

チャーム・ボトムの エキゾチックハドロン物理の最近の発展

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学

内容

1. イントロダクション

- 1.1 ハドロンの基本的性質
- 1.2 なぜ重いハドロンを研究するのか？

2. 重いクォークのスピン対称性と有効理論

- 2.1 スピン対称性とハドロンスペクトロスコピー
- 2.2 重いクォークの有効理論
- 2.3 重いハドロンの有効理論

3. 重いエキゾチックハドロン -ハドロン相互作用の観点から-

- 3.1 なぜエキゾチックハドロンが面白いのか？
- 3.2 チャームメソン： X, Y, Z_c
- 3.3 ボトムメソン： Z_b
- 3.4 チャームペンタクォーク： P_c, P_{cs}
- 3.5 ダブルチャームメソン： T_{cc}
- 3.6 フルチャームメソン： X_{cc}
- 3.7 反応論—重イオン衝突によるエキゾチックハドロン生成—

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2. Theoretical framework for heavy hadrons

Heavy-light meson

2. Theoretical framework for heavy hadrons

D meson

\bar{D} meson

D^* meson

\bar{D}^* meson

\bar{B} meson

B meson

\bar{B}^* meson

B^* meson

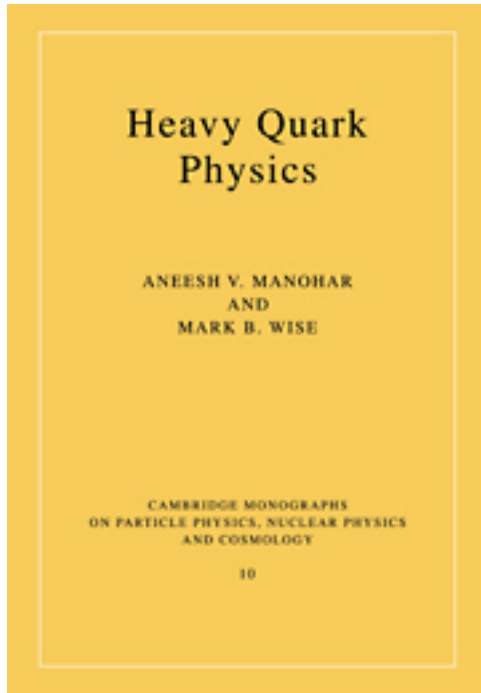
2. Theoretical framework for heavy hadrons

$J^P=0^-$	D meson	\bar{D} meson
$J^P=1^-$	D^* meson	\bar{D}^* meson
$J^P=0^-$	\bar{B} meson	B meson
$J^P=1^-$	\bar{B}^* meson	B^* meson

2. Theoretical framework for heavy hadrons

$J^P=0^-$	D meson ($\bar{q}c$)	\bar{D} meson ($q\bar{c}$)
$J^P=1^-$	D^* meson ($\bar{q}c$)	\bar{D}^* meson ($q\bar{c}$)
$J^P=0^-$	\bar{B} meson ($\bar{q}b$)	B meson ($q\bar{b}$)
$J^P=1^-$	\bar{B}^* meson ($\bar{q}b$)	B^* meson ($q\bar{b}$)

2. Theoretical framework for heavy hadrons Heavy quark effective theory (HQET)




Heavy Quark Physics

A. V. Manohar and M. W. Wise

Cambridge University Press 2000

2. Theoretical framework for heavy hadrons

Heavy quark effective theory (HQET)

 **PHYSICS REPORTS**


Elsevier Physics Reports 245 (1994) 259–395

Heavy-quark symmetry

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Contents:

0. Preface	262	4. Hadronic matrix elements	321
1. Heavy-quark symmetry	264	4.1. Covariant representation of states	321
1.1. The physical picture	264	4.2. Meson decay form factors	325
1.2. An effective theory	266	4.3. Implications of Luke's theorem	332
1.3. Spectroscopic implications	269	4.4. Second-order power corrections and the anatomy of $\delta_{1/2}$	334
1.4. Weak decay form factors	270	4.5. Nonperturbative techniques	336
2. Heavy-quark effective theory	278	4.6. Meson decay constants	340
2.1. Effective field theories	278	4.7. Heavy-to-light transitions and the determination of V_{cb}	344
2.2. Derivation of the effective Lagrangian	279	5. QCD sum rules	347
2.3. The $1/m_Q$ expansion	282	5.1. Nonperturbative techniques	347
2.4. Hadron masses in the effective theory	284	5.2. Properties of the Isgur–Wise function	349
2.5. Reparameterization invariance	285	5.3. The sum rules of SVZ	352
2.6. The residual mass term	287	5.4. Meson masses and decay constants	356
3. Renormalization	289	5.5. The Isgur–Wise function	362
3.1. Matching	289	5.6. Power corrections	367
3.2. The renormalized Lagrangian	290	5.7. Predictions for the meson form factors	371
3.3. Composite operators, short-distance expansion, and the renormalization group	294	6. Phenomenology	373
3.4. Heavy-light currents	298	6.1. Heavy-to-light transitions	373
3.5. Leading-power corrections to the effective Lagrangian	301	6.2. Model-independent determination of V_{cb}	379
3.6. Power corrections to heavy-light currents	303	6.3. Asymmetries	383
3.7. Flavor-conserving heavy-quark currents	306	6.4. Some model-dependent results	386
3.8. Flavor-changing heavy-quark currents	310	7. Concluding remarks	388
3.9. Power corrections to heavy-quark currents	316	References	391
3.10. Summary and numerical results	319		

 **PHYSICS REPORTS**

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Phenomenology of heavy meson chiral lagrangians

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Contents

1. Introduction	148	5.2. $B \rightarrow \pi$ semileptonic decays	185
2. Heavy quark and chiral symmetry	151	5.3. $B \rightarrow V$ semileptonic decays	190
2.1. Heavy quark effective theory	151	6. Radiative decays	199
2.2. Chiral symmetry	155	6.1. Flavour conserving radiative decays: $D^* \rightarrow D\gamma$	199
2.3. A chiral lagrangian for heavy mesons	157	6.2. Weak radiative decay: $B \rightarrow \ell\nu\gamma$	204
2.4. Light-vecor resonances	158	6.3. Weak radiative decays: $B \rightarrow V\gamma$	206
2.5. The chiral lagrangian for the positive parity states	161	6.4. Weak radiative decays: $B \rightarrow Ke^+e^-$, $B \rightarrow K^*e^+e^-$	212
3. Strong interactions	161	7. Symmetries for heavy quarkonium states	214
3.1. Theoretical estimates of g	163	7.1. Non-relativistic QCD description	215
3.2. Chiral corrections to g	165	7.2. Heavy quarkonium effective theory	216
3.3. Hyperfine splitting	166	7.3. Heavy-meson effective theory	217
3.4. Strong decays of positive parity states	170	8. Heavy quarkonium decays	219
4. $B \rightarrow D$ decays and chiral dynamics	174	8.1. Radiative decays	220
4.1. Chiral corrections	175	8.2. Hadronic transitions in heavy quarkonia	221
4.2. The $B \rightarrow D^{(*)}\pi l\nu$ decay	177	Appendix A	228
4.3. The heavy-to-light effective current	178	Appendix B	230
4.4. Chiral corrections for f_{D^*}/f_D	179	Appendix C	231
5. Heavy-to-light semileptonic exclusive decays	182	References	232
5.1. Form factors	182		

2. Theoretical framework for heavy hadrons

- Heavy mass of charm hadron

$$m_c = 1.3 \text{ GeV}, m_b = 4.7 \text{ GeV} \gg \Lambda_{\text{QCD}} = \text{a few } 100 \text{ MeV}$$

- Heavy Quark Symmetry (HQS)

