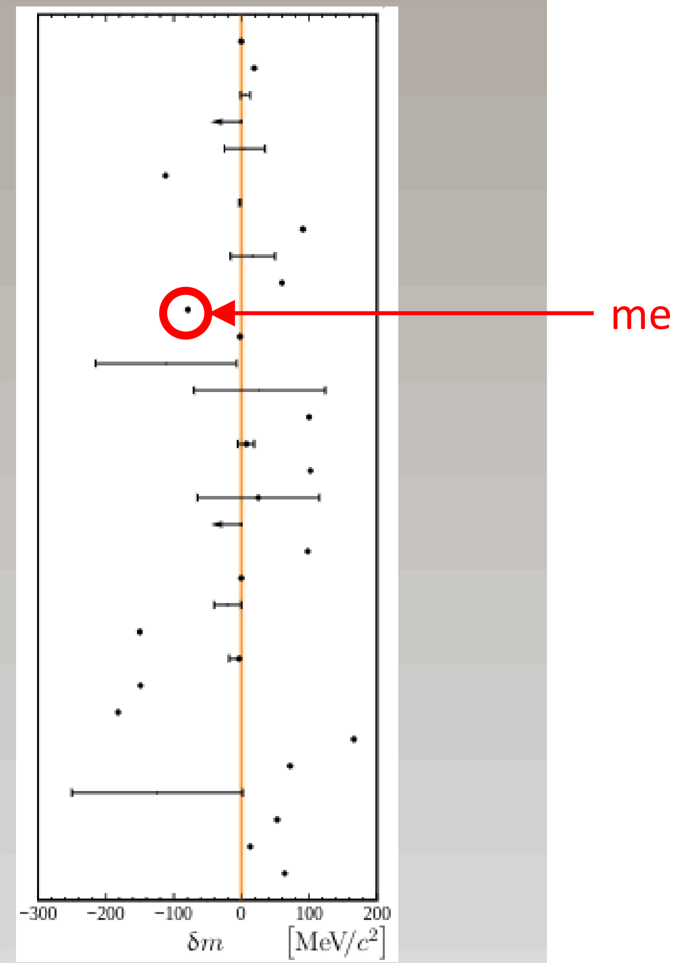
3. Heavy exotic hadrons -X, Y, Z hadrons- T_{cc}

Double charm tetraquark

Ivan Polyakov (2021)

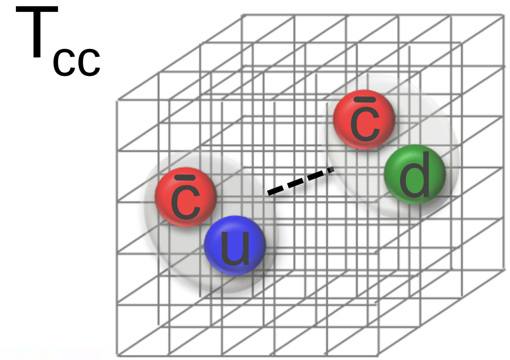
Theory predictions

Reference		Year	$\delta' m$ [MeV/c^2]
J. Carlson, L. Heller and J. A. Tjon	[36]	1987	~ 0
B. Silvestre-Brac and C. Semay	[37]	1993	$+19$
C. Semay and B. Silvestre-Brac	[38]	1994	$[-1, +13]$
S. Pepin, F. Stancu, M. Genovese and J. M. Richard	[39]	1996	< 0
B. A. Gelman and S. Nussinov	[40]	2002	$[-25, +35]$
J. Vijande, F. Fernandez, A. Valcarce, A. and B. Silvestre-Brac	[41]	2003	-112
D. Janc and M. Rosina	[42]	2004	$[-3, -1]$
F. Navarra, M. Nielsen and S. H. Lee	[43]	2007	$+91$
J. Vijande, E. Weissman, A. Valcarce	[44]	2007	$[-16, +50]$
D. Ebert, R. N. Faustov, V. O. Galkin and W. Lucha	[45]	2007	$+60$
S. H. Lee and S. Yasui	[46]	2009	-79
Y. Yang, C. Deng, J. Ping and T. Goldman	[47]	2009	-1.8
G.-Q. Feng, X.-H. Guo and B.-S. Zou	[48]	2013	-215
Y. Ikeda, B. Charron, S. Aoki, T. Doi, T. Hatsuda, T. Inoue, N. Ishii, K. Murano, H. Nemura and K. Sasaki	[49]	2013	$[-70, +124]$
S.-Q. Luo, K. Chen, X. Liu, Y.-R. Liu and S.-L. Zhu	[50]	2017	$+100$
M. Karliner and J. Rosner	[51]	2017	$7 \pm 12 \to 1$
E. J. Eichten and C. Quigg	[52]	2017	$+102$
Z. G. Wang	[53]	2017	$+25 \pm 90$
G. K. C. Cheung, C. E. Thomas, J. J. Dudek and R. G. Edwards	[54]	2017	$\lesssim 0$
W. Park, S. Noh and S. H. Lee	[55]	2018	$+98$
A. Francis, R. J. Hudspith, R. Lewis and K. Maltman	[56]	2018	~ 0
P. Junnarkar, N. Mathur and M. Padmanath	[57]	2018	$[-40, 0]$
C. Deng, H. Chen and J. Ping	[58]	2018	-150
M.-Z. Liu, T.-W. Wu, V. Pavon Valderrama, J.-J. Xie and L.-S. Geng	[59]	2019	-3^{+4}_{-15}
G. Yang, J. Ping and J. Segovia	[60]	2019	-149
Y. Tan, W. Lu and J. Ping	[61]	2020	-182
Q.-F. Lü, D.-Y. Chen and Y.-B. Dong	[62]	2020	$+166$
E. Braaten, L.-P. He and A. Mohapatra	[63]	2020	$+72$
D. Gao, D. Jia, Y.-J. Sun, Z. Zhang, W.-N. Liu and Q. Mei	[64]	2020	$[-250, +2]$
J.-B. Cheng, S.-Y. Li, Y.-R. Liu, Z.-G. Si, T. Yao	[65]	2020	$+53$
S. Noh, W. Park and S. H. Lee	[66]	2021	$+13$
R. N. Faustov, V. O. Galkin and E. M. Savchenko	[67]	2021	$+64$



https://indico.desy.de/event/28202/contributions/105627/attachments/67806/84639/EPS-HEP_2021_Polyakov_v5.pdf

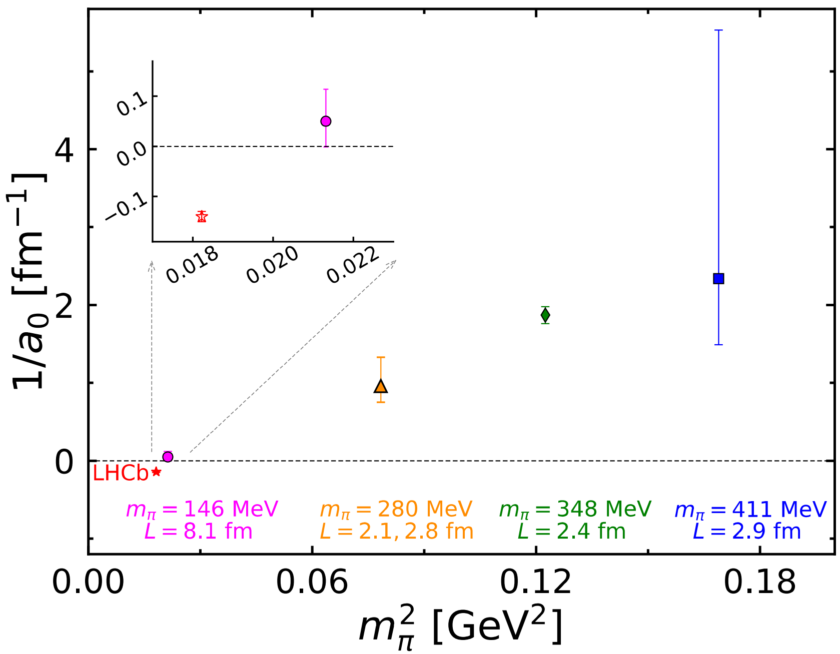
0. Introduction to exotic hadrons

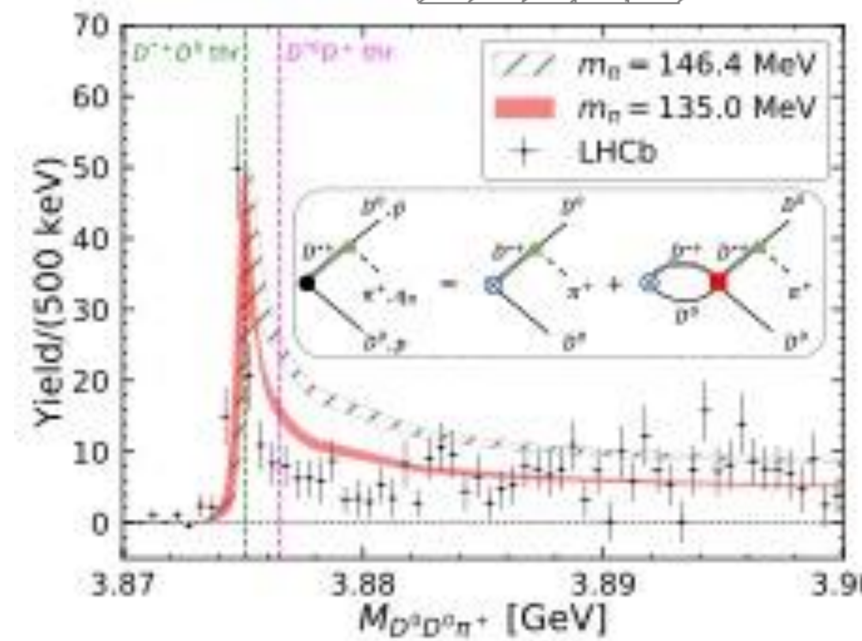


Lattice QCD study of T_{cc} near physical point

Y. Ikeda, et al. (HAL Collaboration), PLB729, 85 (2014) : $m_\pi = 410, 700$ MeV

Y. Lyu, et al. (HAL Collaboration), 2302.04505: $m_\pi = 135$ MeV (near physical point)





$E_{\text{pole}} = -45$ keV

“virtual state”

