チャーム・ボトムの
エキゾチックハドロン物理の最近の発展

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内容

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Heavy-light meson

2. Theoretical framework for heavy hadrons

D meson $\bar{D}$ meson

D^* meson $\bar{D}^*$ meson

$\bar{B}$ meson B meson

$\bar{B}^*$ meson B^* meson

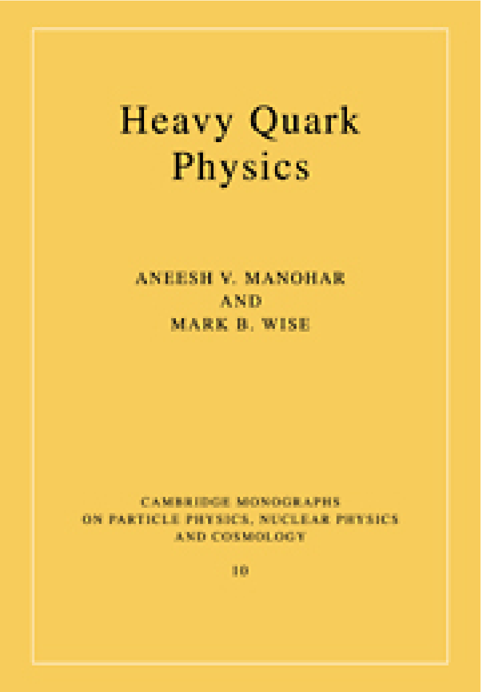
2. Theoretical framework for heavy hadrons

$J^P=0^-$	D meson	$\bar{D}$ meson
$J^P=1^-$	D^* meson	$\bar{D}^*$ meson
$J^P=0^-$	$\bar{B}$ meson	B meson
$J^P=1^-$	$\bar{B}^*$ meson	B^* meson

2. Theoretical framework for heavy hadrons

$J^P=0^-$	D meson ($\bar{q}c$)	$\bar{D}$ meson ($q\bar{c}$)
$J^P=1^-$	D^* meson ($\bar{q}c$)	$\bar{D}^*$ meson ($q\bar{c}$)
$J^P=0^-$	$\bar{B}$ meson ($\bar{q}b$)	B meson ($q\bar{b}$)
$J^P=1^-$	$\bar{B}^*$ meson ($\bar{q}b$)	B^* meson ($q\bar{b}$)

2. Theoretical framework for heavy hadrons
Heavy quark effective theory (HQET)



Heavy Quark Physics

A. V. Manohar and M. W. Wise

Cambridge University Press 2000

2. Theoretical framework for heavy hadrons
Heavy quark effective theory (HQET)

PHYSICS REPORTS

ELSEVIER

Physics Reports 245 (1994) 259–395

Heavy-quark symmetry

Matthias Neubert[1]

Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309, USA

Received February 1994; editor: R. Slansky

Contents:

PHYSICS REPORTS

ELSEVIER

Physics Reports 281 (1997) 145–238

Phenomenology of heavy meson chiral lagrangians

R. Casalbuoni[a], A. Deandrea[b], N. Di Bartolomeo[b], R. Gatto[b], F. Feruglio[c], G. Nardulli[d]

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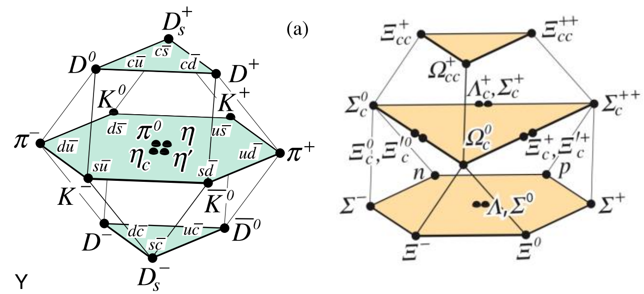
Contents

2. Theoretical framework for heavy hadrons

SU(4) flavor symmetry？？？



16.3 Heavy quarks

H. Georgi, *Lie Algebras in Particle Physics* (1999, 2019)

As you probably know, there are other quarks besides the u, d and s quarks. The c (for charm) quark, the b (for bottom or beauty) quark and the t (for top or truth) quark have all been seen in high energy collisions. They are all unstable, decaying back into the lighter quarks very quickly. But the c and b quarks last long enough to bind into hadrons just as the lighter quarks do. So there is now a rich phenomenology of particles containing c and b quarks. These particle had not been seen, and were barely even imagined when Gell-Mann first explored the consequences of $SU(3)$ symmetry.

2. Theoretical framework for heavy hadrons

- Heavy mass of charm hadron

m_c=1.3 GeV, m_b=4.7 GeV >> Λ_{QCD}=a few 100MeV

- Heavy Quark Symmetry (HQS)

Hadron spin = light quark spin x heavy quark spin

2. Theoretical framework for heavy hadrons
Heavy quark effective theory

Separating QCD Lagrangian to heavy quark part and light quark part

$$\mathcal{L}_{\rm QCD} = \mathcal{L}_{\rm heavy} + \mathcal{L}_{\rm light}$$

$$\mathcal{L}_{\rm heavy} = \sum_{Q} \bar{Q}(i\not{D} - m_Q)Q.$$

We focus on the *heavy* part.

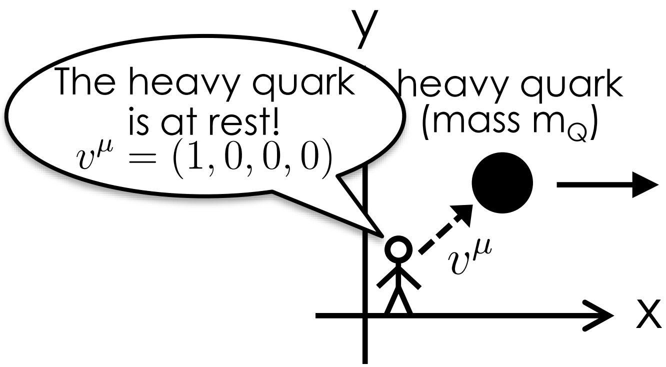
$$\mathcal{L}_{\rm light} = -\frac{1}{4}G^{a}_{\mu\nu}G^{a\mu\nu} + \sum_{q} \bar{q}(i\not{D} - m_q)q$$

gluons + light quarks

2. Theoretical framework for heavy hadrons
Heavy quark effective theory

$$\mathcal{L}_{\rm heavy} = \sum_{Q} \bar{Q}(i\not{D} - m_Q)Q$$

① On-mass-shell part and off-mass-shell part



$$p^\mu = m_Q v^\mu + k^\mu$$

on-mass shell | off-mass shell

on-mass-shell condition

$$v^\mu v_\mu = 1 \quad v^0 > 0$$

off-mass-shell condition

$$k^\mu \ll m_Q$$

② Effective field heavy quark (positive-energy and negative-energy)

Q: $$Q_v(x) = \frac{1+\not{v}}{2} e^{im_Q v\cdot x} Q(x)$$

$$\not{v}\, Q_v(x) = Q_v(x)$$

Qbar: $$\mathcal{Q}_v = \frac{1-\not{v}}{2} e^{im_Q v\cdot x} Q(x)$$

$$\not{v}\, \mathcal{Q}_v = -\mathcal{Q}_v$$

Projection operator

$$\frac{1+\not{v}}{2} \rightarrow \frac{1+\gamma^0}{2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

rest frame $v^\mu = (1, \vec{0})$

We consider only positive-energy part in the effective theory.