Hadron spin = light quark spin x heavy quark spin

2. Theoretical framework for heavy hadrons

Heavy quark effective theory

Separating QCD Lagrangian to heavy quark part and light quark part

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{heavy}} + \mathcal{L}_{\text{light}}$$

$$\mathcal{L}_{\text{heavy}} = \sum_Q \bar{Q}(i\not{D} - m_Q)Q$$

We focus on the *heavy* part.

$$\mathcal{L}_{\text{light}} = -\frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu} + \sum_q \bar{q}(i\not{D} - m_q)q$$

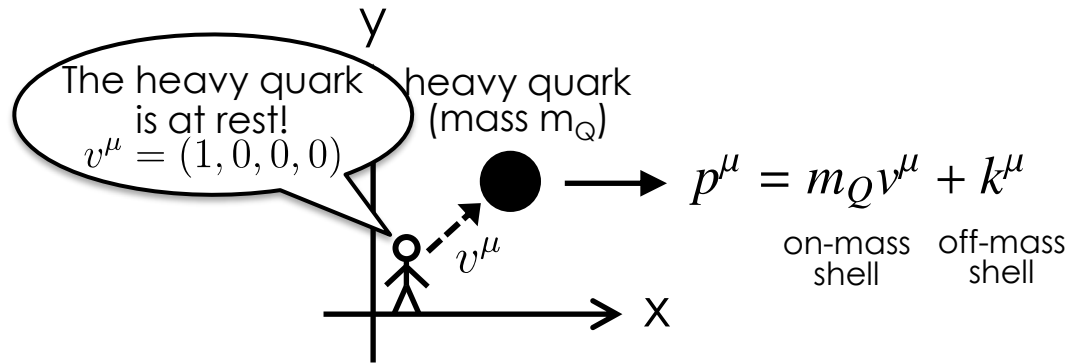
gluons + light quarks

2. Theoretical framework for heavy hadrons

Heavy quark effective theory

$$\mathcal{L}_{\text{heavy}} = \sum_Q \bar{Q}(i\not{D} - m_Q)Q$$

① On-mass-shell part and off-mass-shell part



on-mass-shell condition
 $v^\mu v_\mu = 1 \quad v^0 > 0$

off-mass-shell condition
 $k^\mu \ll m_Q$

② Effective field heavy quark (positive-energy and negative-energy)

Q: $Q_v(x) = \frac{1 + \not{v}}{2} e^{im_Q v \cdot x} Q(x)$

$$\not{v} Q_v(x) = Q_v(x)$$

Q^{bar}: $\bar{Q}_v = \frac{1 - \not{v}}{2} e^{im_Q v \cdot x} \bar{Q}(x)$

$$\not{v} \bar{Q}_v = -\bar{Q}_v$$

Projection operator

$$\frac{1 + \not{v}}{2} \rightarrow \frac{1 + \gamma^0}{2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

rest frame $v^\mu = (1, \vec{0})$

We consider only positive-energy part in the effective theory.

2. Theoretical framework for heavy hadrons

Heavy quark effective theory

$$\mathcal{L}_{\text{heavy}} = \sum_Q \bar{Q}(i\not{D} - m_Q)Q$$

3 steps to get HQ effective Lagrangian

- ① Expressing $\mathcal{L}_{\text{heavy}}$ in terms of Q_v (heavy quark) and \bar{Q}_v (anti heavy quark).

$$\mathcal{L}_{\text{heavy}} = \bar{Q}_v \underline{v \cdot iD} Q_v - \bar{Q}_v (v \cdot iD + 2m_Q) Q_v + \bar{Q}_v i\not{D}_\perp Q_v + \bar{Q}_v i\not{D}_\perp Q_v$$

no mass term
(m_Q was absorbed in Q_v)

$$D_\perp^\mu = D^\mu - v^\mu v \cdot D$$

- ② Eliminating \bar{Q}_v (anti heavy quark) by using equation of motion.

$$(v \cdot iD + 2m_Q) Q_v = i\not{D}_\perp Q_v$$

- ③ We get HQ effective Lagrangian!

$$\begin{aligned} \mathcal{L}_{\text{heavy}} &= \bar{Q}_v \left(v \cdot iD + i\not{D}_\perp \frac{1}{v \cdot iD + 2m_Q} i\not{D}_\perp \right) Q_v \\ &= \bar{Q}_v v \cdot iD Q_v + \bar{Q}_v \frac{(iD_\perp)^2}{2m_Q} Q_v - g_s \bar{Q}_v \frac{\sigma_{\mu\nu} G^{\mu\nu}}{4m_Q} Q_v + \mathcal{O}(1/m_Q^2) \end{aligned}$$

2. Theoretical framework for heavy hadrons

Heavy quark effective theory

$$\mathcal{L}_{\text{heavy}} = \sum_Q \bar{Q}(i\not{D} - m_Q)Q$$

HQ effective Lagrangian (N_h heavy flavors)

$$\mathcal{L}_{\text{heavy}} = \sum_{Q=1}^{N_h} \left(\bar{Q}_v v \cdot iD Q_v + \bar{Q}_v \frac{(iD_{\perp})^2}{2m_Q} Q_v - c(\mu) g_s \bar{Q}_v \frac{\sigma_{\mu\nu} G^{\mu\nu}}{4m_Q} Q_v + \mathcal{O}(1/m_Q^2) \right)$$

3 Key Points

1. Velocity rearrangement

2. Radiative corrections

3. Heavy quark spin symmetry ($m_Q \rightarrow \infty$)

2. Theoretical framework for heavy hadrons

Heavy quark effective theory

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specific v^μ — 1. Velocity rearrangement

① LO: Superposition of various v^μ

$$\mathcal{L}_{\text{heavy}}^{\text{total}} = \sum \mathcal{L}_{\text{heavy},v} \quad p^\mu = m_Q v^\mu + k^\mu$$

② Change of v^μ and k^μ

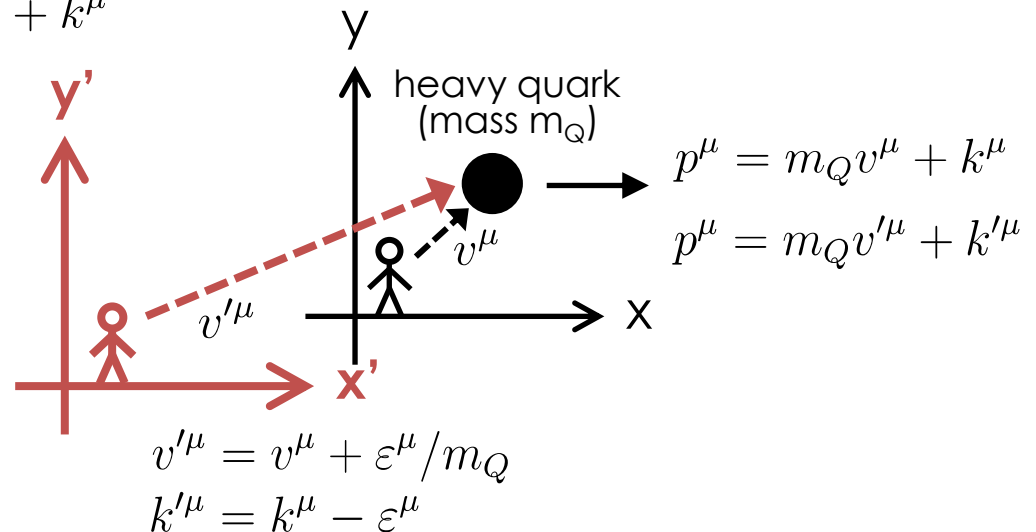
$$\begin{aligned} v^\mu &\rightarrow v^\mu + \varepsilon^\mu / m_Q \\ k^\mu &\rightarrow k^\mu - \varepsilon^\mu \end{aligned} \quad v \cdot \varepsilon = 0$$