Mass spectrum

Cf. LHCb (2022): below $D^{*+}D^0$ threshold

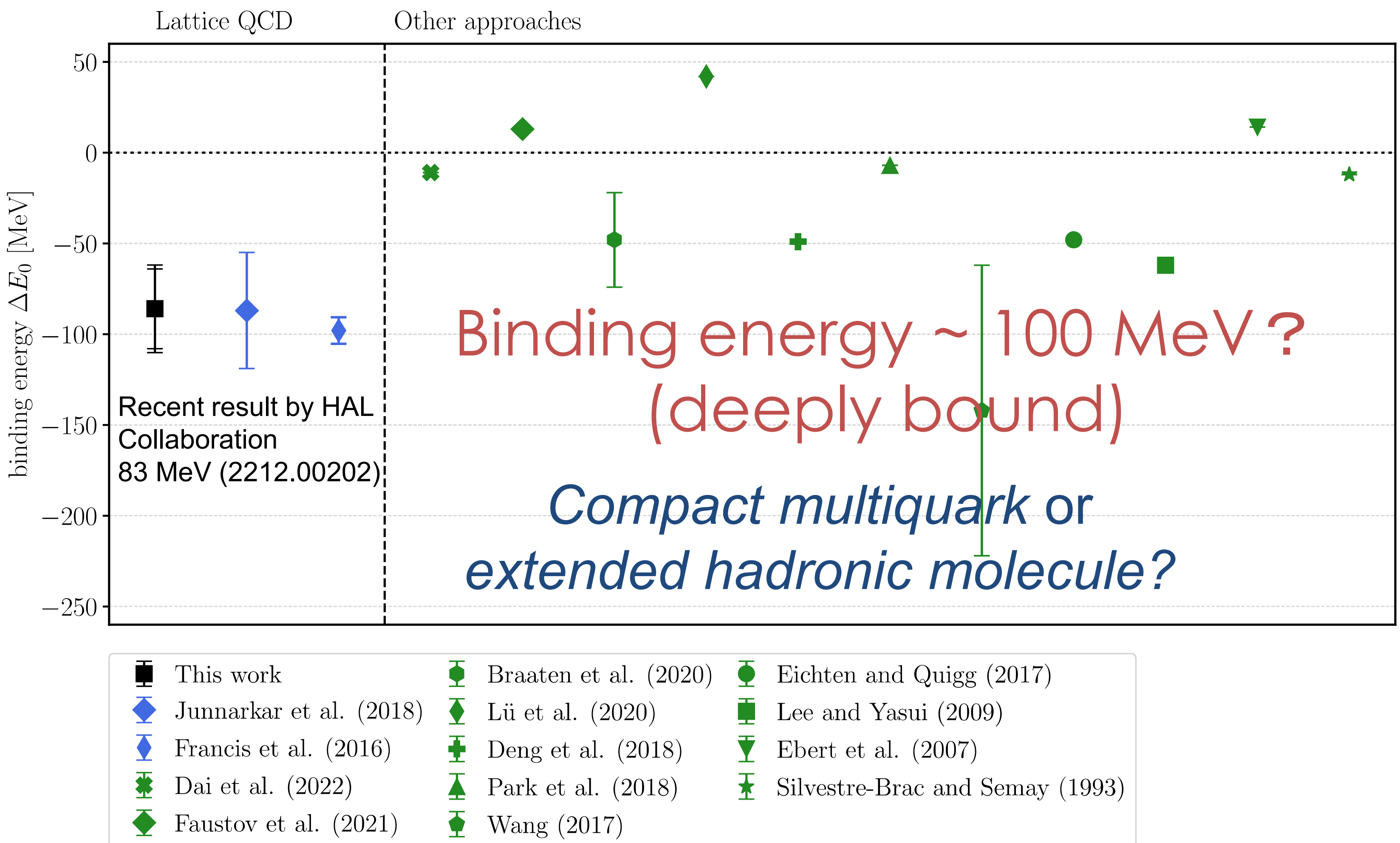
$$\delta m_{\text{BW}} = -\underline{273} \pm 61 \pm 5\,^{+11}_{-14}\ \underline{\text{keV}}\,c^{-2},$$

$$\Gamma_{\text{BW}} = \underline{410} \pm 165 \pm 43\,^{+18}_{-38}\ \underline{\text{keV}},$$

3. Heavy exotic hadrons -X, Y, Z hadrons-

Recent lattice QCD study on T_{bb}
Meinel, Pflaumer, Wagner,
Phys. Rev. D106, 034507 (2022)

T_{bb}
*Double **bottom** tetraquark*



Future experimental studies

3. Heavy exotic hadrons -X, Y, Z hadrons-

Fully

Charm

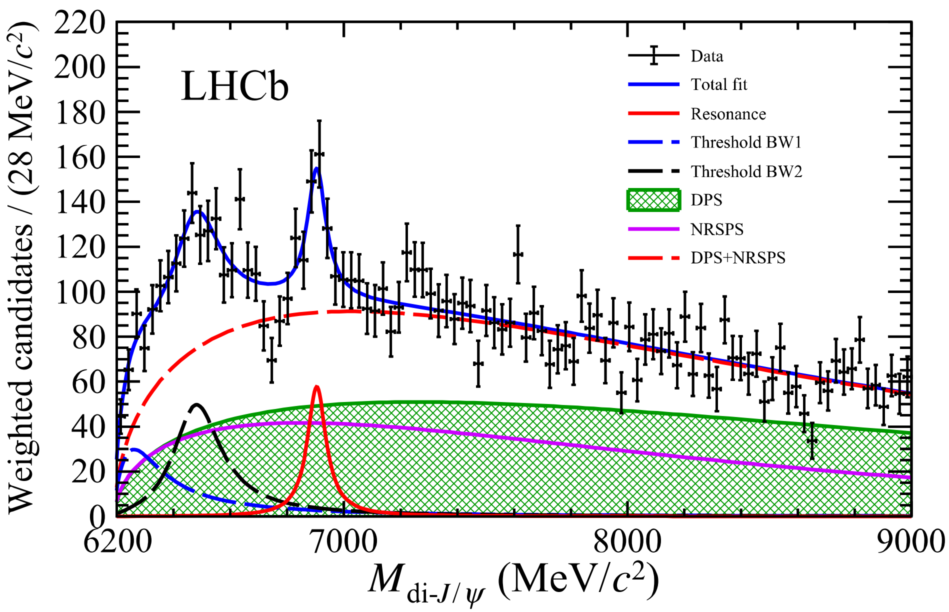
Tetraquark

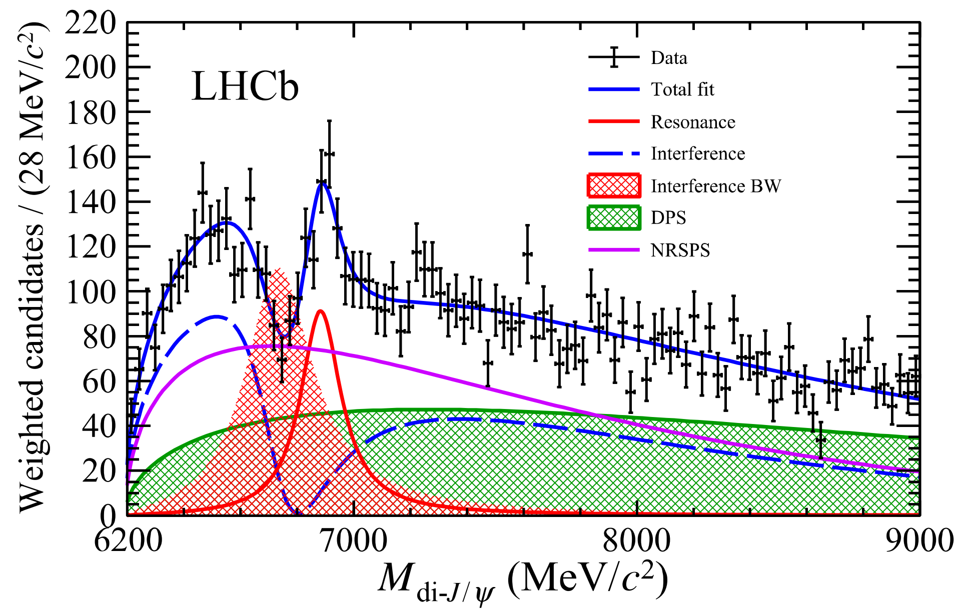
3. Heavy exotic hadrons -X, Y, Z hadrons-

LHCb, Science Bulletin 65 (2020) 1983

$X_{cc}(6900)$ *Four-charm* resonance

$cc\bar{c}\bar{c}$





Assuming no interference…

$$m[X(6900)] = 6905 \pm 11 \pm 7\ \mathrm{MeV}/c^2$$
$$\Gamma[X(6900)] = 80 \pm 19 \pm 33\ \mathrm{MeV}$$

Assuming interference…

$$m[X(6900)] = 6886 \pm 11 \pm 11\ \mathrm{MeV}/c^2$$
$$\Gamma[X(6900)] = 168 \pm 33 \pm 69\ \mathrm{MeV}$$