2. Theoretical framework for heavy hadrons
Heavy quark effective theory

$$\mathcal{L}_{\text{heavy}} = \sum_Q \bar{Q}(i\not{D} - m_Q)Q$$

3 steps to get HQ effective Lagrangian

① Expressing L_{heavy} in terms of Q_v (heavy quark) and $\mathcal{Q}_v$ (anti heavy quark).

$$\mathcal{L}_{\text{heavy}} = \underline{\bar{Q}_v v\cdot iD Q_v} - \bar{\mathcal{Q}}_v(v\cdot iD + 2m_Q)\mathcal{Q}_v + \bar{Q}_v i\not{D}_\perp \mathcal{Q}_v + \bar{\mathcal{Q}}_v i\not{D}_\perp Q_v$$

no mass term

(m_Q was absorbed in Q_v)

$$D_\perp^\mu = D^\mu - v^\mu v\cdot D$$

② Eliminating $\mathcal{Q}_v$ (anti heavy quark) by using equation of motion.

$$(v\cdot iD + 2m_Q)\mathcal{Q}_v = i\not{D}_\perp Q_v$$

③ We get HQ effective Lagrangian!

$$\mathcal{L}_{\text{heavy}} = \bar{Q}_v\left(v\cdot iD + i\not{D}_\perp \frac{1}{v\cdot iD + 2m_Q} i\not{D}_\perp\right)Q_v$$

$$= \bar{Q}_v v\cdot iD Q_v + \bar{Q}_v \frac{(iD_\perp)^2}{2m_Q} Q_v - g_s \bar{Q}_v \frac{\sigma_{\mu\nu}G^{\mu\nu}}{4m_Q} Q_v + O(1/m_Q^2)$$

2. Theoretical framework for heavy hadrons
Heavy quark effective theory

$$\mathcal{L}_{\text{heavy}} = \sum_{Q} \bar{Q}(i\not{D} - m_Q)Q$$

HQ effective Lagrangian (N$_h$ heavy flavors)

$$\mathcal{L}_{\text{heavy}} = \sum_{Q=1}^{N_h} \left(\bar{Q}_v v \cdot iD Q_v + \bar{Q}_v \frac{(iD_\perp)^2}{2m_Q} Q_v - c(\mu) g_s \bar{Q}_v \frac{\sigma_{\mu\nu} G^{\mu\nu}}{4m_Q} Q_v + O(1/m_Q^2) \right)$$

3 Key Points

1. Velocity rearrangement

2. Radiative corrections

3. Heavy quark spin symmetry ($m_Q \rightarrow \infty$)

2. Theoretical framework for heavy hadrons
Heavy quark effective theory

$$\mathcal{L}_{\text{heavy}} = \sum_{Q} \bar{Q}(i\not{D} - m_Q)Q$$

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specific v^μ — 1. Velocity rearrangement

① LO: Superposition of various v^μ

$$\mathcal{L}_{\text{heavy}}^{\text{total}} = \sum_v \mathcal{L}_{\text{heavy},v} \quad p^\mu = m_Q v^\mu + k^\mu$$

② Change of v^μ and k^μ

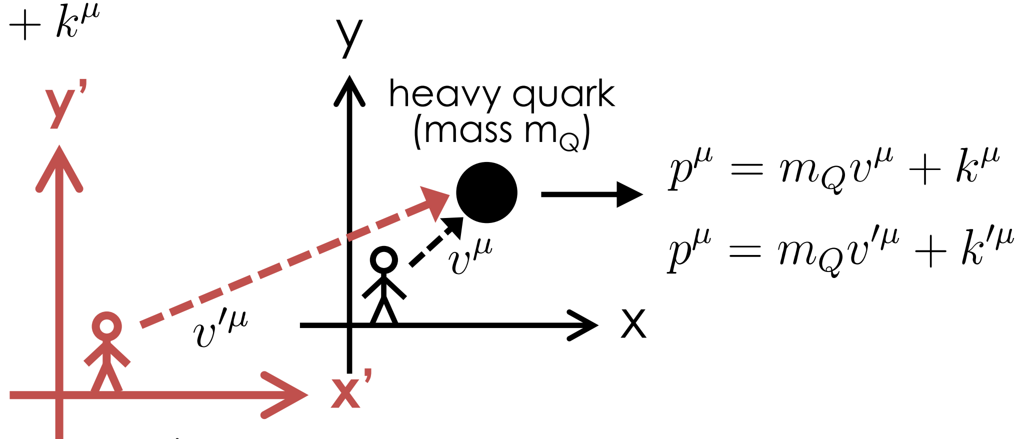
$$v^\mu \to v^\mu + \varepsilon^\mu / m_Q \qquad v \cdot \varepsilon = 0$$
$$k^\mu \to k^\mu - \varepsilon^\mu$$

③ Condition of $v^2 = 1$

$$v^2 \simeq 1 + \mathcal{O}(1/m_Q^2) \quad \not{v} Q_v = Q_v$$

④ Transformation of Q_v

$$Q_v \to e^{i\varepsilon \cdot x} \left(Q_v + \frac{\not{\varepsilon}}{2m_Q} Q_v \right)$$



$$v'^\mu = v^\mu + \varepsilon^\mu / m_Q$$
$$k'^\mu = k^\mu - \varepsilon^\mu$$

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Heavy quark effective theory

$$\mathcal{L}_{\text{heavy}} = \sum_Q \bar{Q}(i\not{D} - m_Q)Q$$

HQ effective Lagrangian (N_h heavy flavors)

$$\mathcal{L}_{\text{heavy}} = \sum_{Q=1}^{N_h}\left(\bar{Q}_v v\cdot iDQ_v + \bar{Q}_v\frac{(iD_\perp)^2}{2m_Q}Q_v - c(\mu)g_s\bar{Q}_v\frac{\sigma_{\mu\nu}G^{\mu\nu}}{4m_Q}Q_v + \mathcal{O}(1/m_Q^2)\right)$$

specific v^μ — 1. Velocity rearrangement

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$$\mathcal{L}^{\text{total}}_{\text{heavy}} = \sum_v \mathcal{L}_{\text{heavy},v} \quad p^\mu = m_Q v^\mu + k^\mu$$

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④ Transformation of Q_v

$$Q_v \to e^{i\varepsilon\cdot x}\left(Q_v + \frac{\not{\varepsilon}}{2m_Q}Q_v\right)$$

⑤ Transformation of Lagrangian

$$\mathcal{L}_0 \equiv \bar{Q}_v iv\cdot DQ_v$$
$$\mathcal{L}_1 \equiv \bar{Q}_v\frac{(iD_\perp)^2}{2m_Q}Q_v - g\,\bar{Q}_v\frac{\sigma_{\mu\nu}G^{\mu\nu}}{4m_Q}Q_v$$

$$\mathcal{L}_0 \to \mathcal{L}_0 + \frac{1}{m_Q}\bar{Q}_v i\varepsilon\cdot DQ_v$$
$$\mathcal{L}_1 \to \mathcal{L}_1 - \frac{1}{m_Q}\bar{Q}_v i\varepsilon\cdot DQ_v$$

⑥ Invariance of HQ effective Lagrangian

$\mathcal{L}_0 + \mathcal{L}_1$: invariant for changing v^μ

We can choose any v-frame.