③ Condition of $v^2 = 1$

$$v^2 \simeq 1 + \mathcal{O}(1/m_Q^2) \quad \psi Q_v = Q_v$$

④ Transformation of Q_v

$$Q_v \rightarrow e^{i\varepsilon \cdot x} \left(Q_v + \frac{\not{\varepsilon}}{2m_Q} Q_v \right)$$



2. Theoretical framework for heavy hadrons

Heavy quark effective theory

$$\mathcal{L}_{\text{heavy}} = \sum_Q \bar{Q}(i\not{D} - m_Q)Q$$

HQ effective Lagrangian (N_h heavy flavors)

$$\mathcal{L}_{\text{heavy}} = \sum_{Q=1}^{N_h} \left(\bar{Q}_v v \cdot iD Q_v + \bar{Q}_v \frac{(iD_{\perp})^2}{2m_Q} Q_v - c(\mu) g_s \bar{Q}_v \frac{\sigma_{\mu\nu} G^{\mu\nu}}{4m_Q} Q_v + \mathcal{O}(1/m_Q^2) \right)$$

specific v^μ — 1. Velocity rearrangement

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④ Transformation of Q_v

$$Q_v \rightarrow e^{i\varepsilon \cdot x} \left(Q_v + \frac{\not{\varepsilon}}{2m_Q} Q_v \right)$$

⑤ Transformation of Lagrangian

$$\mathcal{L}_0 \equiv \bar{Q}_v i v \cdot D Q_v$$

$$\mathcal{L}_1 \equiv \bar{Q}_v \frac{(iD_{\perp})^2}{2m_Q} Q_v - g \bar{Q}_v \frac{\sigma_{\mu\nu} G^{\mu\nu}}{4m_Q} Q_v$$

$$\mathcal{L}_0 \rightarrow \mathcal{L}_0 + \frac{1}{m_Q} \bar{Q}_v i \varepsilon \cdot D Q_v$$

$$\mathcal{L}_1 \rightarrow \mathcal{L}_1 - \frac{1}{m_Q} \bar{Q}_v i \varepsilon \cdot D Q_v$$

⑥ Invariance of HQ effective Lagrangian

$$\mathcal{L}_0 + \mathcal{L}_1: \text{invariant for changing } v^\mu$$

We can choose any v-frame.

2. Theoretical framework for heavy hadrons

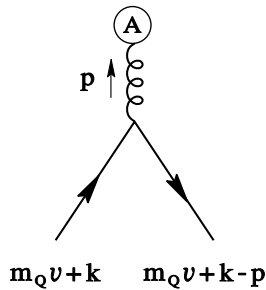
Heavy quark effective theory

$$\mathcal{L}_{\text{heavy}} = \sum_Q \bar{Q}(i\not{D} - m_Q)Q$$

HQ effective Lagrangian (N_h heavy flavors)

$$\mathcal{L}_{\text{heavy}} = \sum_{Q=1}^{N_h} \left(\bar{Q}_v v \cdot iD Q_v + \bar{Q}_v \frac{(iD_{\perp})^2}{2m_Q} Q_v - c(\mu) g_s \bar{Q}_v \frac{\sigma_{\mu\nu} G^{\mu\nu}}{4m_Q} Q_v + \mathcal{O}(1/m_Q^2) \right)$$

2. Radiative corrections



Tree level:

$$c(m_Q) = 1 + \mathcal{O}(\alpha_s(m_Q))$$



Leading logarithmic approximation:

$$c(\mu) = \left(\frac{\alpha_s(m_Q)}{\alpha_s(\mu)} \right)^{9/(33-2N_f)}$$

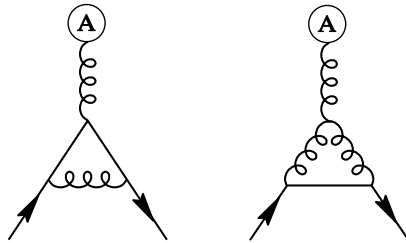


Figure: M. Neubert, Subnucl. Ser. 34, 98 (1997)
(arXiv:9610266 [hep-ph])

2. Theoretical framework for heavy hadrons

Heavy quark effective theory

$$\mathcal{L}_{\text{heavy}} = \sum_Q \bar{Q}(i\not{D} - m_Q)Q$$

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3. Heavy quark spin symmetry ($m_Q \rightarrow \infty$)

① Leading order of $1/m_Q$

$$\mathcal{L}_0 \equiv \bar{Q}_v i v \cdot D Q_v$$

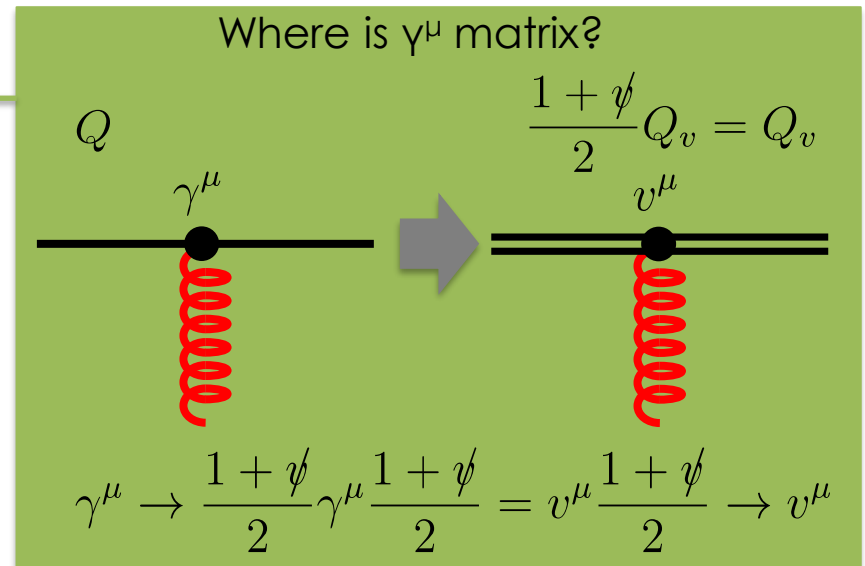
② Heavy quark spin (HQS) transformation

$$Q_v \rightarrow S Q_v \quad S \in SU(2)_{\text{spin}}$$

Lagrangian \mathcal{L}_0 is invariant for HQS:

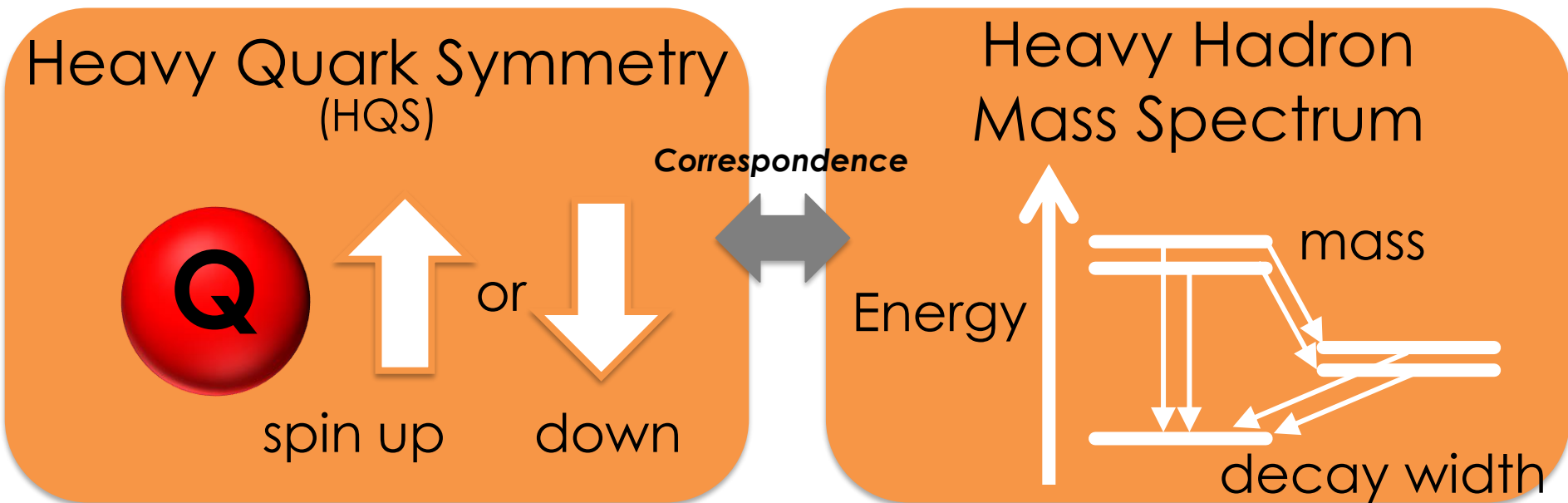
$$\mathcal{L}_0 \rightarrow \mathcal{L}_0$$

③ HQS is a conserved quantity.