Compact state or extended state？

3. Heavy exotic hadrons -X, Y, Z hadrons-

Brief summary of X, Y, Z and P_c

A. Esposito, A. Pilloni, A. D. Plolsa, Phys. Rep. 668, 1 (2017)

State	M (MeV)	Γ (MeV)	J^{PC}	Process (mode)	Experiment (#σ)
$X(3872)$	3871.69 ± 0.17	<1.2	1^{++}	See Tables 11, 12	See Tables 11, 12
$Z_c(3900)^+$	3888.4 ± 1.6	27.9 ± 2.7	1^{+-}	$Y(4260) \to \pi^-(D\bar{D}^*)^+$	BES III [163,257] (>10)
				$Y(4260) \to \pi^-(\pi^+J/\psi)$	BES III [160] (8), Belle [159] (5.2)
					NU group [258] (3.5)
$Z_c(3900)^0$	3893.6 ± 3.7	31 ± 10	1^{+-}	$Y(4260) \to \pi^0(D\bar{D}^*)^0$	BES III [259] (10)
				$Y(4260) \to \pi^0(\pi^0J/\psi)$	BES III [160] (10.4)
					NU group [258] (5.7)
$Z_c'(4020)^+$	4023.9 ± 2.4	10 ± 6	1^{+-}	$e^+e^- \to \pi^-(\pi^+h_c)$	BES III [165] (8.9)
				$e^+e^- \to \pi^-(D^*\bar{D}^*)^+$	BES III [260] (10)
$Z_c'(4020)^0$	4024.5 ± 3.1	$23 \pm 6 \pm 1$	1^{+-}	$e^+e^- \to \pi^0(\pi^0h_c)$	BES III [261] (5)
				$e^+e^- \to \pi^0(D^*\bar{D}^*)^0$	BES III [262] (5.9)
$\chi_{c0}(3915)$	3918.4 ± 1.9	20 ± 5	0^{++}	$B \to K(\omega J/\psi)$	Belle [263] (8), BaBar [264,265] (19)
				$e^+e^- \to e^+e^-(\omega J/\psi)$	Belle [266] (7.7), BaBar [267] (7.6)
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	(0^{-+})	$e^+e^- \to J/\psi\,(D\bar{D}^*)$	Belle [268,269] (6)
$(Y(4008)?)$	3891 ± 42	255 ± 42	1^{--}	$e^+e^- \to (\pi^+\pi^-J/\psi)$	Belle [159,270] (7.4)
	3813^{+62}_{-97}	477^{+78}_{-65}			BES III [111] (np)
$Z(4050)^+$	4051^{+24}_{-43}	82^{+51}_{-55}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle [271] (5.0), BaBar [272] (1.1)
$(Z(4050)^+?)$	4054 ± 3	45 ± 13	$?^{?-}$	$e^+e^- \to (\psi(2S)\,\pi^+)\,\pi^-$	Belle [273] (3.5)
$X(4140)$	$4146.5^{+6.4}_{-5.3}$	83^{+30}_{-25}	1^{++}	$B^+ \to (J/\psi\phi)K^+$	LHCb [139,140] (8.4), see Table 16
$X(4160)$	4156^{+29}_{-25}	139^{+113}_{-65}	(0^{-+})	$e^+e^- \to J/\psi\,(D^*\bar{D}^*)$	Belle [269] (5.5)
$Z(4200)^+$	4196^{+35}_{-30}	370^{+99}_{-110}	1^{+-}	$\bar{B}^0 \to K^-(\pi^+J/\psi)$	Belle [274] (7.2)
$Y(4220)$	4218.4 ± 4.1	66.0 ± 9.0	1^{--}	$e^+e^- \to (\pi^+\pi^-h_c)$	BES III [111] (np)
$Y(4230)$	4230 ± 8	38 ± 12	1^{--}	$e^+e^- \to (\chi_{c0}\omega)$	BES III [275] (>9)
$Z(4250)^+$	4248^{+185}_{-45}	177^{+321}_{-72}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle [271] (5.0), BaBar [272] (2.0)
$Y(4260)$	4251 ± 9	120 ± 12	1^{--}	$e^+e^- \to (\pi\pi J/\psi)$	BaBar [276,277] (8), CLEO [278,279] (11)
					Belle [159,270] (15), BES III [160] (np)
				$e^+e^- \to (f_0(980)J/\psi)$	BaBar [277] (np), Belle [159] (np)
				$e^+e^- \to (\pi^-Z_c(3900)^+)$	BES III [160] (8), Belle [159] (5.2)
				$e^+e^- \to (\gamma\,X(3872))$	BES III [113] (5.3)
$X(4274)$	4273^{+19}_{-9}	56^{+13}_{-16}	1^{++}	$B^+ \to (J/\psi\phi)K^+$	LHCb [139,140] (6.0), see Table 16
$(X(4350)?)$	$4350.6^{+4.6}_{-5.1}$	13^{+18}_{-10}	$0/2^{?+}$	$e^+e^- \to e^+e^-(\phi J/\psi)$	Belle [280] (3.2)
$Y(4360)$	4346.2 ± 6.3	102.3 ± 9.9	1^{--}	$e^+e^- \to (\pi^+\pi^-\psi(2S))$	Belle [273,281] (8), BaBar [282] (np)
$P_c(4380)^+$	4380 ± 30	205 ± 88	$3/2^-$	$\Lambda_b^0 \to K^-(J/\psi\,p)$	LHCb [117] (9)
$Y(4390)$	4391.6 ± 6.4	139.5 ± 16.1	1^{--}	$e^+e^- \to (\pi^+\pi^-h_c)$	BES III [111] (10)

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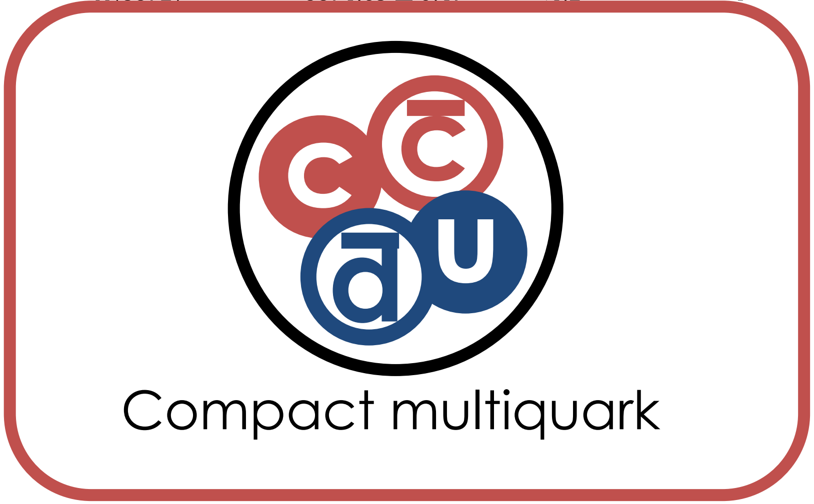
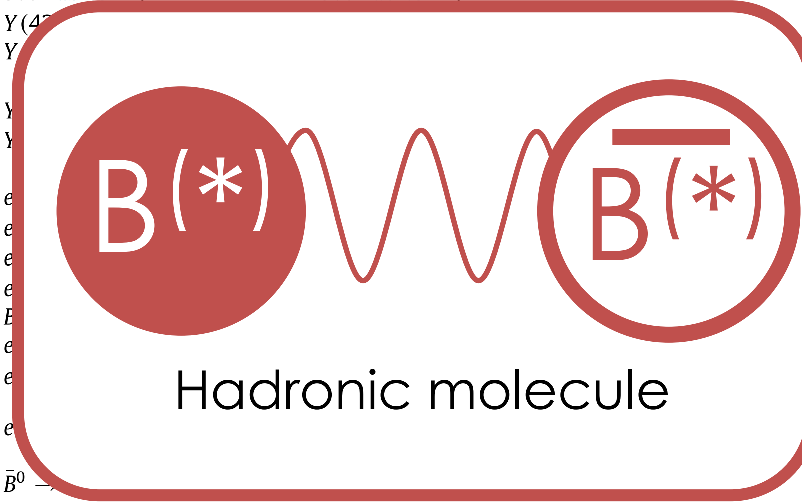
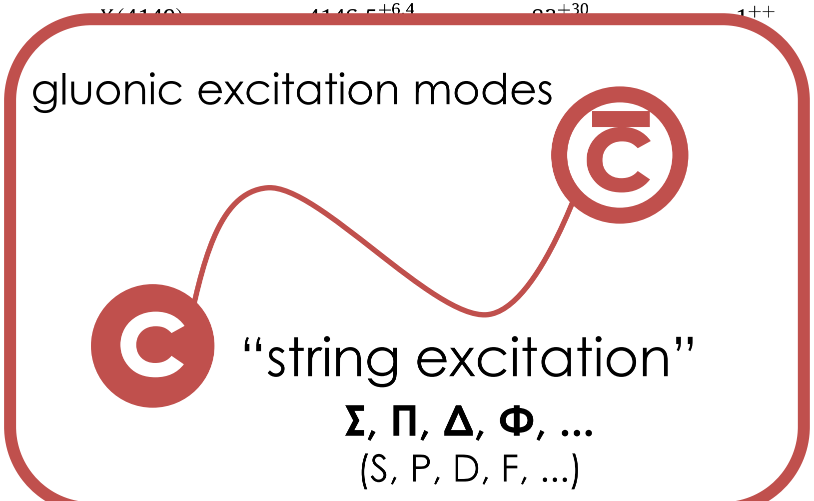
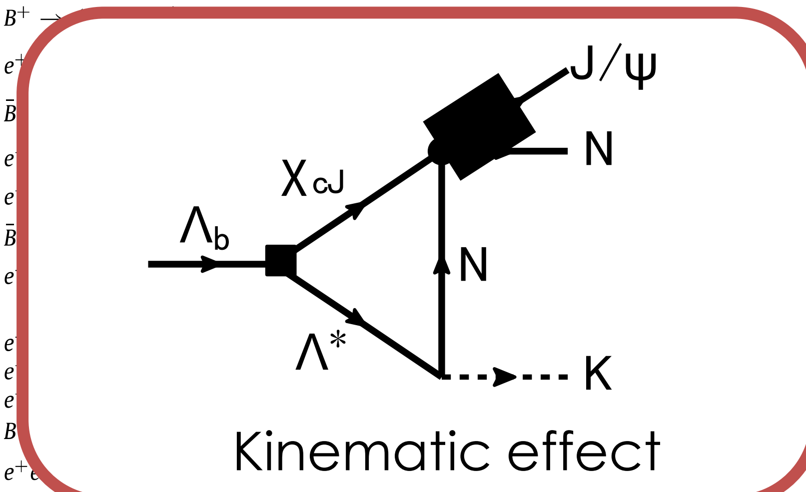








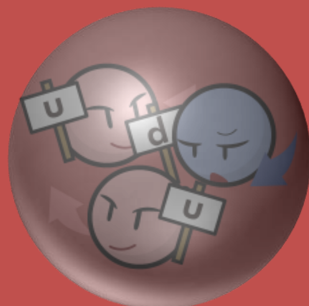
3. Heavy exotic hadrons -X, Y, Z hadrons-

Researches continue....

3. Heavy exotic hadrons -X, Y, Z hadrons-

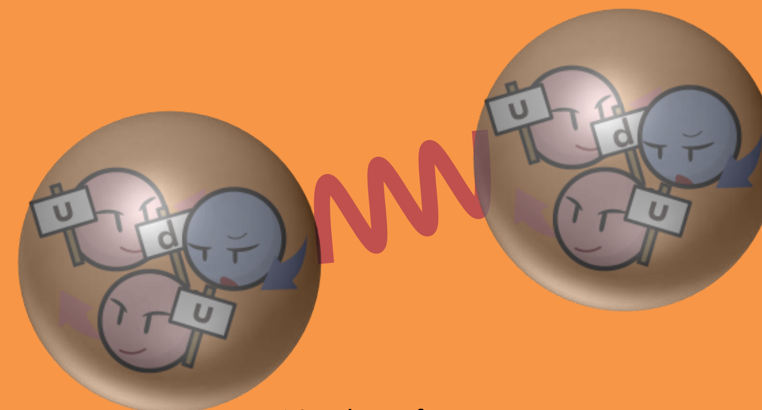
Fundamental 4 Questions in Hadron Physics

① Why are quarks confined?



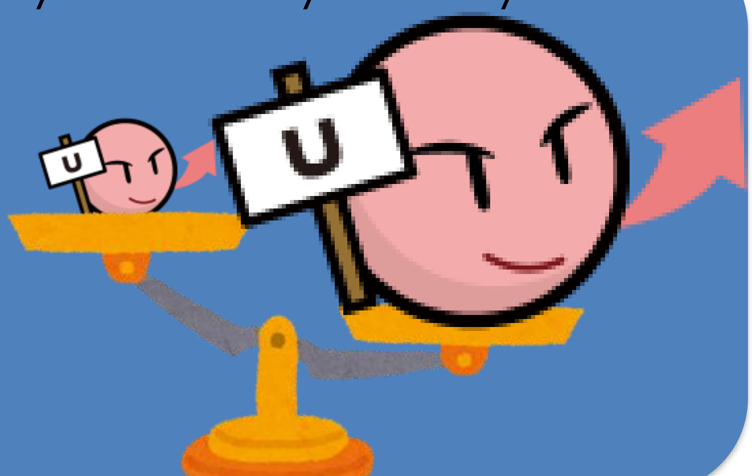
Confined quarks

② What is hadron interaction?