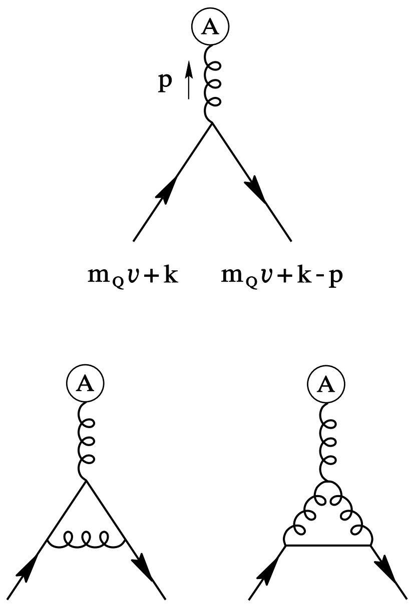
2. Theoretical framework for heavy hadrons
Heavy quark effective theory

$$\mathcal{L}_{\text{heavy}} = \sum_Q \bar{Q}(i\not{D} - m_Q)Q$$

HQ effective Lagrangian (N_h heavy flavors)

$$\mathcal{L}_{\text{heavy}} = \sum_{Q=1}^{N_h}\left(\bar{Q}_v v\cdot iDQ_v + \bar{Q}_v\frac{(iD_\perp)^2}{2m_Q}Q_v - c(\mu)g_s\bar{Q}_v\frac{\sigma_{\mu\nu}G^{\mu\nu}}{4m_Q}Q_v + \mathcal{O}(1/m_Q^2)\right)$$

2. Radiative corrections



Tree level:

$$c(m_Q) = 1 + \mathcal{O}(\alpha_s(m_Q))$$

Leading logarithmic approximation:

$$c(\mu) = \left(\frac{\alpha_s(m_Q)}{\alpha_s(\mu)}\right)^{9/(33-2N_f)}$$

Figure: M. Neubert, Subnucl. Ser. 34, 98 (1997)
(arXiv:9610266 [hep-ph])

2. Theoretical framework for heavy hadrons
Heavy quark effective theory

$$\mathcal{L}_{\text{heavy}} = \sum_{Q} \bar{Q}(i\not{D} - m_Q)Q$$

HQ effective Lagrangian (N_h heavy flavors)

$$\mathcal{L}_{\text{heavy}} = \sum_{Q=1}^{N_h} \left(\bar{Q}_v v \cdot iD Q_v + \bar{Q}_v \frac{(iD_\perp)^2}{2m_Q} Q_v - c(\mu) g_s \bar{Q}_v \frac{\sigma_{\mu\nu} G^{\mu\nu}}{4m_Q} Q_v + \mathcal{O}(1/m_Q^2) \right)$$

3. Heavy quark spin symmetry ($m_Q \to \infty$)

① Leading order of $1/m_Q$

$$\mathcal{L}_0 \equiv \bar{Q}_v i v \cdot D Q_v$$

② Heavy quark spin (HQS) transformation

$$Q_v \to S Q_v \qquad S \in \mathrm{SU}(2)_{\text{spin}}$$

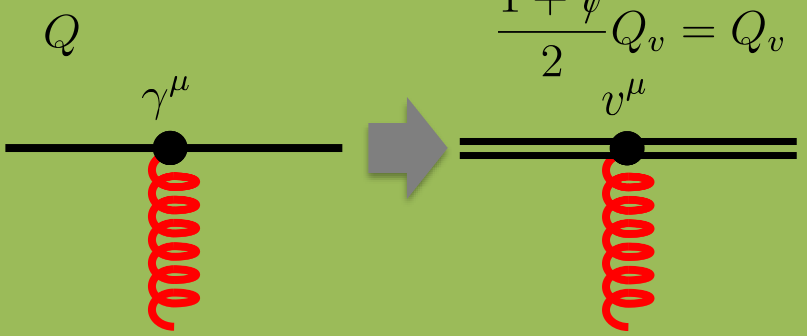
Lagrangian L_0 is invariant for HQS:

$$\mathcal{L}_0 \to \mathcal{L}_0$$

③ HQS is a conserved quantity.

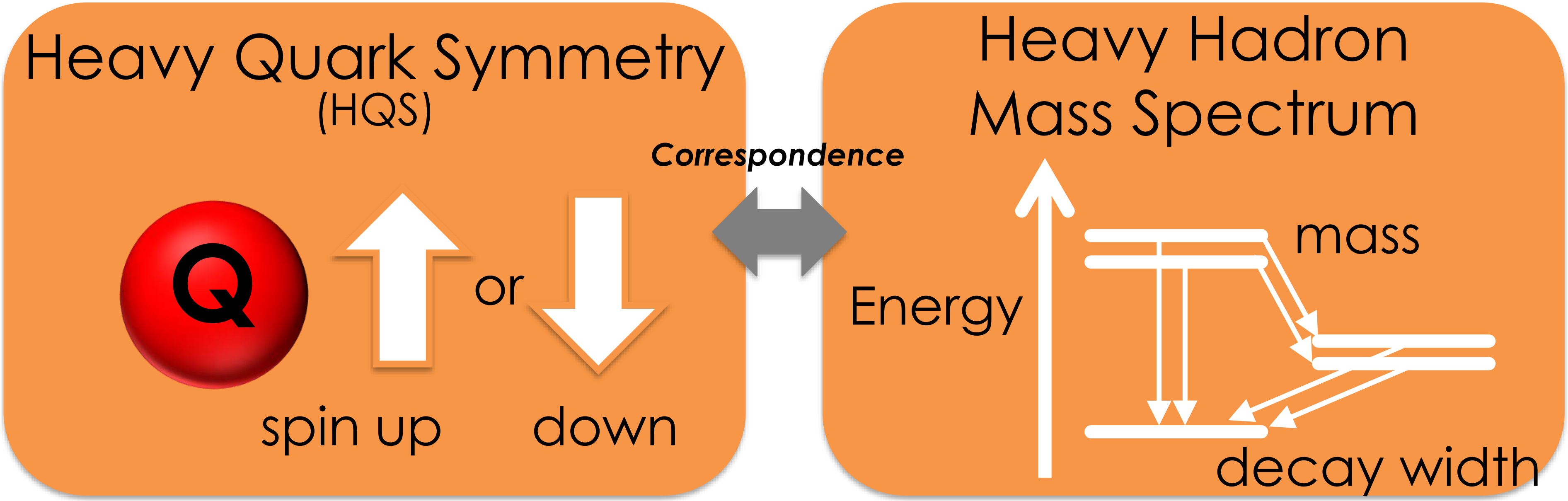
Where is γ^μ matrix?