2. Theoretical framework for heavy hadrons

Heavy quark symmetry



2. Theoretical framework for heavy hadrons

Heavy quark symmetry

“ Qq^{bar} ” meson



HQS ($S=1/2$)
conserved



Spin J
conserved



Light spin j
conserved!

$$\vec{J} = \vec{S} + \vec{j}$$

→ **Mass degeneracy for $J=j \pm 1/2$**

2. Theoretical framework for heavy hadrons

Heavy quark symmetry

“Qqq” baryon



HQS ($S=1/2$)
conserved



Spin J
conserved



Light spin j
conserved!

$$\vec{J} = \vec{S} + \vec{j}$$

→ **Mass degeneracy for $J=j \pm 1/2$**

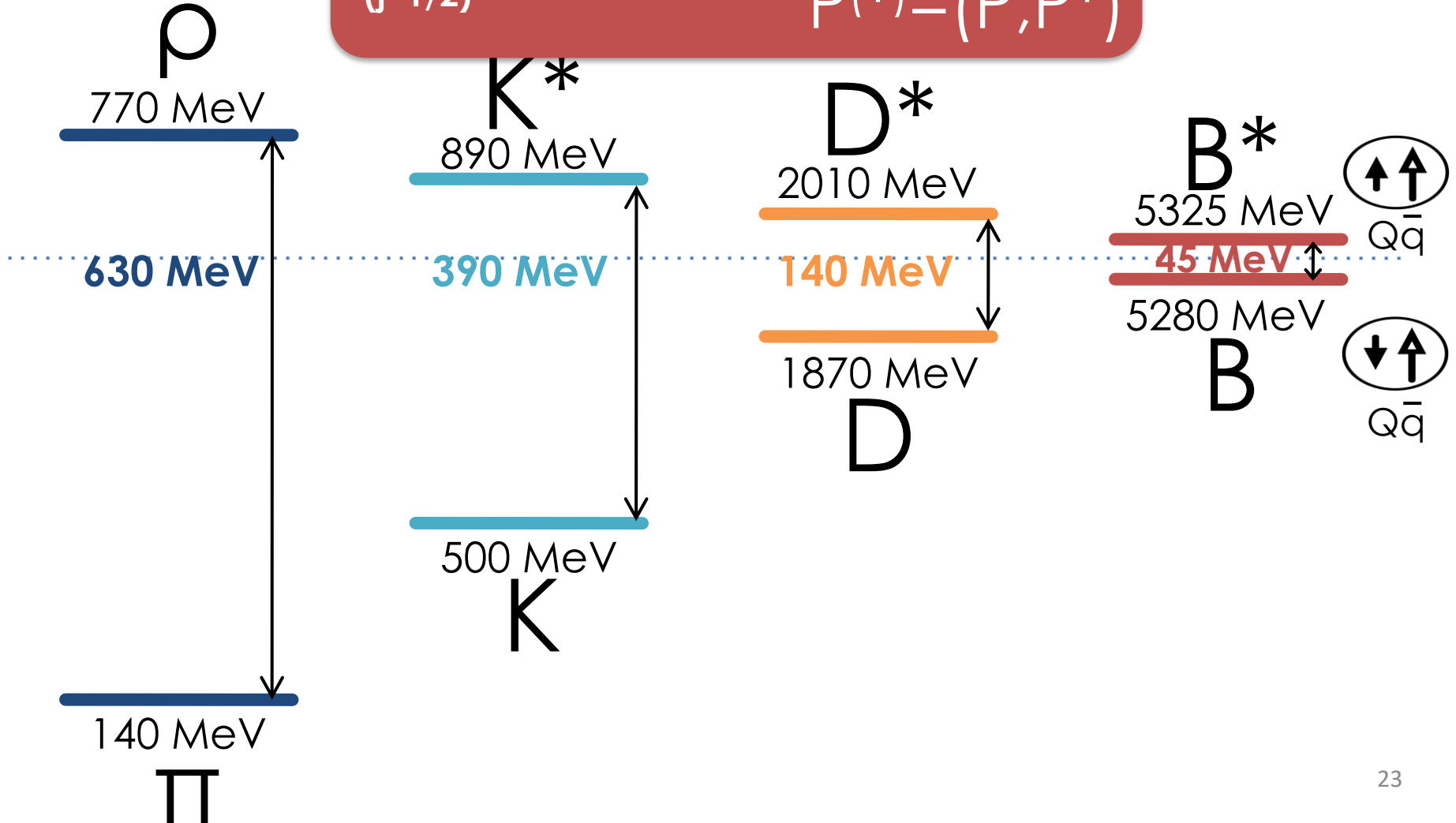
2. Theoretical framework for heavy hadrons

Heavy quark symmetry

$J = 1$
 $J = 0$
 $(j=1/2)$

\rightarrow **HQS doublet**
 $P^{(*)} = (P, P^*)$

@large m_Q



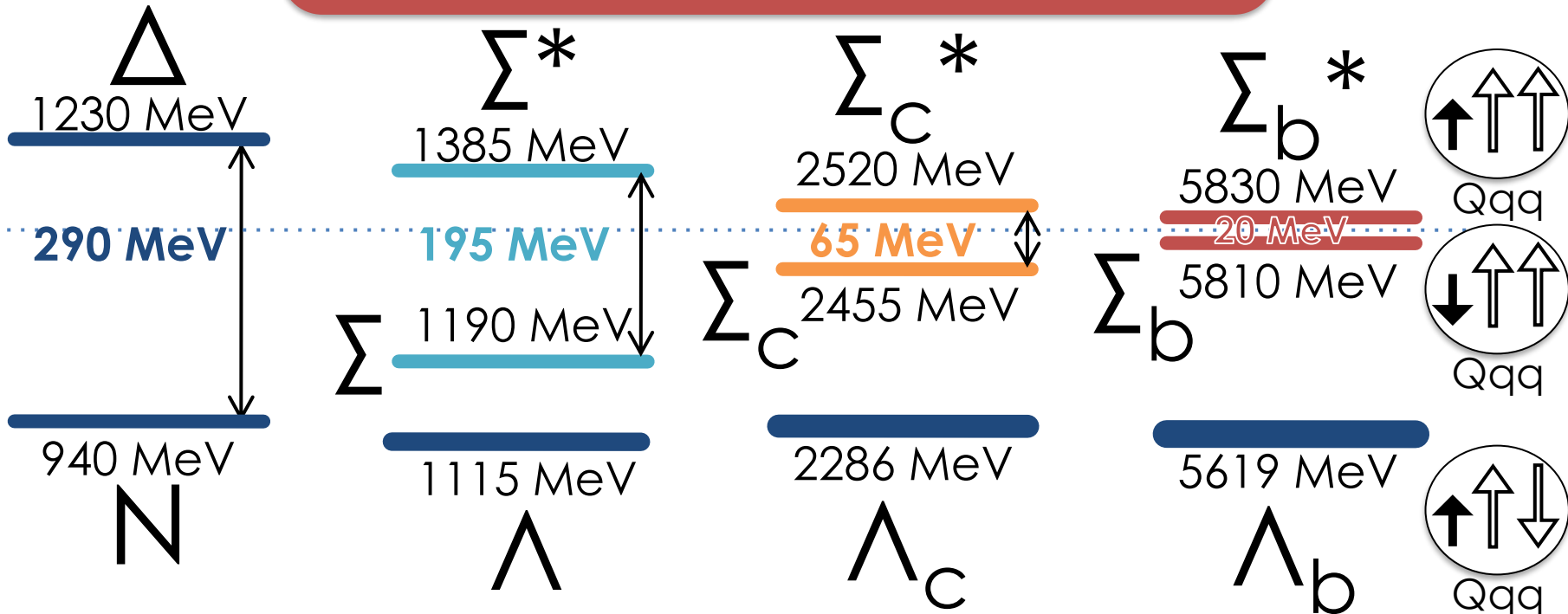
2. Theoretical framework for heavy hadrons