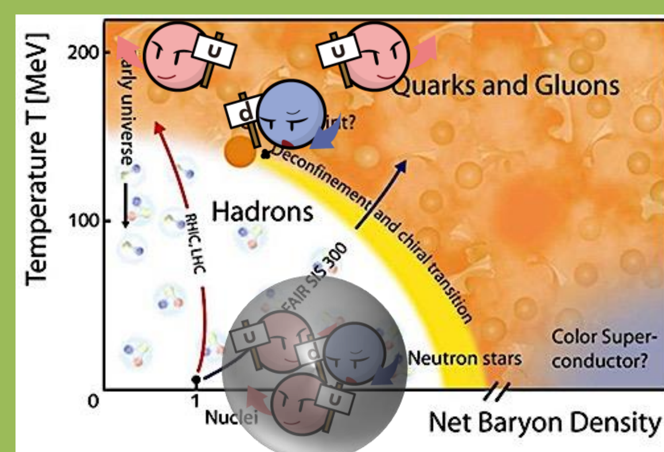


Nuclear force

③ Why is chiral symmetry broken?

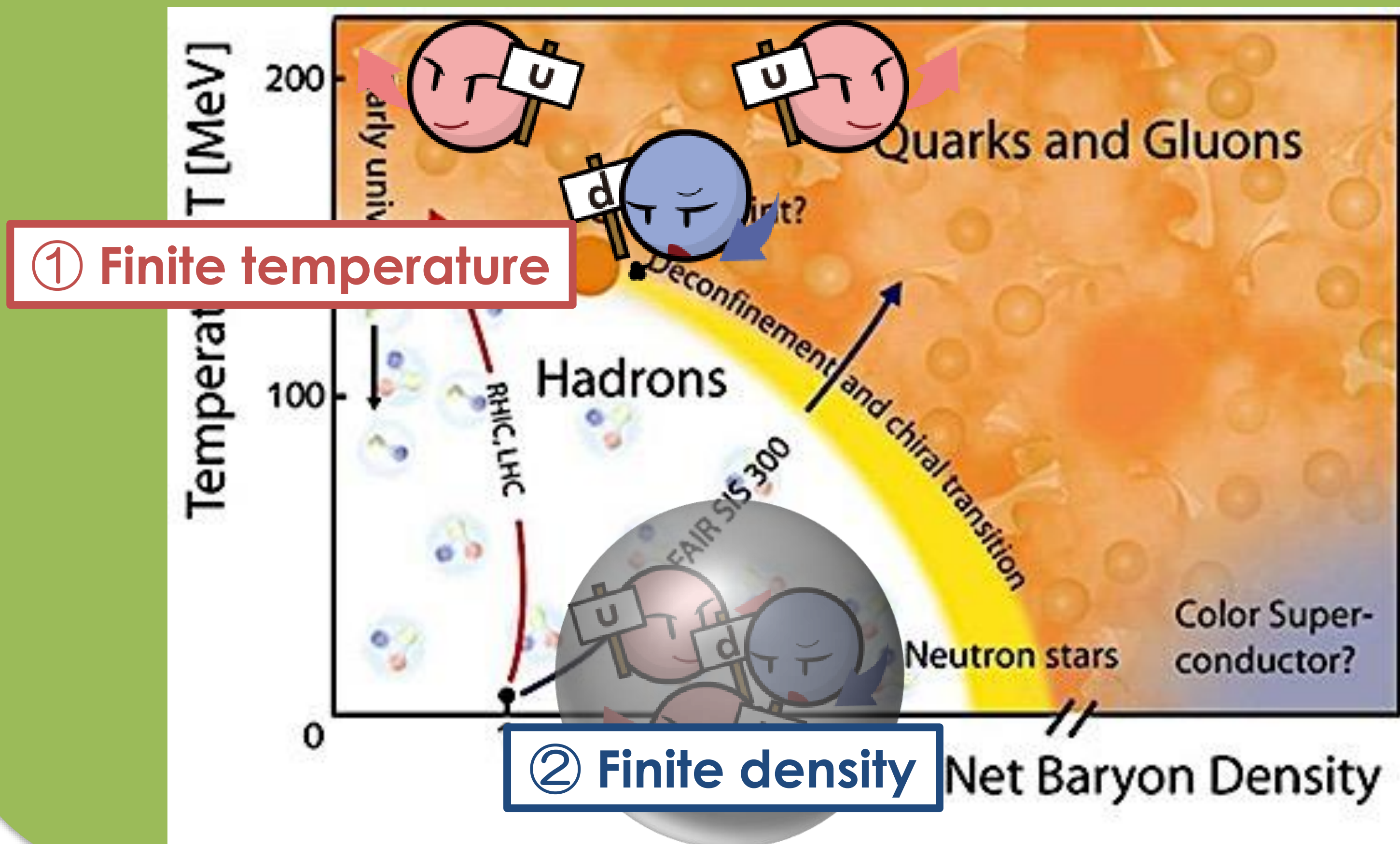


④ What is phase diagram?



3. Heavy exotic hadrons -X, Y, Z hadrons-

④ What is phase diagram?

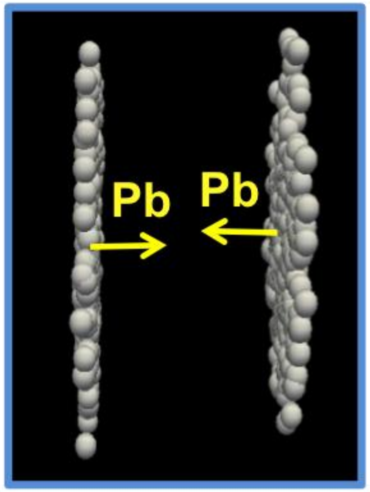


3. Heavy exotic hadrons -X, Y, Z hadrons-

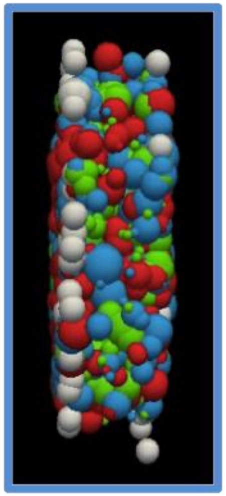
time-evolution in relativistic heavy ion collisions

→ time

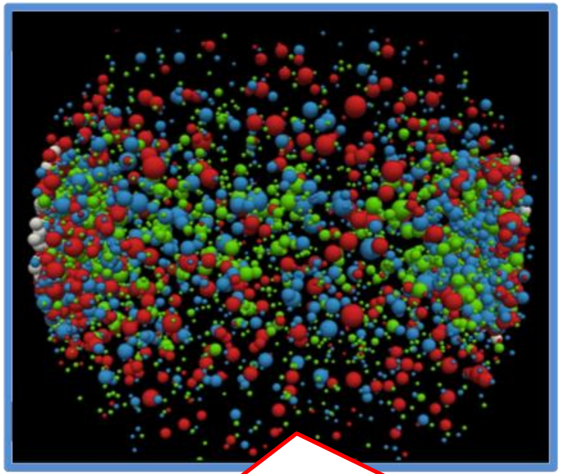
Pre-reaction



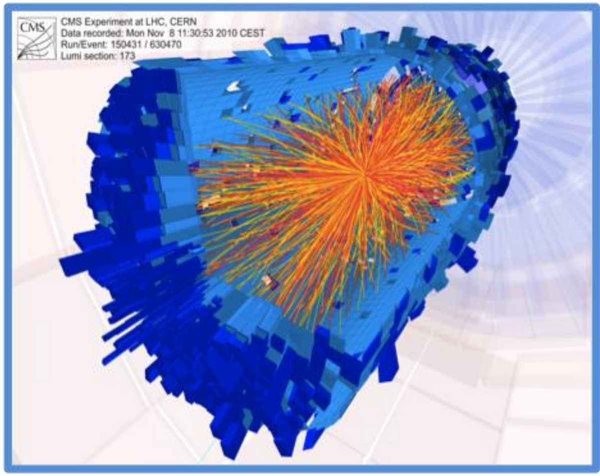
QGP



Hadronization



Detection



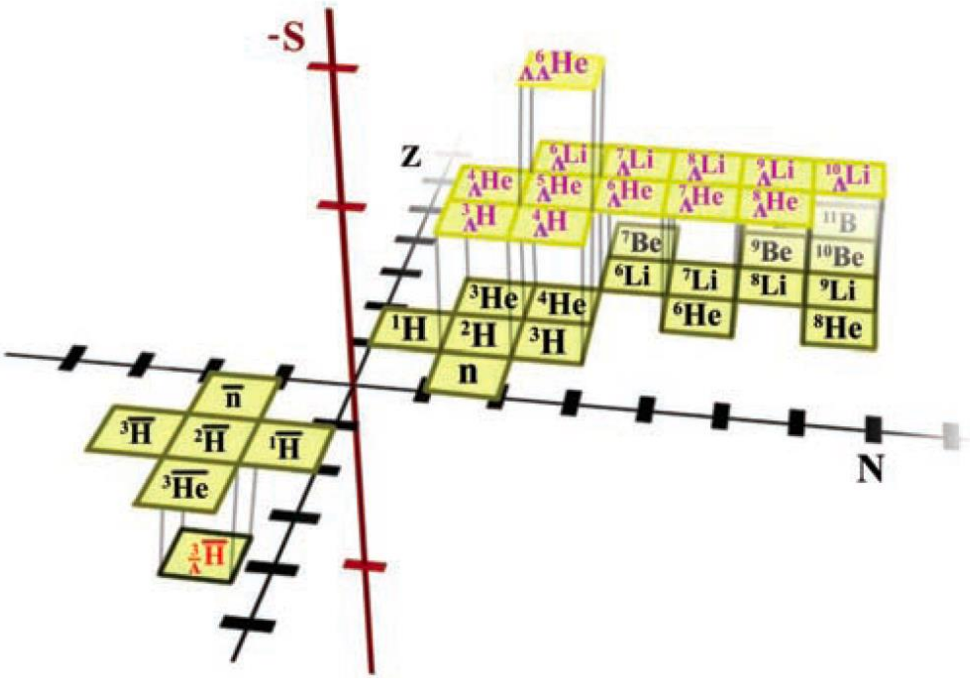
Production of exotic hadrons?