$$\frac{1+\not{v}}{2} Q_v = Q_v$$



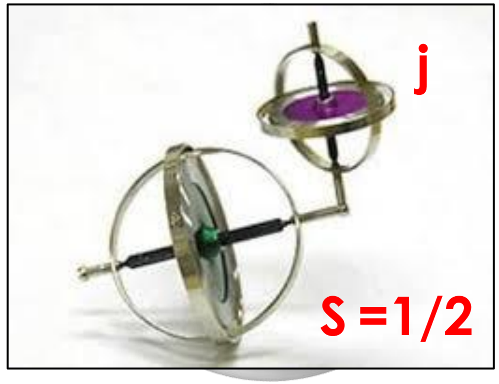
$$\gamma^\mu \to \frac{1+\not{v}}{2} \gamma^\mu \frac{1+\not{v}}{2} = v^\mu \frac{1+\not{v}}{2} \to v^\mu$$

2. Theoretical framework for heavy hadrons
Heavy quark symmetry



2. Theoretical framework for heavy hadrons
Heavy quark symmetry

"Qq^{bar}" meson



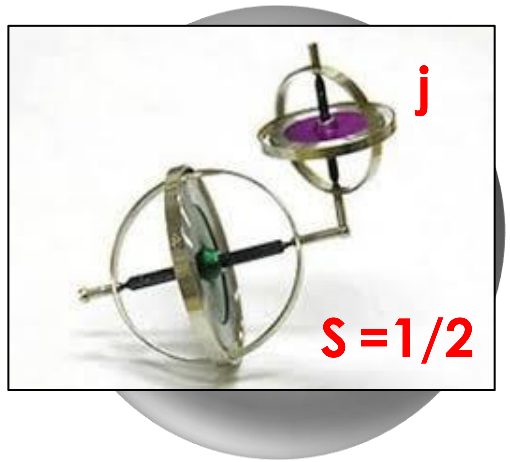
HQS (S=1/2) conserved ⇦ Spin J conserved ⇨ Light spin j **conserved!**

$$\vec{J} = \vec{S} + \vec{j}$$

→ **Mass degeneracy for J=j±1/2**

2. Theoretical framework for heavy hadrons
Heavy quark symmetry

"Qqq" baryon



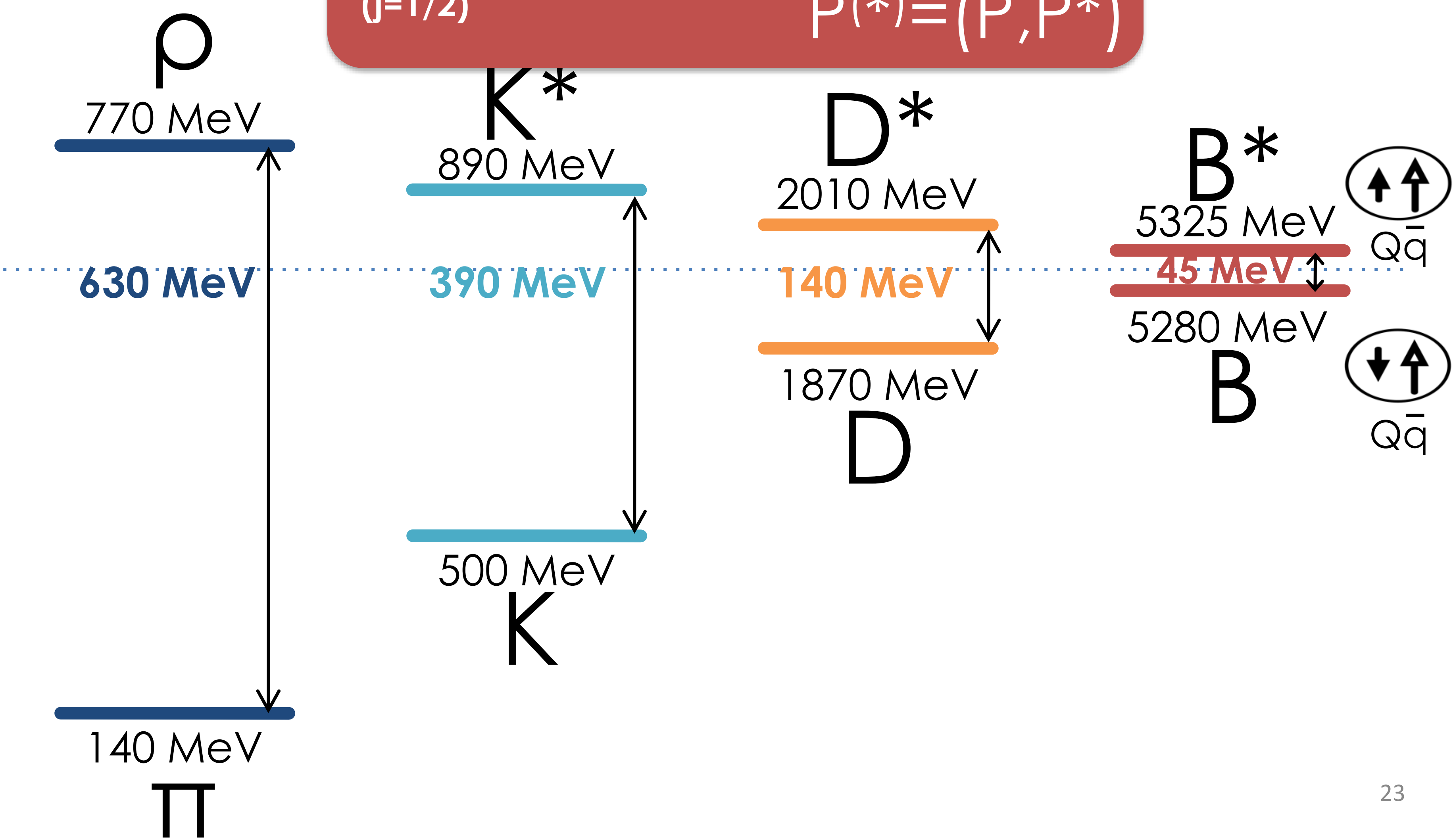
HQS (S=1/2) conserved ← Spin J conserved → Light spin j **conserved!**