Heavy quark symmetry

$J = 3/2$
 $J = 1/2$
 $(j=1)$

\rightarrow **HQS doublet**
 $\Sigma_Q^{(*)} = (\Sigma_Q, \Sigma_Q^*)$

@large m_Q



$J = 1/2$
 $(j=0)$

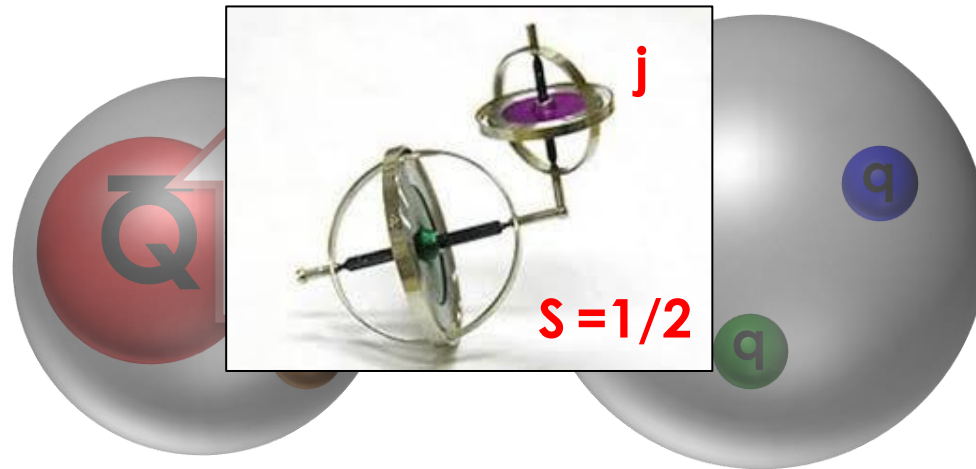
\rightarrow **HQS singlet**
 (Λ_Q)

@large m_Q

2. Theoretical framework for heavy hadrons

Heavy quark symmetry

“ $Q^{\text{bar}}q$ ” meson + “ qqq ” baryon



HQS ($S=1/2$)
conserved



Spin J
conserved



Light spin j
conserved!

$$\vec{J} = \vec{S} + \vec{j}$$

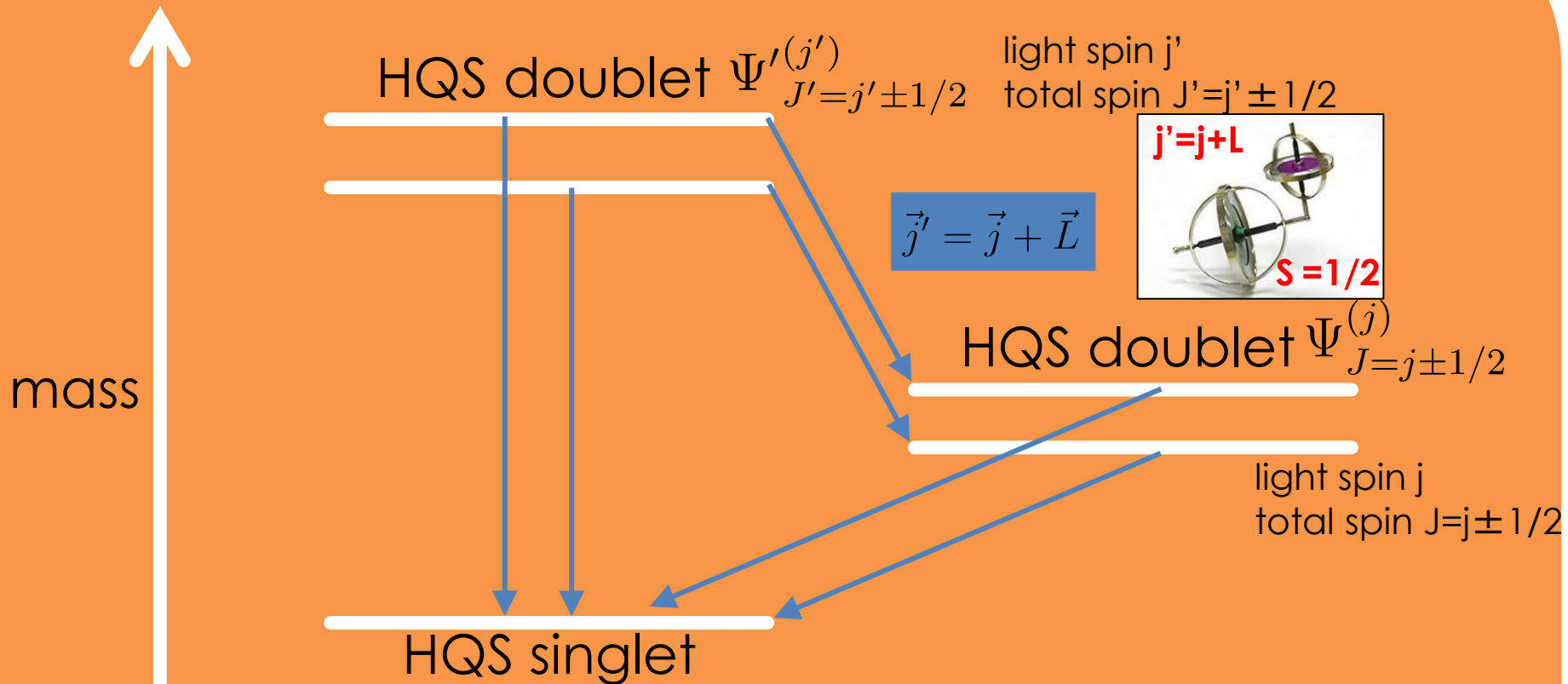
→ **Mass degeneracy for $J=j \pm 1/2$**

Example: Single-heavy hadronic molecules

2. Theoretical framework for heavy hadrons

Heavy quark symmetry

Heavy hadron mass spectrum (example)



Decay width:

N. Isgur and M. B. Wise, Phys. Rev. Lett. 66, 1130 (1991)

$$\Gamma \left[\Psi'_{J'}^{(j')} \rightarrow \Psi_J^{(j)} + \pi \right] \propto (2j+1)(2J'+1) \left\{ \begin{matrix} L & j' & j \\ 1/2 & J & J' \end{matrix} \right\}^2 + O(1/M)$$

Cf. NLO of $1/m_Q$: S. Yasui, Phys. Rev. D91, 014031 (2015)