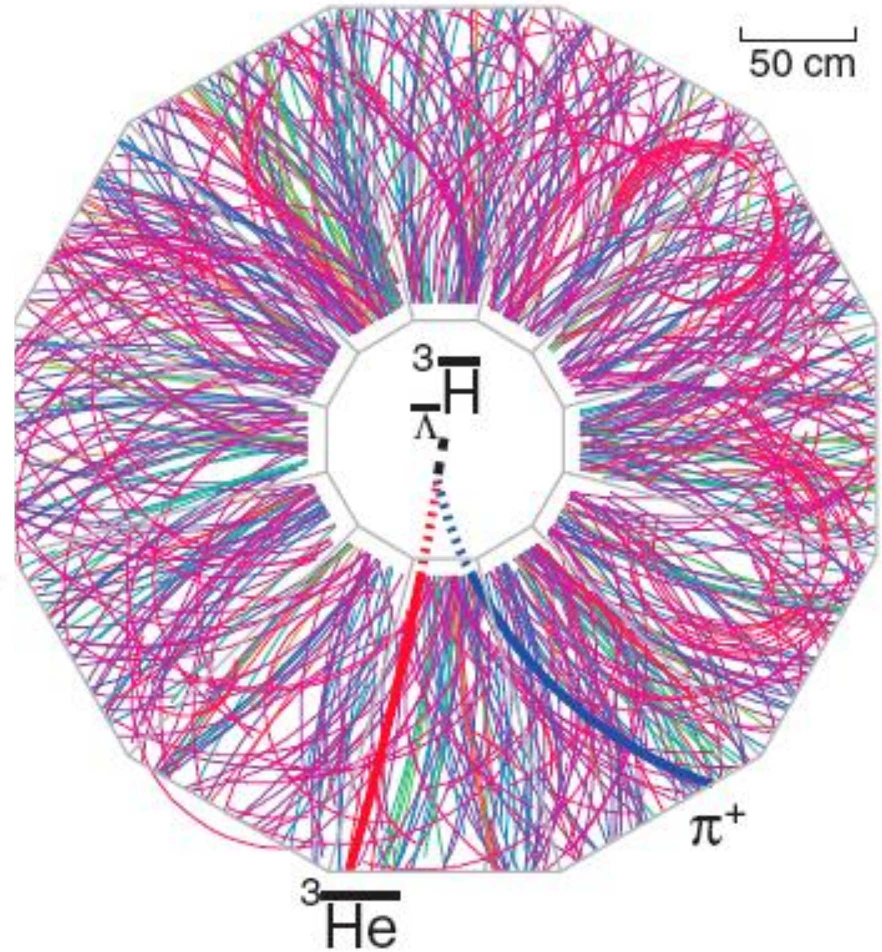
More quark number than e^+e^- and pp → Can we see rare events？
(e.x. 20 cc^{bar} from Pb+Pb, collision in LHC)

What is the hadron production mechanism？
How much are yields of hadrons？

3. Heavy exotic hadrons -X, Y, Z hadrons-

Observation of "anti-hypernuclei"





STAR, Science 328, 58 (2010)

Possible to produce quark many-body systems?

3. Heavy exotic hadrons -X, Y, Z hadrons-

PRL **106,** 212001 (2011) PHYSICAL REVIEW LETTERS week ending 27 MAY 2011

Identifying Multiquark Hadrons from Heavy Ion Collisions

Sungtae Cho,[1] Takenori Furumoto,[2,3] Tetsuo Hyodo,[4] Daisuke Jido,[2] Che Ming Ko,[5] Su Houng Lee,[1,2] Marina Nielsen,[6] Akira Ohnishi,[2] Takayasu Sekihara,[2,7] Shigehiro Yasui,[8] and Koichi Yazaki[2,3]

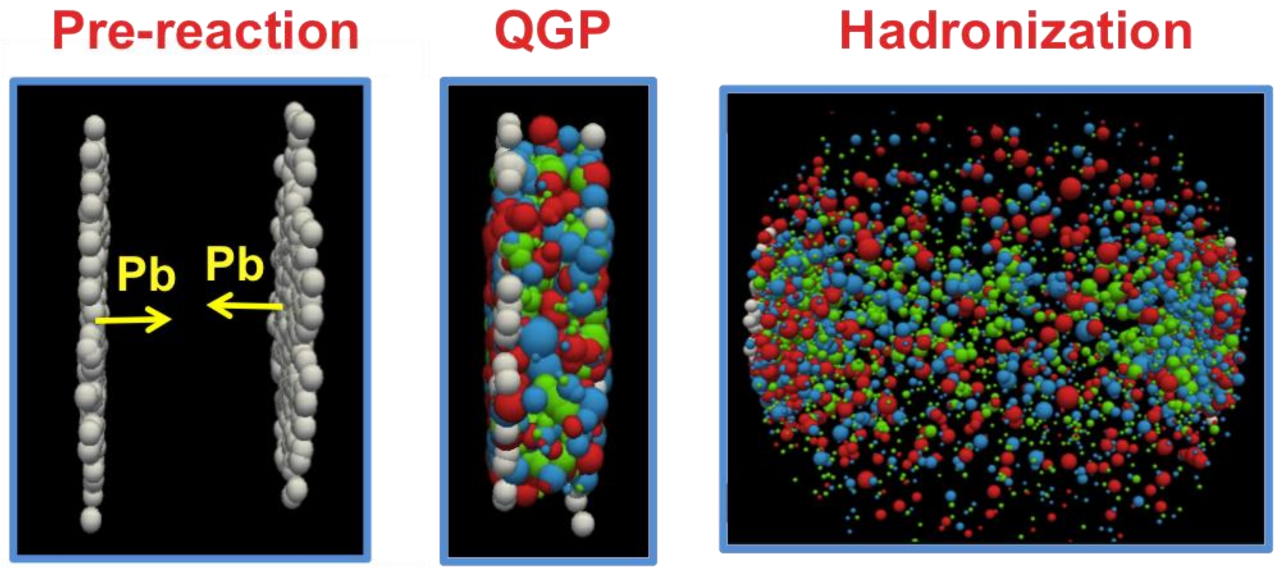
(ExHIC Collaboration)

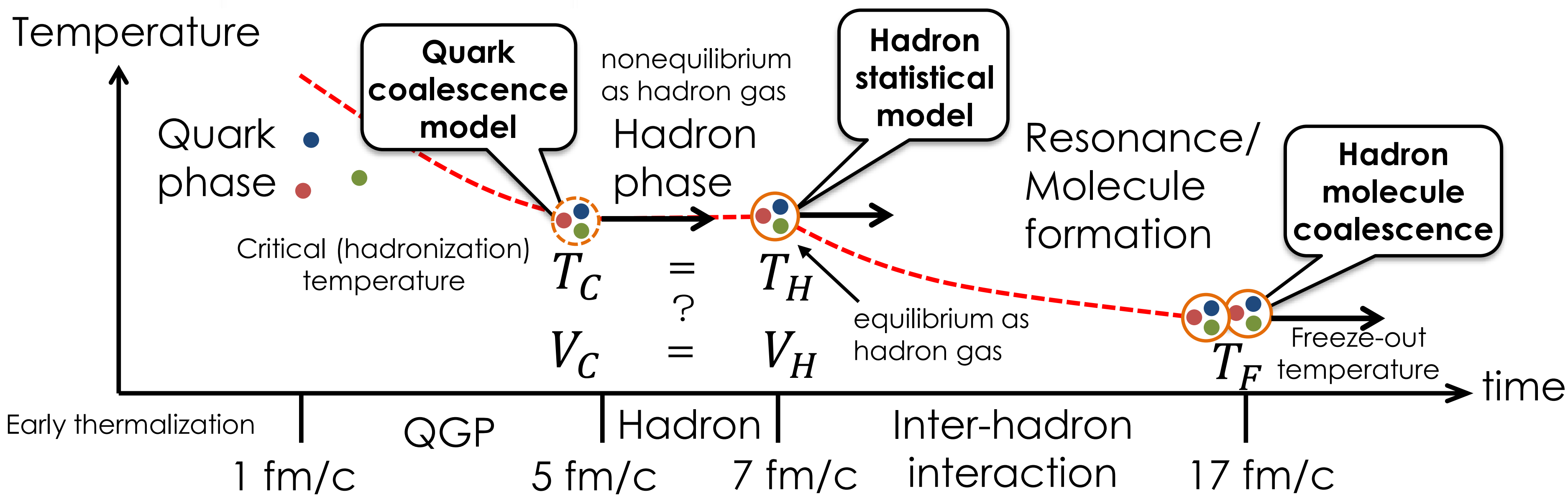
[1]*Institute of Physics and Applied Physics, Yonsei University, Seoul 120-749, Korea*
[2]*Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*
[3]*RIKEN Nishina Center, Hirosawa 2-1, Wako, Saitama 351-0198, Japan*
[4]*Department of Physics, Tokyo Institute of Technology, Meguro 152-8551, Japan*
[5]*Cyclotron Institute and Department of Physics and Astronomy, Texas A&M University, College Station, Texas 77843, USA*
[6]*Instituto de Física, Universidade de São Paulo, C.P. 66318, 05389-970 São Paulo, SP, Brazil*
[7]*Department of Physics, Graduate School of Science, Kyoto University, Kyoto 606-8502, Japan*
[8]*Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization (KEK), 1-1, Oho, Ibaraki 305-0801, Japan*
(Received 10 November 2010; published 24 May 2011)

Cf. ExHIC collaboration: Phys. Rev. C84 (2011) 064910; Prog. Part. Nucl. Phys. 95 (2017) 279 (review)

3. Heavy exotic hadrons -X, Y, Z hadrons-

Production process of exotic hadrons





Free quark gas, First order, Evolution with entropy conservation, ...

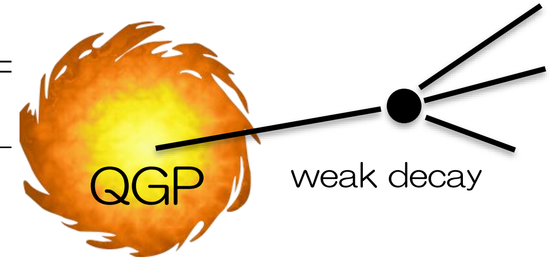
L. W. Chen, V. Greco, C. M. Ko, S. H. Lee, W. Liu, PLB **601**, 34 (2004)

3. Heavy exotic hadrons -X, Y, Z hadrons-

What is the production yield for each hadron？

Exotic hadrons to be explored

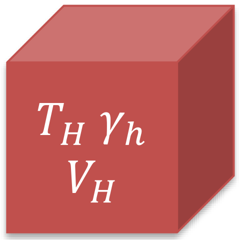
Particle	m (MeV)	Decay mode	
Mesons			
$f_0(980)$	980	$\pi\pi$ (Strong decay)	
$a_0(980)$	980	$\eta\pi$ (Strong decay)	
$K(1460)$	1460	$K\pi\pi$ (Strong decay)	
$D_s(2317)$	2317	$D_s\pi$ (Strong decay)	
T_{cc}^{1}[a]	3797	$K^+\pi^- + K^+\pi^- + \pi^-$	weak decay
$X(3872)$	3872	$J/\psi\pi\pi$ (Strong decay)	
$Z^+(4430)$[b]	4430	$J/\psi\pi$ (Strong decay)	
T_{cb}^{0}[a]	7123	$K^+\pi^- + K^+\pi^-$	weak decay
Baryons			
$\Lambda(1405)$	1405	$\pi\Sigma$ (Strong decay)	
$\Theta^+(1530)$[b]	1530	KN (Strong decay)	
$\bar{K}KN$[a]	1920	$K\pi\Sigma$, $\pi\eta N$ (Strong decay)	
$\bar{D}N$[a]	2790	$K^+\pi^-\pi^- + p$	weak decay
$\bar{D}^*N$[a]	2919	$\bar{D} + N$ (Strong decay)	
Θ_{cs}[a]	2980	$\Lambda + K^+\pi^-$	weak decay
BN[a]	6200	$K^+\pi^-\pi^- + \pi^+ + p$	weak decay
B^*N[a]	6226	$B + N$ (Strong decay)	
Dibaryons			
H[a]	2245	$\Lambda\Lambda$ (Strong decay)	
$\bar{K}NN$[b]	2352	ΛN (Strong decay)	
$\Omega\Omega$[a]	3228	$\Lambda K^- + \Lambda K^-$	weak decay
H_c^{++}[a]	3377	$\Lambda K^-\pi^+\pi^+ + p$	weak decay
$\bar{D}NN$[a]	3734	$K^+\pi^- + d$, $K^+\pi^-\pi^- + p + p$	weak decay
BNN[a]	7147	$K^+\pi^- + d$, $K^+\pi^- + p + p$	weak decay



Weak decay; some decays outside the fireball (QGP).