$$\vec{J} = \vec{S} + \vec{j}$$

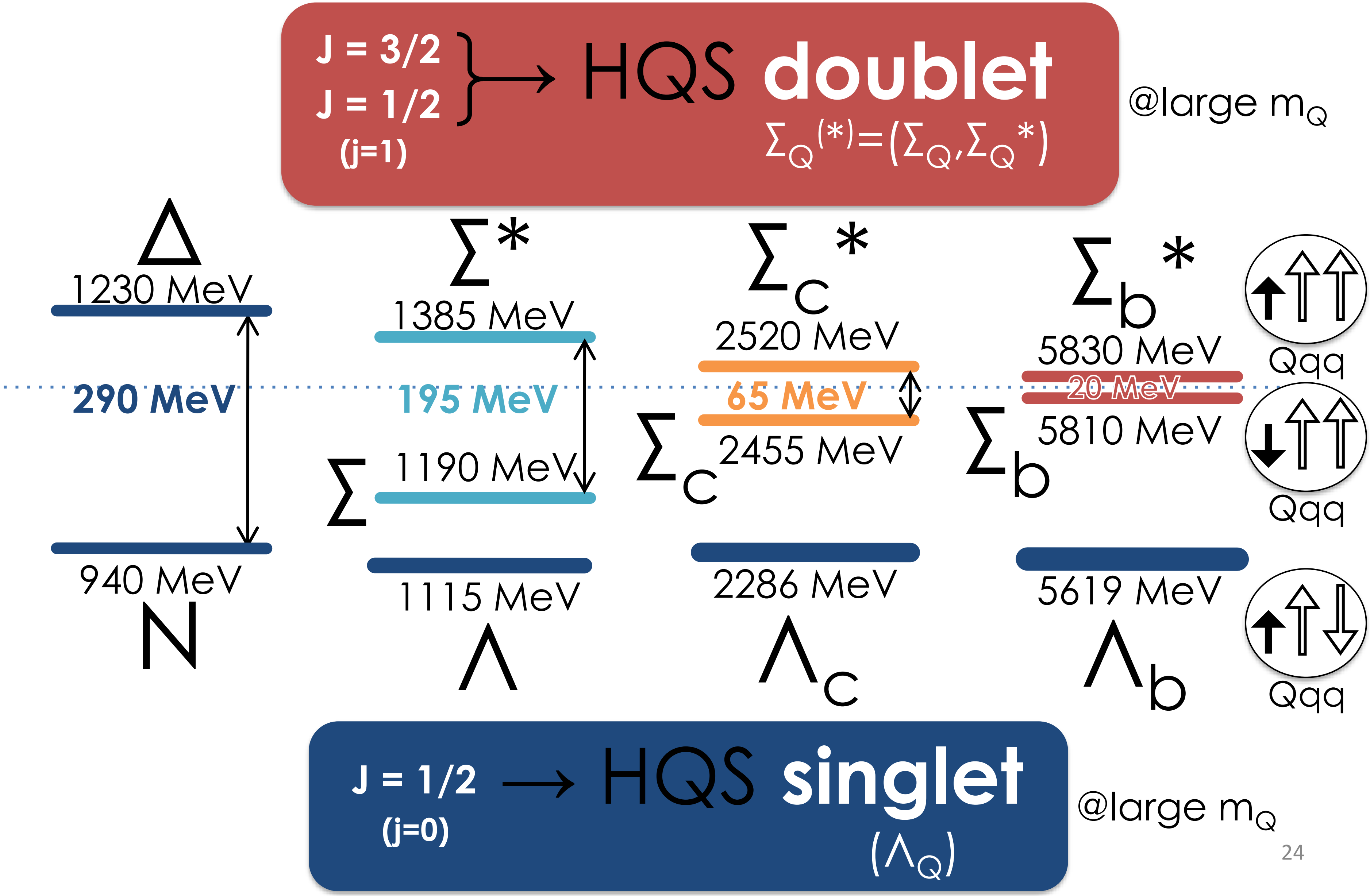
→ **Mass degeneracy for J=j±1/2**

2. Theoretical framework for heavy hadrons
Heavy quark symmetry

J = 1
J = 0
(j=1/2)
→ HQS **doublet** @large m_Q
$P^{(*)}=(P,P^*)$

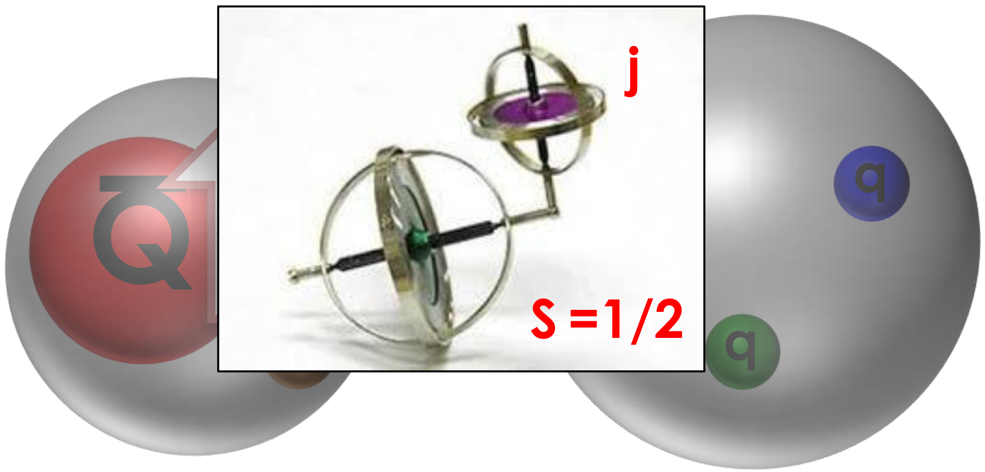


2. Theoretical framework for heavy hadrons
Heavy quark symmetry



2. Theoretical framework for heavy hadrons
Heavy quark symmetry

"$Q^{bar}q$" meson + "qqq" baryon



HQS (S=1/2) conserved ← Spin J conserved → Light spin j **conserved!**

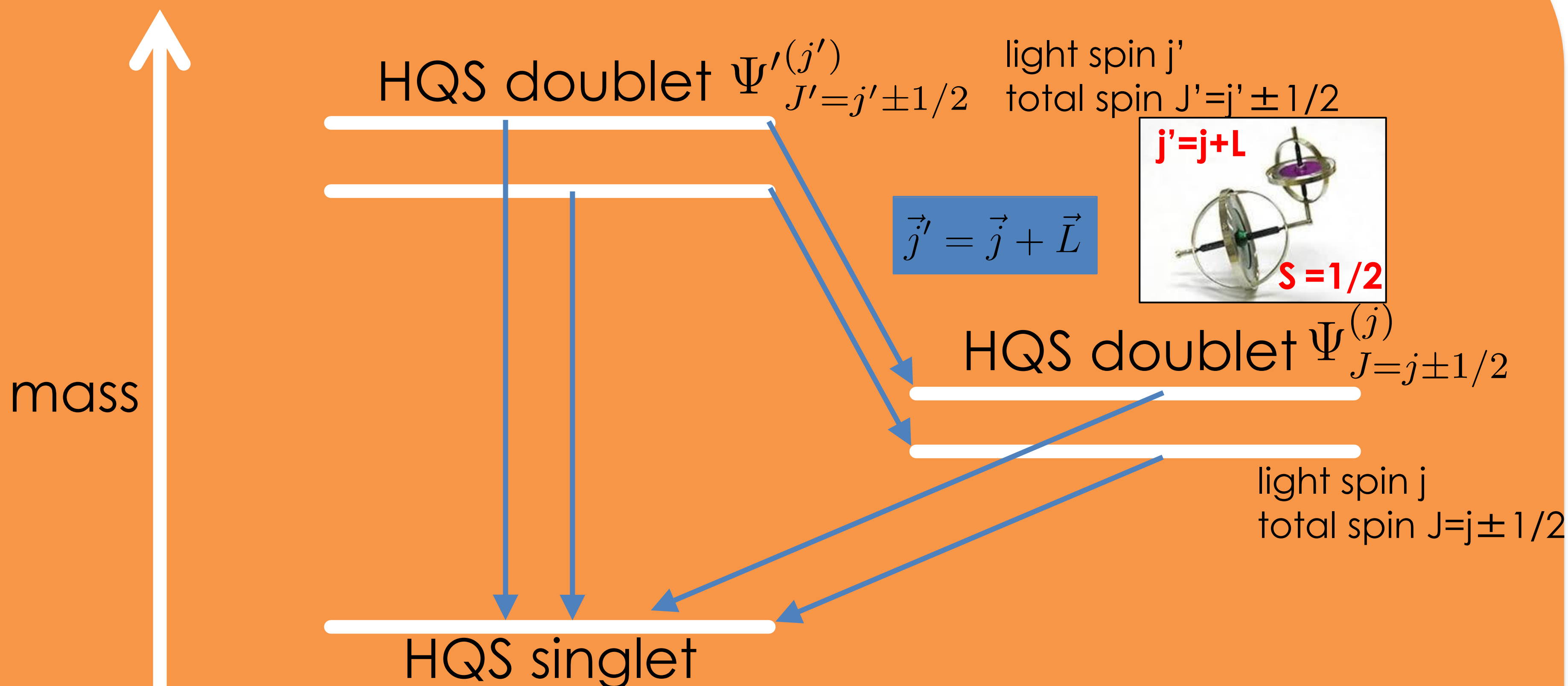
$$\vec{J} = \vec{S} + \vec{j}$$

→ Mass degeneracy for J=j±1/2

Example: Single-heavy hadronic molecules

2. Theoretical framework for heavy hadrons
Heavy quark symmetry

Heavy hadron mass spectrum (example)



Deacy width:

N. Isgur and M. B. Wise, Phys. Rev. Lett. 66, 1130 (1991)

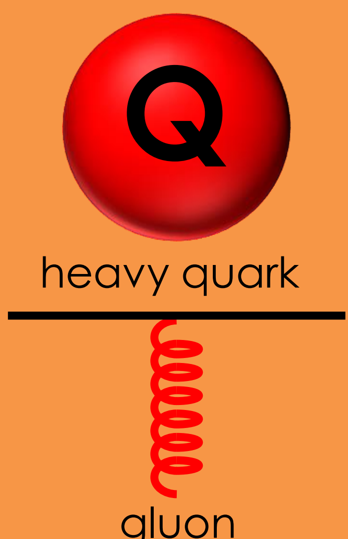
$$\Gamma\left[\Psi'^{(j')}_{J'} \to \Psi^{(j)}_{J} + \pi\right] \propto (2j+1)(2J'+1)\left|\begin{Bmatrix} L & j' & j \\ 1/2 & J & J' \end{Bmatrix}\right|^2 + \mathcal{O}(1/M)$$