2. Theoretical framework for heavy hadrons

Heavy hadron effective theory

Heavy Quark Effective Theory

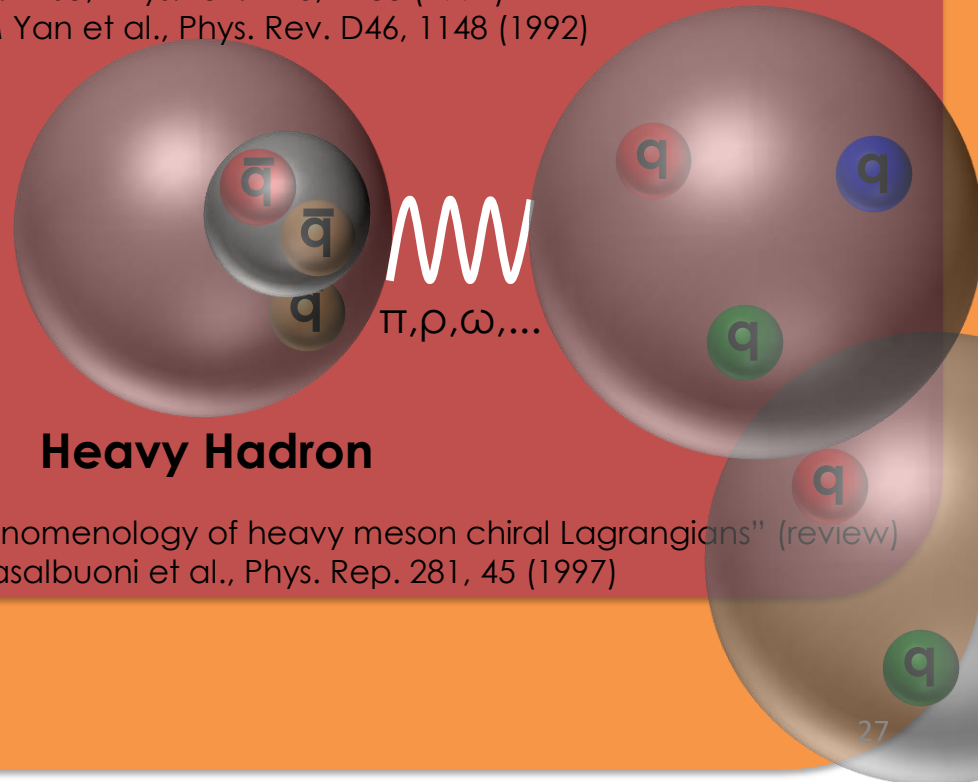
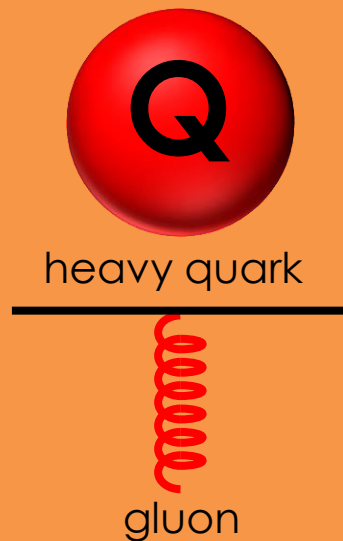
$$\mathcal{L}_{\text{heavy}} = \sum_{Q=1}^{N_h} \left(\bar{Q}_v v \cdot iD Q_v + \bar{Q}_v \frac{(iD_{\perp})^2}{2m_Q} Q_v - c(\mu) g_s \bar{Q}_v \frac{\sigma_{\mu\nu} G^{\mu\nu}}{4m_Q} Q_v + \mathcal{O}(1/m_Q^2) \right)$$

Heavy Hadron Effective Theory

G. Burdman, J.F. Donoghue, Phys. Lett. B280, 287 (1992)

M.B. Wise, Phys. Rev. D45, 2188 (1992)

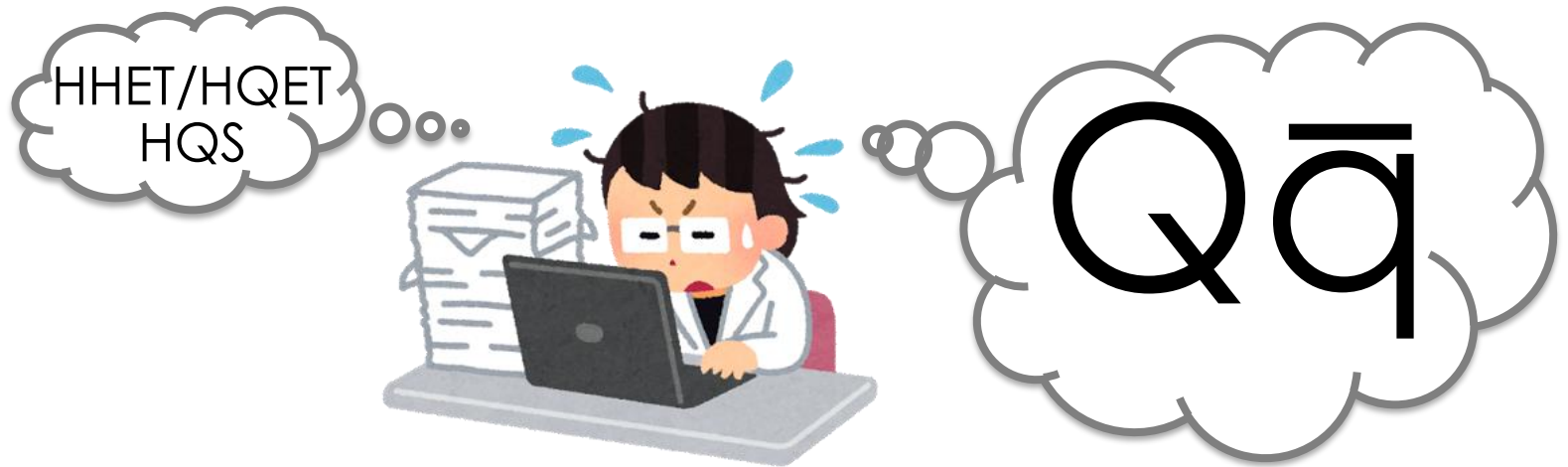
T.-M. Yan et al., Phys. Rev. D46, 1148 (1992)



Cf. "Phenomenology of heavy meson chiral Lagrangians" (review)
R. Casalbuoni et al., Phys. Rep. 281, 45 (1997)

2. Theoretical framework for heavy hadrons Heavy hadron effective theory

Let us construct “heavy hadron effective theory”
based on the heavy quark effective theory!



2. Theoretical framework for heavy hadrons Heavy hadron effective theory

Let us construct “heavy hadron effective theory”
based on the heavy quark effective theory!



Heavy-light meson: Qq^{bar} : $u_{Q\alpha}\bar{v}_{q\beta}$

2. Theoretical framework for heavy hadrons

Heavy hadron effective theory

Heavy-light meson: Qq^{bar} : $u_{Q\alpha}\bar{v}_{q\beta}$ $\not{v}u_Q = u_Q$ $\bar{v}_q\not{v} = \bar{v}_q$

① Constructing effective field at rest frame $v^\mu = (1, \vec{0})$

	spin up	spin down
Q:	$u_{Q\alpha}^{(1)} = \delta_{1\alpha}$	$u_{Q\alpha}^{(2)} = \delta_{2\alpha}$
q ^{bar} :	$v_{q\alpha}^{(2)} = -\delta_{4\alpha}$	$v_{q\alpha}^{(1)} = +\delta_{3\alpha}$