3. Heavy exotic hadrons -X, Y, Z hadrons-

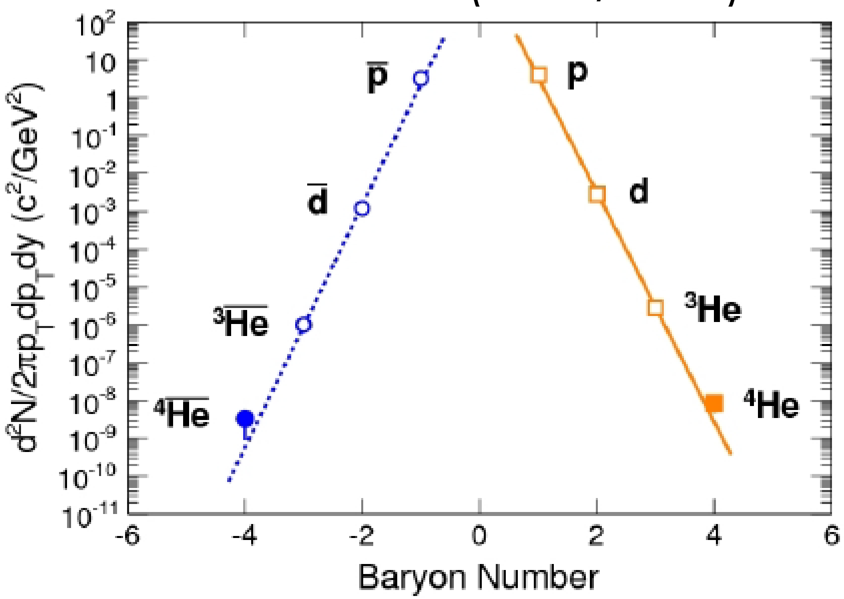
Statistical model



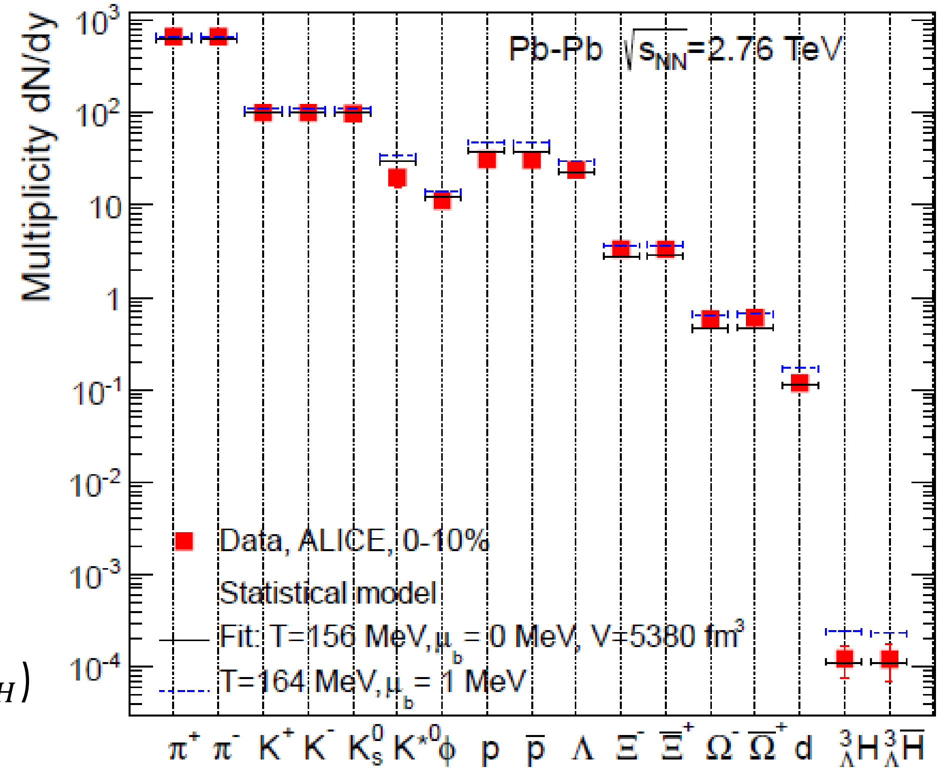
$$N_h^{\mathrm{stat}} = V_H \frac{g_h}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\gamma_h^{-1} e^{E_h/T_H} \pm 1}$$

A. Andronic et al.,
Nucl. Phys. A772, 167 (2006)

antmatter (RHIC/STAR)



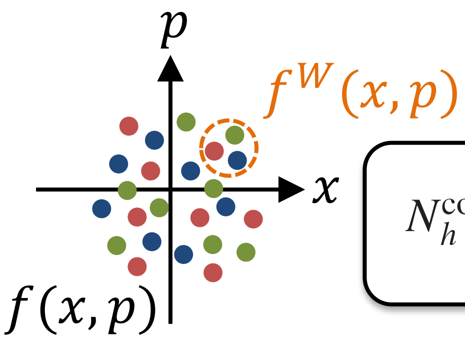
Normal hadrons (LHC/ALICE)



Key point:
Almost equilibrium state (temperature T_H)
Chemical-freezeout (fugacity γ_h)
Uniform volume (V_H)
Those parameters are determined to reproduce normal hadrons.

What's about exotic hadrons？

3. Heavy exotic hadrons -X, Y, Z hadrons-



Coalescence model

$$N_h^{\mathrm{coal}} \simeq g_h \prod_{j=1}^{n} \frac{N_j}{g_j} \prod_{i=1}^{n-1} \frac{(4\pi\sigma_i^2)^{3/2}}{V(1+2\mu_i T\sigma_i^2)} \left[\frac{4\mu_i T\sigma_i^2}{3(1+2\mu_i T\sigma_i^2)}\right]^{l_i}$$

Key point: convolution of wave functions and thermal distributions in phase space (x, p)

$$N_h^{\mathrm{coal}} = g_h \int \left[\prod_{i=1}^{n} \frac{1}{g_i} \frac{p_i \cdot d\sigma_i}{(2\pi)^3} \frac{\mathrm{d}^3\mathbf{p}_i}{E_i} f(x_i, p_i)\right] \times f^W(x_1, \ldots, x_n : p_1, \ldots, p_n).$$

① distribution function $f(x_i, p_i)$ for particle i

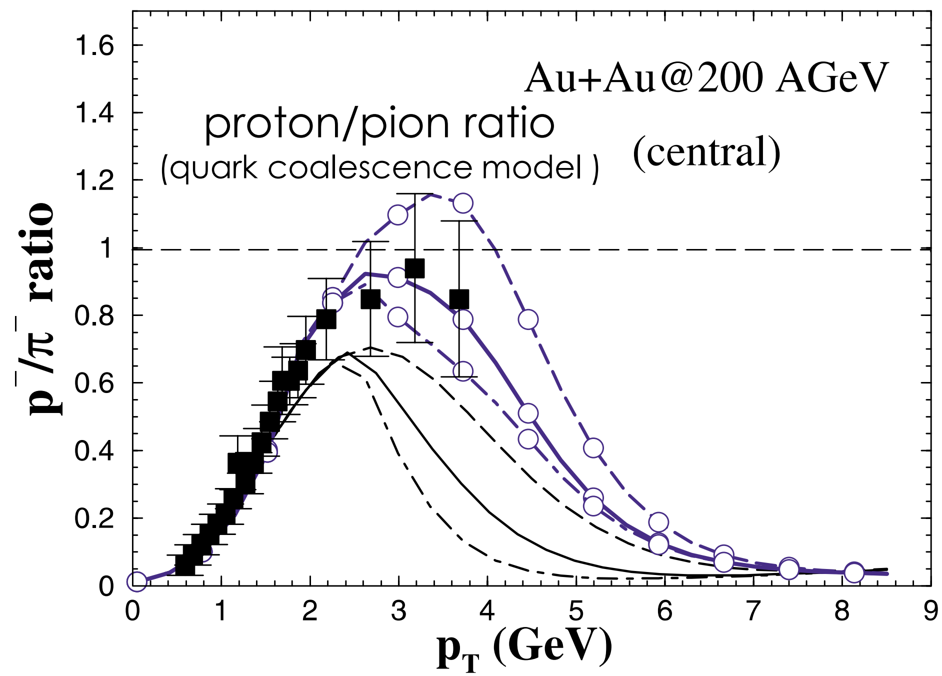
$$\int p_i \cdot d\sigma_i \frac{d^3\mathbf{p}_i}{(2\pi)^3 E_i} f(x_i, p_i) = N_i$$

② Wigner function

$$f^W(x_1, \ldots, x_n : p_1, \ldots, p_n) = \int \prod_{i=1}^{n} dy_i e^{ip_i y_i} \psi^*(x_1 + y_1/2, \ldots, x_n + y_n/2) \times \psi(x_1 - y_1/2, \ldots, x_n - y_n/2),$$

Harmonic oscillator wave function (frequency ω) is used.
The value of ω is determined by normal hadron productions.

V. Greco, C. M. Ko, P. Levai, PRL90, 202302 (2003)
L. W. Chen et al., PLB 601, 34 (2004)
L. W. Chen et al., PRC 76, 014906 (2007)



What's about exotic hadrons?

The same formula can be applied to quark/hadron-molecule coalescence.

3. Heavy exotic hadrons -X, Y, Z hadrons-

Parameters in statistical/coalescence models

	RHIC		LHC (2.76 TeV)		LHC (5.02 TeV)		RHIC	LHC (5 TeV)
	Sc. 1	Sc. 2	Sc. 1	Sc. 2	Sc. 1	Sc. 2	Refs [14,15]	
T_H (MeV)	162		156				175	
V_H (fm^3)	2100		5380				1908	5152
μ_B (MeV)	24		0				20	0
μ_s (MeV)	10		0				10	0
γ_c	22		39		50		6.40	15.8
γ_b	4.0×10^7		8.6×10^8		1.4×10^9		2.2×10^6	3.3×10^7
T_C (MeV)	162	166	156	166	156	166	175	
V_C (fm^3)	2100	1791	5380	3533	5380	3533	1000	2700
ω (MeV)	590	608	564	609	564	609	550	
ω_s (MeV)	431	462	426	502	426	502	519	
ω_c (MeV)	222	244	219	278	220	279	385	
ω_b (MeV)	183	202	181	232	182	234	338	
$N_u = N_d$	320	302	700	593	700	593	245	662
$N_s = N_{\bar{s}}$	183	176	386	347	386	347	150	405
$N_c = N_{\bar{c}}$	4.1		11		14		3	20
$N_b = N_{\bar{b}}$	0.03		0.44		0.71		0.02	0.8

Scenario 1: $T_C = T_H$, $V_C = V_H$

ω: the hadron yields from the coalescence model at T_C
= the hadron yields from the statistical modal at T_H.