Cf. NLO of $1/m_Q$: S. Yasui, Phys. Rev. D91, 014031 (2015)

2. Theoretical framework for heavy hadrons
Heavy hadron effective theory

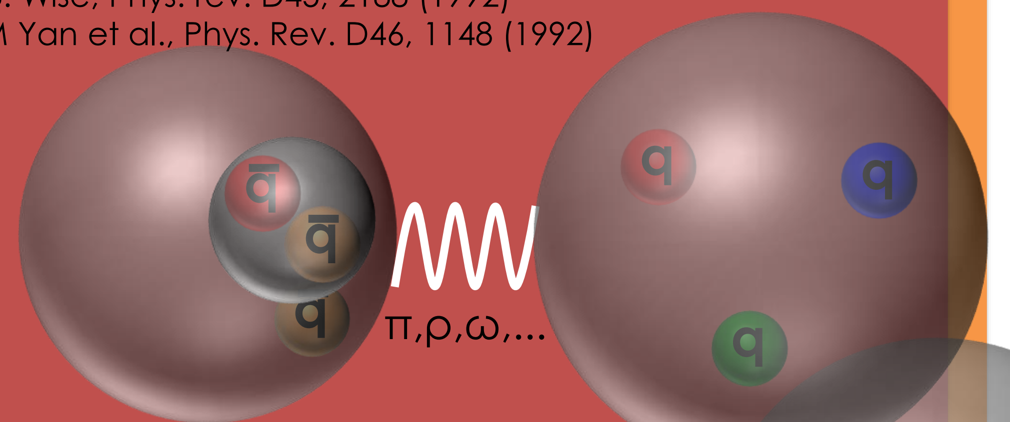
Heavy Quark Effective Theory

$$\mathcal{L}_{\text{heavy}} = \sum_{Q=1}^{N_h} \left(\bar{Q}_v v \cdot iDQ_v + \bar{Q}_v \frac{(iD_\perp)^2}{2m_Q} Q_v - c(\mu) g_s \bar{Q}_v \frac{\sigma_{\mu\nu} G^{\mu\nu}}{4m_Q} Q_v + O(1/m_Q^2) \right)$$



Heavy Hadron Effective Theory

G. Burdman, J.F. Donoghue, Phys. Lett. B280, 287 (1992)
M.B. Wise, Phys. rev. D45, 2188 (1992)
T.-M Yan et al., Phys. Rev. D46, 1148 (1992)



Heavy Hadron

Cf. "Phenomenology of heavy meson chiral Lagrangians" (review)
R. Casalbuoni et al., Phys. Rep. 281, 45 (1997)

2. Theoretical framework for heavy hadrons
Heavy hadron effective theory

Let us construct "heavy hadron effective theory"

based on the heavy quark effective theory!

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Heavy hadron effective theory

Let us construct "heavy hadron effective theory"

based on the heavy quark effective theory!