A. F. Falk, H. Georgi, B. Grinstein, and M. B. Wise, Nucl. Phys. B343, 1 (1990)
A. F. Falk, Nucl. Phys. B378, 79 (1992)

→ See next page

4 × 4 dim. Dirac matrix form for Lorentz covariance

	rest frame	v-frame
spin 0:	$u_Q^{(1)}\bar{v}_q^{(1)} + u_Q^{(2)}\bar{v}_q^{(2)} = \frac{1 + \gamma^0}{2}\gamma_5$	$\rightarrow \frac{1 + \not{v}}{2}\gamma_5$
	$u_Q^{(1)}\bar{v}_q^{(2)} = \frac{1 + \gamma^0}{2}\not{\epsilon}^{(+)}$	$\rightarrow \frac{1 + \not{v}}{2}\not{\epsilon}^{(+)}$
spin 1:	$\frac{1}{\sqrt{2}}(u_Q^{(1)}\bar{v}_q^{(1)} - u_Q^{(2)}\bar{v}_q^{(2)}) = \frac{1 + \gamma^0}{2}\not{\epsilon}^{(0)}$	$\rightarrow \frac{1 + \not{v}}{2}\not{\epsilon}^{(0)}$
	$u_Q^{(2)}\bar{v}_q^{(1)} = \frac{1 + \gamma^0}{2}\not{\epsilon}^{(-)}$	$\rightarrow \frac{1 + \not{v}}{2}\not{\epsilon}^{(-)}$
polarization vector: $\epsilon^{(\pm)\mu} = \frac{1}{\sqrt{2}}(0, 1, \pm i, 0)$		$\epsilon^{(0)\mu} = (0, 0, 0, 1)$

2. Theoretical framework for heavy hadrons

Heavy hadron effective theory

SU(2) spin doublet representation of **particle** and **antiparticle** (review)

① SU(2) operator $U = \begin{pmatrix} \alpha & \beta \\ -\beta^* & \alpha^* \end{pmatrix} \quad |\alpha|^2 + |\beta|^2 = 1$

② Special property $U = e^{-i\pi\frac{\sigma_2}{2}} U^* e^{i\pi\frac{\sigma_2}{2}} \quad e^{-i\pi\frac{\sigma_2}{2}} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

③ Fermion (**2**) and antifermion (**2***)

$$\begin{pmatrix} a'_\uparrow \\ a'_\downarrow \end{pmatrix} = U \begin{pmatrix} a_\uparrow \\ a_\downarrow \end{pmatrix} \quad \begin{pmatrix} \bar{a}'_\uparrow \\ \bar{a}'_\downarrow \end{pmatrix} = U^* \begin{pmatrix} \bar{a}_\uparrow \\ \bar{a}_\downarrow \end{pmatrix}$$

$U \neq U^* \quad \mathbf{2} \neq \mathbf{2}^* \quad ???$

④ Unitary transformation

$$e^{-i\pi\frac{\sigma_2}{2}} \begin{pmatrix} \bar{a}'_\uparrow \\ \bar{a}'_\downarrow \end{pmatrix} = e^{-i\pi\frac{\sigma_2}{2}} U^* e^{i\pi\frac{\sigma_2}{2}} e^{-i\pi\frac{\sigma_2}{2}} \begin{pmatrix} \bar{a}_\uparrow \\ \bar{a}_\downarrow \end{pmatrix}$$

⑤ Transformation of **2*** representation

$$\begin{pmatrix} -\bar{a}'_\downarrow \\ \bar{a}'_\uparrow \end{pmatrix} = U \begin{pmatrix} -\bar{a}_\downarrow \\ \bar{a}_\uparrow \end{pmatrix}$$

$$\mathbf{2}: \begin{pmatrix} a_\uparrow \\ a_\downarrow \end{pmatrix} = \mathbf{2}^*: \begin{pmatrix} -\bar{a}_\downarrow \\ \bar{a}_\uparrow \end{pmatrix}$$

2. Theoretical framework for heavy hadrons

Heavy hadron effective theory

① Constructing effective field (cont'd)

spin 0:
(pseudoscalar) $P_v \rightarrow \frac{1 + \not{v}}{2} \gamma_5 P_v$

spin 1:
(vector) $P_v^{*\mu} \rightarrow \frac{1 + \not{v}}{2} P_v^*$

Symmetries of effective fields
 $S_Q: SU(2)$ HQS $U_q: U(N_f)_v$

$u_Q \bar{v}_q \rightarrow S_Q (u_Q \bar{v}_q) U_q^\dagger$

$$\begin{aligned} \frac{1 + \not{v}}{2} \gamma_5 P_v &\rightarrow S_Q \frac{1 + \not{v}}{2} \gamma_5 P_v U_q^\dagger \\ \frac{1 + \not{v}}{2} P_v^* &\rightarrow S_Q \frac{1 + \not{v}}{2} P_v^* U_q^\dagger \end{aligned}$$

“Superposed” field of spin 0 and 1

$$H_v(x) = \frac{1 + \not{v}}{2} \left(P_{v\mu}^*(x) \gamma^\mu + i P_v(x) \gamma_5 \right)$$

$H_v \rightarrow S_Q H_v U_q^\dagger$

2. Theoretical framework for heavy hadrons

Heavy hadron effective theory

① Constructing effective field (cont'd)

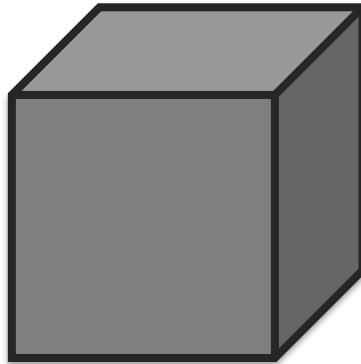
Note: normalization of the state

Original state : $\langle H(p') | H(p) \rangle = 2E_{\vec{p}} (2\pi)^3 \delta^3(\vec{p} - \vec{p}')$

HHET state : $\langle H_v(\vec{k}') | H_v(\vec{k}) \rangle = 2v^0 (2\pi)^3 \delta^3(\vec{k} - \vec{k}')$

Q. How many particles do exist in volume V ?

**Not 1 even for
"one-particle"
state!**



$2E_p (2v^0)$ particles per unit volume

← **Important**
(Don't forget this...)

This FACT is important for constructing the inter-particle potential.

2. Theoretical framework for heavy hadrons

Heavy hadron effective theory

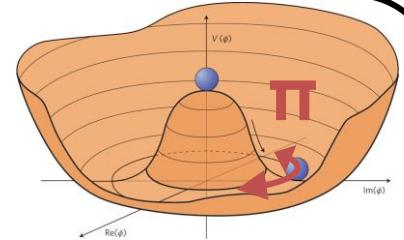
② Constructing effective Lagrangian (leading order of $m_Q \rightarrow \infty$)

$$\mathcal{L}_{\text{heavy-light}} = \text{Tr} \bar{H}_v \not{v} \cdot i D H_v + g \text{Tr} \bar{H}_v H_v \gamma_\mu \gamma_5 A^\mu + O(1/M)$$

chiral covariant derivative: $D^\mu H_v = \partial^\mu H_v - i V^\mu H_v$ **invariant** under **HQS** and **chiral symmetry**

Non-linear chiral transformation

Non-linear rep. of π field: $\xi = \exp(i\phi / \sqrt{2}f_\pi)$ $\phi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$



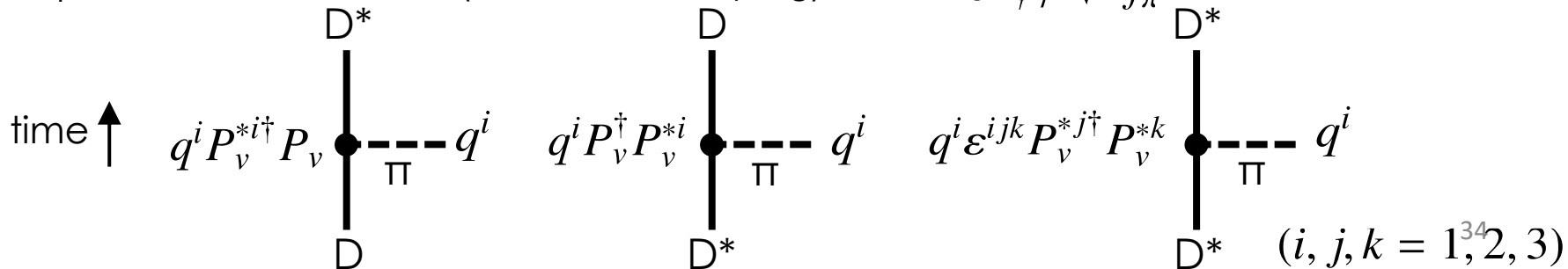
Vector current: $V^\mu(x) = \frac{i}{2}(\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger)$
(even # of π)

Axial-vector current: $A^\mu(x) = \frac{i}{2}(\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger)$
(odd # of π)

$$V^\mu(x) \rightarrow U_q V^\mu(x) U_q^\dagger + i U_q \partial^\mu U_q^\dagger$$

$$A^\mu \rightarrow U_q A^\mu U_q^\dagger$$

Example of vertex structure (axial-vector coupling) $A^\mu \simeq -\partial^\mu \phi / \sqrt{2}f_\pi$ We will see details later.