Scenario 2: ω: the hadron yields at T_C to reproduce RHIC/LHC data

T_C and V_C :the hadron yields from the coalescence model at T_C
= the hadron yields from the statistical modal at T_H.

3. Heavy exotic hadrons -X, Y, Z hadrons- numerical results (# per collision)

Particle	Scenario 1		Scenario 2		Mol.	Stat.
	$q\bar{q}/qqq$	Multiquark	$q\bar{q}/qqq$	Multiquark		
RHIC						
$f_0(980)$	2.1 (0.7)	3.9×10^{-2}	2.1 (0.7)	4.0×10^{-2}	1.7	3.5
$a_0(980)$	6.4	1.2×10^{-1}	6.4	1.2×10^{-1}	5.2	10
$K(1460)$	–	5.8×10^{-2}	–	5.7×10^{-2}	1.3×10^{-1}	6.3×10^{-1}
$\Lambda(1405)$	4.7×10^{-1}	2.3×10^{-2}	4.5×10^{-1}	2.4×10^{-2}	7.3×10^{-1}	8.6×10^{-1}
$\Delta\Delta$	–	4.2×10^{-3}	–	5.3×10^{-3}	–	1.8×10^{-2}
$\Lambda\Lambda$-$N\Xi$ (H)	–	4.7×10^{-4}	–	5.0×10^{-4}	1.6×10^{-3}	4.9×10^{-3}
$N\Omega$	–	1.7×10^{-3}	–	1.9×10^{-3}	1.4×10^{-3}	6.7×10^{-3}
LHC (2.76 TeV)						
$f_0(980)$	4.3 (1.2)	5.4×10^{-2}	4.1 (1.2)	6.0×10^{-2}	3.2	6.6
$a_0(980)$	13	1.6×10^{-1}	12	1.8×10^{-1}	9.5	20
$K(1460)$	–	8.2×10^{-2}	–	8.0×10^{-2}	1.9×10^{-1}	1.0
$\Lambda(1405)$	7.5×10^{-1}	2.9×10^{-2}	7.0×10^{-1}	3.2×10^{-2}	1.1	1.4
$\Delta\Delta$	–	5.8×10^{-3}	–	1.0×10^{-2}	–	1.9×10^{-2}
$\Lambda\Lambda$-$N\Xi$ (H)	–	5.0×10^{-4}	–	6.1×10^{-4}	1.8×10^{-3}	5.9×10^{-3}
$N\Omega$	–	1.8×10^{-3}	–	2.3×10^{-3}	1.6×10^{-3}	7.8×10^{-3}
LHC (5.02 TeV)						
$f_0(980)$	4.3 (1.2)	5.4×10^{-2}	4.1 (1.2)	6.0×10^{-2}	3.2	6.6
$a_0(980)$	13	1.6×10^{-1}	12	1.8×10^{-1}	9.5	20
$K(1460)$	–	8.2×10^{-2}	–	8.0×10^{-2}	1.9×10^{-1}	1.0
$\Lambda(1405)$	7.5×10^{-1}	2.9×10^{-2}	7.0×10^{-1}	3.2×10^{-2}	1.1	1.4
$\Delta\Delta$	–	5.8×10^{-3}	–	1.0×10^{-2}	–	1.9×10^{-2}
$\Lambda\Lambda$-$N\Xi$ (H)	–	5.0×10^{-4}	–	6.1×10^{-4}	1.8×10^{-3}	5.9×10^{-3}
$N\Omega$	–	1.8×10^{-3}	–	2.3×10^{-3}	1.6×10^{-3}	7.8×10^{-3}

Particle	Scenario 1		Scenario 2		Mol.	Stat.
	$q\bar{q}/qqq$	Multiquark	$q\bar{q}/qqq$	Multiquark		
RHIC						
$\Theta(1530)$	–	6.7×10^{-3}	–	6.7×10^{-3}	–	5.0×10^{-1}
$\bar{K}KN$	–	5.0×10^{-3}	–	5.1×10^{-3}	4.2×10^{-2}	1.2×10^{-1}
$\bar{K}NN$	7.3×10^{-4}	2.7×10^{-5}	7.4×10^{-4}	2.9×10^{-5}	3.9×10^{-3}	5.8×10^{-3}
$\Omega\Omega$	–	8.2×10^{-6}	–	9.4×10^{-6}	–	1.5×10^{-5}
LHC (2.76 TeV)						
$\Theta(1530)$	–	8.2×10^{-3}	–	8.5×10^{-3}	–	6.8×10^{-1}
$\bar{K}KN$	–	6.0×10^{-3}	–	6.6×10^{-3}	5.1×10^{-2}	1.5×10^{-1}
$\bar{K}NN$	7.9×10^{-4}	2.3×10^{-5}	8.6×10^{-4}	3.0×10^{-5}	3.9×10^{-3}	6.3×10^{-3}
$\Omega\Omega$	–	7.6×10^{-6}	–	1.2×10^{-5}	–	1.8×10^{-5}
LHC (5.02 TeV)						
$\Theta(1530)$	–	8.2×10^{-3}	–	8.5×10^{-3}	–	6.8×10^{-1}
$\bar{K}KN$	–	6.0×10^{-3}	–	6.6×10^{-3}	5.2×10^{-2}	1.5×10^{-1}
$\bar{K}NN$	7.9×10^{-4}	2.3×10^{-5}	8.6×10^{-4}	3.0×10^{-5}	3.9×10^{-3}	6.3×10^{-3}
$\Omega\Omega$	–	7.6×10^{-6}	–	1.2×10^{-5}	–	1.8×10^{-5}

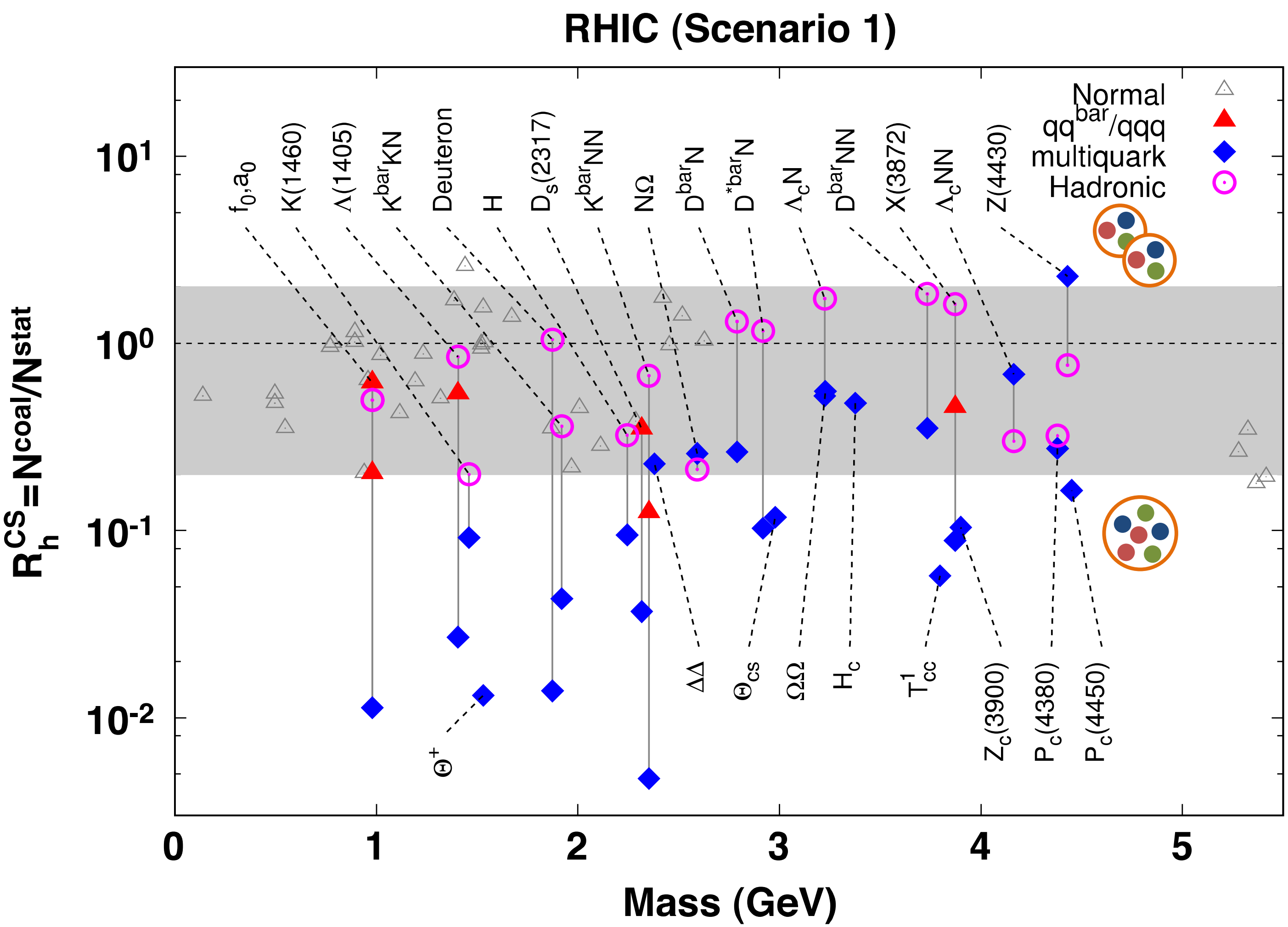
The yields of exotic hadrons are much smaller than those of normal hadrons (1~10), but not negligible.