Heavy-light meson: Qq^{bar}: $u_{Q\alpha}\bar{v}_{q\beta}$

2. Theoretical framework for heavy hadrons
Heavy hadron effective theory

Heavy-light meson: Qq$^{\text{bar}}$: $u_{Q\alpha}\bar{v}_{q\beta}$ $\quad \not{v} u_Q = u_Q \quad \bar{v}_q \not{v} = \bar{v}_q$

① Constructing effective field at rest frame $v^\mu = (1, \vec{0})$

	spin up	spin down
Q:	$u^{(1)}_{Q\alpha} = \delta_{1\alpha}$	$u^{(2)}_{Q\alpha} = \delta_{2\alpha}$
q$^{\text{bar}}$:	$v^{(2)}_{q\alpha} = -\delta_{4\alpha}$	$v^{(1)}_{q\alpha} = +\delta_{3\alpha}$

A. F. Falk, H. Georgi, B. Grinstein, and M. B. Wise, Nucl. Phys. B343, 1 (1990)
A. F. Falk, Nucl. Phys. B378, 79 (1992)

→ **See next page**

4×4 dim. Dirac matrix form for Lorentz covariance

rest frame → v-frame

spin 0:
$$u^{(1)}_Q \bar{v}^{(1)}_q + u^{(2)}_Q \bar{v}^{(2)}_q = \frac{1+\gamma^0}{2}\gamma_5 \;\to\; \frac{1+\not{v}}{2}\gamma_5$$

spin 1:
$$u^{(1)}_Q \bar{v}^{(2)}_q = \frac{1+\gamma^0}{2}\not{\epsilon}^{(+)} \;\to\; \frac{1+\not{v}}{2}\not{\epsilon}^{(+)}$$
$$\frac{1}{\sqrt{2}}\left(u^{(1)}_Q \bar{v}^{(1)}_q - u^{(2)}_Q \bar{v}^{(2)}_q\right) = \frac{1+\gamma^0}{2}\not{\epsilon}^{(0)} \;\to\; \frac{1+\not{v}}{2}\not{\epsilon}^{(0)}$$
$$u^{(2)}_Q \bar{v}^{(1)}_q = \frac{1+\gamma^0}{2}\not{\epsilon}^{(-)} \;\to\; \frac{1+\not{v}}{2}\not{\epsilon}^{(-)}$$

polarization vector: $\epsilon^{(\pm)\mu} = \frac{1}{\sqrt{2}}(0, 1, \pm i, 0) \quad \epsilon^{(0)\mu} = (0, 0, 0, 1)$

2. Theoretical framework for heavy hadrons
Heavy hadron effective theory

SU(2) spin doublet representation of particle and antiparticle (review)

① SU(2) operator $U = \begin{pmatrix} \alpha & \beta \\ -\beta^* & \alpha^* \end{pmatrix}$ $|\alpha|^2 + |\beta|^2 = 1$

② Special property $U = e^{-i\pi\frac{\sigma_2}{2}} U^* e^{i\pi\frac{\sigma_2}{2}}$ $e^{-i\pi\frac{\sigma_2}{2}} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

③ Fermion (**2**) and antifermion (**2***)

$$\begin{pmatrix} a'_\uparrow \\ a'_\downarrow \end{pmatrix} = U \begin{pmatrix} a_\uparrow \\ a_\downarrow \end{pmatrix} \qquad \begin{pmatrix} \bar{a}'_\uparrow \\ \bar{a}'_\downarrow \end{pmatrix} = U^* \begin{pmatrix} \bar{a}_\uparrow \\ \bar{a}_\downarrow \end{pmatrix}$$

$U \neq U^*$ **2≠2* ???**

④ Unitary transformation

$$e^{-i\pi\frac{\sigma_2}{2}} \begin{pmatrix} \bar{a}'_\uparrow \\ \bar{a}'_\downarrow \end{pmatrix} = e^{-i\pi\frac{\sigma_2}{2}} U^* e^{i\pi\frac{\sigma_2}{2}} e^{-i\pi\frac{\sigma_2}{2}} \begin{pmatrix} \bar{a}_\uparrow \\ \bar{a}_\downarrow \end{pmatrix}$$

⑤ Transformation of **2*** representation

$$\begin{pmatrix} -\bar{a}'_\downarrow \\ \bar{a}'_\uparrow \end{pmatrix} = U \begin{pmatrix} -\bar{a}_\downarrow \\ \bar{a}_\uparrow \end{pmatrix}$$

2: $\begin{pmatrix} a_\uparrow \\ a_\downarrow \end{pmatrix}$ = **2***: $\begin{pmatrix} -\bar{a}_\downarrow \\ \bar{a}_\uparrow \end{pmatrix}$

2. Theoretical framework for heavy hadrons
Heavy hadron effective theory

① Constructing effective field (cont'd)

spin 0: (pseudoscalar) $P_v \to \frac{1+\not v}{2}\gamma_5 P_v$

spin 1: (vector) $P_v^{*\mu} \to \frac{1+\not v}{2}\not P_v^*$

Symmetries of effective fields

S_Q: SU(2) HQS U_q: $U(N_f)_V$

$$u_Q \bar v_q \to S_Q \left(u_Q \bar v_q\right) U_q^\dagger$$

$$\frac{1+\not v}{2}\gamma_5 P_v \to S_Q \frac{1+\not v}{2}\gamma_5 P_v U_q^\dagger$$

$$\frac{1+\not v}{2}\not P_v^* \to S_Q \frac{1+\not v}{2}\not P_v^* U_q^\dagger$$

"Superposed" field of spin 0 and 1

$$H_v(x) = \frac{1+\not v}{2}\left(P_{v\mu}^*(x)\gamma^\mu + iP_v(x)\gamma_5\right)$$

$$H_v \to S_Q H_v U_q^\dagger$$

2. Theoretical framework for heavy hadrons
Heavy hadron effective theory

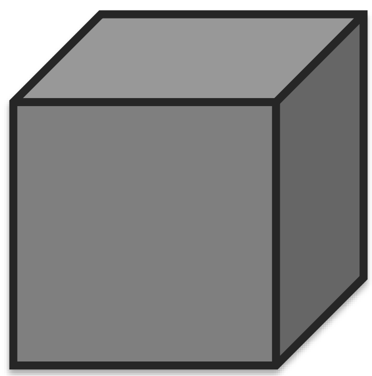
① Constructing effective field (cont'd)

Note: normalization of the state

Original state : $\langle H(p')|H(p)\rangle = 2E_{\vec{p}}\,(2\pi)^3\delta^3(\vec{p}-\vec{p}')$

HHET state : $\langle H_v(\vec{k}')|H_v(\vec{k})\rangle = 2v^0\,(2\pi)^3\delta^3(\vec{k}-\vec{k}')$

Q. How many particles do exist in volume V ?



Not 1 even **for "one-particle" state!**

$2E_p$ $(2v^0)$ particles *per unit volume* ← **Important** **(Don't forget this...)**

This FACT is important for constructing the inter-particle potential.