2. Theoretical framework for heavy hadrons

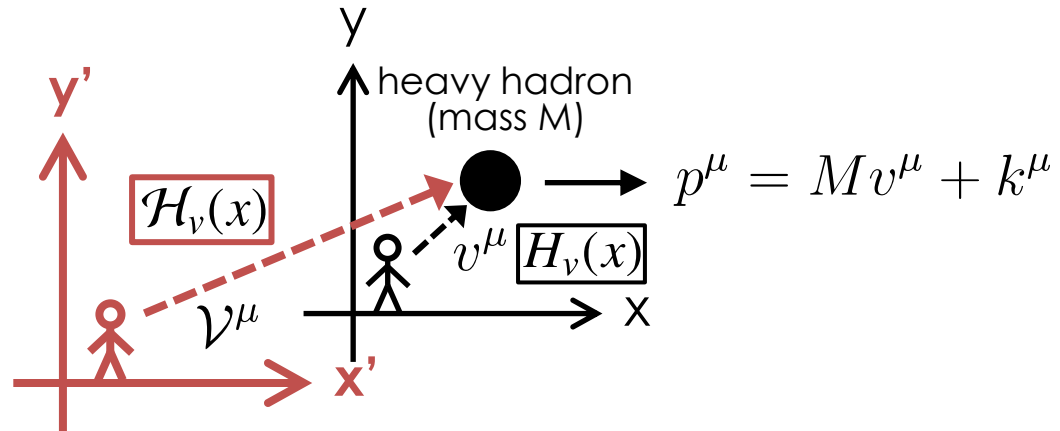
Heavy hadron effective theory

③ Including NLO of $1/M$

Lorentz boost: $v^\mu \rightarrow \mathcal{V}^\mu$

$$\mathcal{V}^\mu = \frac{v^\mu + iD^\mu/M}{|v^\mu + iD^\mu/M|}$$

$$\mathcal{V}_\mu \mathcal{V}^\mu = 1$$



Lorentz boost: $v^\mu \rightarrow \mathcal{V}^\mu$ up to $O(1/M)$

$$v^\mu \rightarrow \mathcal{V}^\mu = v^\mu + \frac{1}{M} (iD^\mu - v^\mu v \cdot iD) + O(1/M^2)$$

$$\mathcal{V}_\mu \mathcal{V}^\mu = 1 + O(1/M^2)$$

New effective field including $O(1/M)$

$$H_v(x) \rightarrow \mathcal{H}_v(x) = H_v(x) + \frac{1}{2M} \left(i\vec{D} H_v(x) - H_v(x) i\overleftarrow{D} - 2v \cdot iD H_v(x) \right) + O(1/M^2)$$

2. Theoretical framework for heavy hadrons

Heavy hadron effective theory

③ Including NLO of $1/M$ (cont'd)

Lorentz boost:

$$w^\mu = v^\mu + q^\mu / M$$



$$\mathcal{H}_w(x) = e^{iq \cdot x} \mathcal{H}_v(x) + \mathcal{O}(1/M^2)$$

“covariant” transformation

✓ construction of interaction term
invariant under velocity rearrangement

Possible form of current: $\text{Tr} \bar{\mathcal{H}}_v \Gamma \mathcal{H}_v \Gamma'$

Γ, Γ' : Dirac matrices
(S, V, T, A, P)

Example of axial-vector current

LO:

$$g \text{Tr} \bar{H}_v H_v \gamma_\mu \gamma_5 A^\mu + \mathcal{O}(1/M)$$

LO: unknown coupling
 g



NLO: $\mathcal{L}_{\pi H_v H_v}^{\text{LO+NLO}} = \left(g + \frac{g_1}{M} \right) \text{Tr} \bar{H}_v H_v \gamma_\mu \gamma_5 A^\mu$

NLO: unknown couplings
 $g \quad g_1 \quad g_2$

$$+ \frac{g}{2M} \left(\text{Tr} v \cdot i D \bar{H}_v H_v \gamma_\mu \gamma_5 A^\mu - \text{Tr} \bar{H}_v v \cdot i D H_v \gamma_\mu \gamma_5 A^\mu \right)$$

$$+ \frac{g}{4M} \epsilon_{\mu\nu\rho\sigma} \left(\text{Tr} i D^\nu \bar{H}_v H_v \sigma^{\rho\sigma} A^\mu - \text{Tr} \bar{H}_v i D^\nu H_v \sigma^{\rho\sigma} A^\mu \right)$$

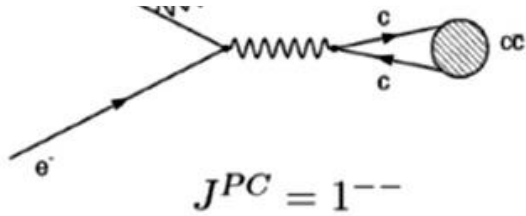
$$+ \frac{g_2}{M} \text{Tr} \bar{H}_v \gamma_\mu \gamma_5 H_v A^\mu + \mathcal{O}(1/M^2) \leftarrow \text{HQE breaking term } (g_2)$$

2. Theoretical framework for heavy hadrons

Experiment \rightleftharpoons Theory

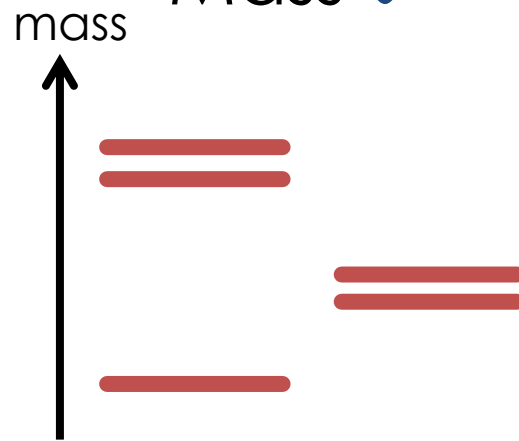
Hadron Spectroscopy

Production

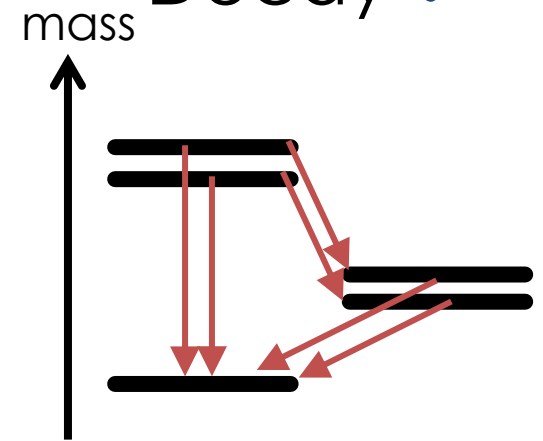


$$JPC = 1^{--}$$

Mass ✓



Decay ✓



Do you have questions?