Particle	Scenario 1		Scenario 2		Mol.	Stat.
	$q\bar{q}/qqq$	Multiquark	$q\bar{q}/qqq$	Multiquark		
RHIC						
$D_s(2317)$	2.3×10^{-2}	2.4×10^{-3}	2.3×10^{-2}	2.5×10^{-3}	6.5×10^{-3}	6.6×10^{-2}
$X(3872)$	5.4×10^{-4}	5.0×10^{-5}	5.6×10^{-4}	5.3×10^{-5}	9.1×10^{-4}	5.7×10^{-4}
$Z_c(3900)$	–	1.5×10^{-4}	–	1.6×10^{-4}	–	1.5×10^{-3}
$Z_c(4430)$	–	1.5×10^{-4}	–	1.6×10^{-5}	5.0×10^{-5}	6.5×10^{-5}
$Z_b(10610)$	–	2.0×10^{-9}	–	2.1×10^{-9}	–	2.1×10^{-8}
$Z_b(10650)$	–	2.0×10^{-9}	–	2.1×10^{-9}	–	1.6×10^{-8}
$X(5568)$	–	5.1×10^{-5}	–	5.2×10^{-5}	–	2.3×10^{-3}
$P_c(4380)$	–	2.5×10^{-5}	–	2.6×10^{-5}	2.9×10^{-5}	9.2×10^{-5}
$P_c(4450)$	–	1.5×10^{-5}	–	1.5×10^{-5}	–	9.1×10^{-5}
LHC (2.76 TeV)						
$D_s(2317)$	5.2×10^{-2}	4.3×10^{-3}	5.0×10^{-2}	4.5×10^{-3}	1.4×10^{-2}	1.5×10^{-1}
$X(3872)$	1.6×10^{-3}	1.1×10^{-4}	1.7×10^{-3}	1.3×10^{-4}	2.7×10^{-3}	1.7×10^{-3}
$Z_c(3900)$	–	3.4×10^{-4}	–	4.0×10^{-4}	–	4.3×10^{-3}
$Z_c(4430)$	–	3.4×10^{-4}	–	4.0×10^{-4}	1.4×10^{-4}	1.7×10^{-4}
$Z_b(10610)$	–	1.3×10^{-7}	–	1.5×10^{-7}	–	1.9×10^{-6}
$Z_b(10650)$	–	1.3×10^{-7}	–	1.5×10^{-7}	–	1.5×10^{-6}
$X(5568)$	–	5.0×10^{-4}	–	5.2×10^{-4}	–	3.1×10^{-2}
$P_c(4380)$	–	5.0×10^{-5}	–	5.8×10^{-5}	6.4×10^{-5}	2.1×10^{-4}
$P_c(4450)$	–	2.9×10^{-5}	–	3.2×10^{-5}	–	2.0×10^{-4}
LHC (5.02 TeV)						
$D_s(2317)$	6.5×10^{-2}	5.4×10^{-3}	6.4×10^{-2}	5.7×10^{-3}	1.8×10^{-2}	1.9×10^{-1}
$X(3872)$	2.5×10^{-3}	1.8×10^{-4}	2.7×10^{-3}	2.1×10^{-4}	4.5×10^{-3}	2.8×10^{-3}
$Z_c(3900)$	–	5.4×10^{-4}	–	6.4×10^{-4}	–	7.1×10^{-3}
$Z_c(4430)$	–	5.4×10^{-4}	–	6.4×10^{-4}	2.3×10^{-4}	2.8×10^{-4}
$Z_b(10610)$	–	3.4×10^{-7}	–	3.9×10^{-7}	–	5.0×10^{-6}
$Z_b(10650)$	–	3.4×10^{-7}	–	3.9×10^{-7}	–	3.9×10^{-6}
$X(5568)$	–	7.9×10^{-4}	–	8.2×10^{-4}	–	5.0×10^{-2}
$P_c(4380)$	–	7.9×10^{-5}	–	9.3×10^{-5}	1.0×10^{-4}	3.4×10^{-4}
$P_c(4450)$	–	4.7×10^{-5}	–	5.0×10^{-5}	–	3.4×10^{-4}

Particle	Scenario 1		Scenario 2		Mol.	Stat.
	$q\bar{q}/qqq$	Multiquark	$q\bar{q}/qqq$	Multiquark		
RHIC						
T^1_{cc}	–	5.0×10^{-5}	–	5.3×10^{-5}	–	8.9×10^{-4}
$\bar{D}N$	–	2.6×10^{-3}	–	2.6×10^{-3}	1.3×10^{-2}	1.0×10^{-2}
$\bar{D}^*N$	–	9.8×10^{-4}	–	9.3×10^{-4}	1.1×10^{-2}	9.6×10^{-3}
Θ_{cs}	–	7.4×10^{-4}	–	7.4×10^{-4}	–	6.4×10^{-3}
H_c	–	2.7×10^{-4}	–	2.8×10^{-4}	–	5.7×10^{-4}
$\bar{D}NN$	–	1.8×10^{-5}	–	1.8×10^{-5}	9.4×10^{-5}	5.1×10^{-5}
Λ_cN	–	1.5×10^{-3}	–	1.5×10^{-3}	5.0×10^{-3}	2.9×10^{-3}
Λ_cNN	–	6.7×10^{-6}	–	6.7×10^{-6}	2.9×10^{-6}	9.8×10^{-6}
T^0_{cb}	–	9.3×10^{-8}	–	9.9×10^{-8}	–	1.6×10^{-6}
LHC (2.76 TeV)						
T^1_{cc}	–	1.1×10^{-4}	–	1.3×10^{-4}	–	2.7×10^{-3}
$\bar{D}N$	–	4.3×10^{-3}	–	4.2×10^{-3}	2.3×10^{-2}	1.9×10^{-2}
$\bar{D}^*N$	–	1.6×10^{-3}	–	1.3×10^{-3}	2.0×10^{-2}	1.8×10^{-2}
Θ_{cs}	–	1.2×10^{-3}	–	1.2×10^{-3}	–	1.2×10^{-2}
H_c	–	3.8×10^{-4}	–	4.0×10^{-4}	–	8.6×10^{-4}
$\bar{D}NN$	–	2.0×10^{-5}	–	2.0×10^{-5}	1.1×10^{-4}	6.7×10^{-5}
Λ_cN	–	2.2×10^{-3}	–	2.2×10^{-3}	7.0×10^{-3}	4.3×10^{-3}
Λ_cNN	–	6.7×10^{-6}	–	6.5×10^{-6}	2.7×10^{-6}	9.9×10^{-6}
T^0_{cb}	–	1.1×10^{-6}	–	1.3×10^{-6}	–	2.7×10^{-5}
LHC (5.02 TeV)						
T^1_{cc}	–	1.8×10^{-4}	–	2.1×10^{-4}	–	4.4×10^{-3}
$\bar{D}N$	–	5.3×10^{-3}	–	5.3×10^{-3}	3.0×10^{-2}	2.4×10^{-2}
$\bar{D}^*N$	–	2.0×10^{-3}	–	1.7×10^{-3}	2.6×10^{-2}	2.3×10^{-2}
Θ_{cs}	–	1.5×10^{-3}	–	1.4×10^{-3}	–	1.6×10^{-2}
H_c	–	4.7×10^{-4}	–	4.9×10^{-4}	–	1.1×10^{-3}
$\bar{D}NN$	–	2.5×10^{-5}	–	2.5×10^{-5}	1.5×10^{-4}	8.6×10^{-5}
Λ_cN	–	2.7×10^{-3}	–	2.7×10^{-3}	9.1×10^{-3}	5.5×10^{-3}
Λ_cNN	–	8.2×10^{-6}	–	8.0×10^{-6}	3.5×10^{-6}	1.3×10^{-5}
T^0_{cb}	–	2.3×10^{-6}	–	2.7×10^{-6}	–	5.6×10^{-5}

3. Heavy exotic hadrons -X, Y, Z hadrons-

The statistical model v.s. the coalescence model



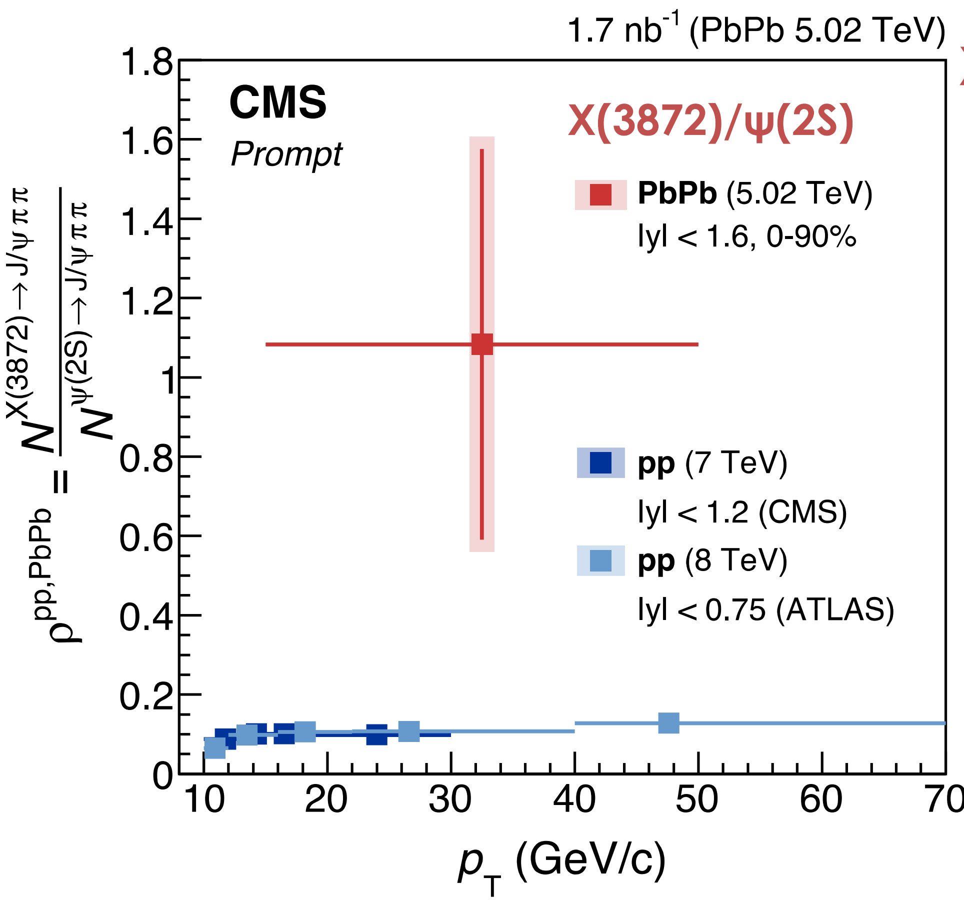
1. The yields of the compact multiquark are relatively suppressed.
2. The yields of the hadronic molecules depend on their spatial sizes.

3. Heavy exotic hadrons -X, Y, Z hadrons-

PHYSICAL REVIEW LETTERS **128,** 032001 (2022)

Evidence for X(3872) in Pb-Pb Collisions and Studies of its Prompt Production at $\sqrt{s_{NN}} = 5.02$ TeV

A. M. Sirunyan *et al.**
CMS Collaboration



X(3872) found in heavy ion collisions!

More produced than pp collisions.

Note: Loosely bound states are difficult to be produced in pp collisions.

Many questions

1. Consistent with ExHIC's prediction？ Supporting the hadronic molecule？
2. How is the p_T dependence related to hadron structure？ (p_T trans. momentum)
   Cf. Choi, Lee, PRC101, 024902 (2020)
3. Elliptic flow by X(3872)？
   Cf. Zhang et al., PRL126, 012301 (2021)
4. What about $\chi_{c1}(2P)$？
   ($\chi_{c1}(2P)$ is coupled to $X(3872)$)
5. Other exotics？

Do you have questions?

Summary of this lecture

1. Exotic heavy hadrons ($X,Y,Z,P_C,T_{CC},\ldots$) are new objects to be studied in hadron physics.

2. Hadron spectroscopy provides us with basic tools to study internal structure of exotic hadrons.

3. Heavy hadrons in nuclear systems are important in understanding the strong interaction.

4. Cooperation between experiments and theory is important (KEK, J-PARC, RHIC, LHC, GSI-FAIR, BES, ...).

Keywords in the lecture

Tetraquark

Exotic

Spectroscopy

Heavy quark symmetry

Charm

XYZ

Heavy hadron effective theory

Bottom

Pentaquark

Spin