2. Theoretical framework for heavy hadrons
Heavy hadron effective theory

② Constructing effective Lagrangian (leading order of $m_Q \to \infty$)

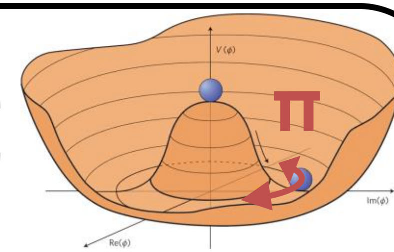
$$\mathcal{L}_{\text{heavy-light}} = \mathrm{Tr}\bar{H}_v v\cdot iDH_v + g\mathrm{Tr}\bar{H}_v H_v \gamma_\mu \gamma_5 A^\mu + \mathcal{O}(1/M)$$

chiral covariant derivative: $D^\mu H_v = \partial^\mu H_v - iV^\mu H_v$

invariant under **HQS** and **chiral symmetry**

Non-linear chiral transformation

Non-linear rep. of π field: $\xi = \exp(i\phi/\sqrt{2}f_\pi)$ $\phi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$



Vector current: $V^\mu(x) = \frac{i}{2}(\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger)$
(even # of π)

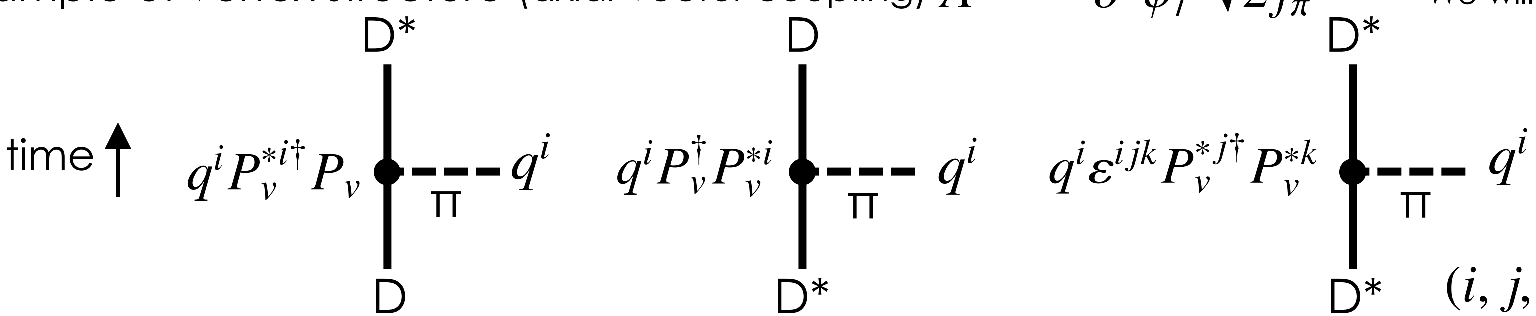
$$V^\mu(x) \to U_q V^\mu(x) U_q^\dagger + iU_q \partial^\mu U_q^\dagger$$

Axial-vector current: $A^\mu(x) = \frac{i}{2}(\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger)$
(odd # of π)

$$A^\mu \to U_q A^\mu U_q^\dagger$$

Example of vertex structure (axial-vector coupling) $A^\mu \simeq -\partial^\mu \phi/\sqrt{2}f_\pi$

We will see details later.



$(i, j, k = 1, 2, 3)$

2. Theoretical framework for heavy hadrons
Heavy hadron effective theory

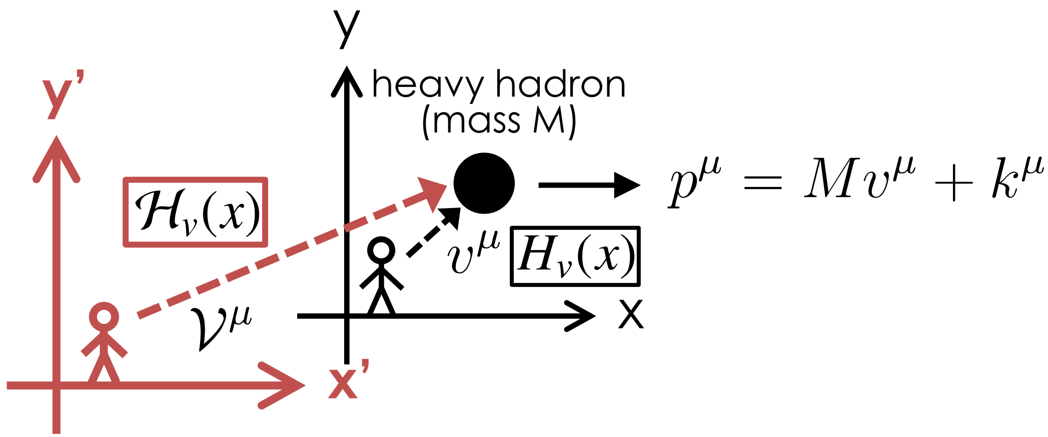
③ Including NLO of 1/M

Cf. "velocity-rearrangement" $O(1/m_Q)$ in HQET

Lorentz boost: $v^\mu \to V^\mu$

$$\mathcal{V}^\mu = \frac{v^\mu + iD^\mu/M}{|v^\mu + iD^\mu/M|}$$

$$\mathcal{V}_\mu \mathcal{V}^\mu = 1$$



Lorentz boost: $v^\mu \to V^\mu$ up to O(1/M)

$$v^\mu \to \mathcal{V}^\mu = v^\mu + \frac{1}{M}\left(iD^\mu - v^\mu v\cdot iD\right) + O(1/M^2)$$

$$\mathcal{V}_\mu \mathcal{V}^\mu = 1 + O(1/M^2)$$

New effective field including O(1/M)

$$H_v(x) \to \mathcal{H}_v(x) = H_v(x) + \frac{1}{2M}\left(i\vec{\not{D}}H_v(x) - H_v(x)i\overleftarrow{\not{D}} - 2v\cdot iDH_v(x)\right) + O(1/M^2)$$

M. E. Luke, A. V. Manohar, Phys. Lett. B286, 348 (1992)
N. Kitazawa, T. Kurimoto, Phys. Lett. B323, 65 (1994)
S. Yasui, K. Sudoh, Phys. Rev. C88,015201(2013)

2. Theoretical framework for heavy hadrons
Heavy hadron effective theory

③ Including NLO of 1/M (cont'd)

"covariant" transformation

Lorentz boost: $w^\mu = v^\mu + q^\mu/M$ ⟺ $\mathcal{H}_w(x) = e^{iq\cdot x}\mathcal{H}_v(x) + \mathcal{O}(1/M^2)$

✓ construction of interaction term
invariant under velocity rearrangement

Possible form of current: $\mathrm{Tr}\,\overline{\mathcal{H}}_v\Gamma\mathcal{H}_v\Gamma'$

Γ, Γ': Dirac matrices (S,V,T,A,P)

Example of axial-vector current

LO: $g\mathrm{Tr}\bar{H}_v H_v \gamma_\mu\gamma_5 A^\mu + \mathcal{O}(1/M)$

LO: unknown coupling g

NLO:

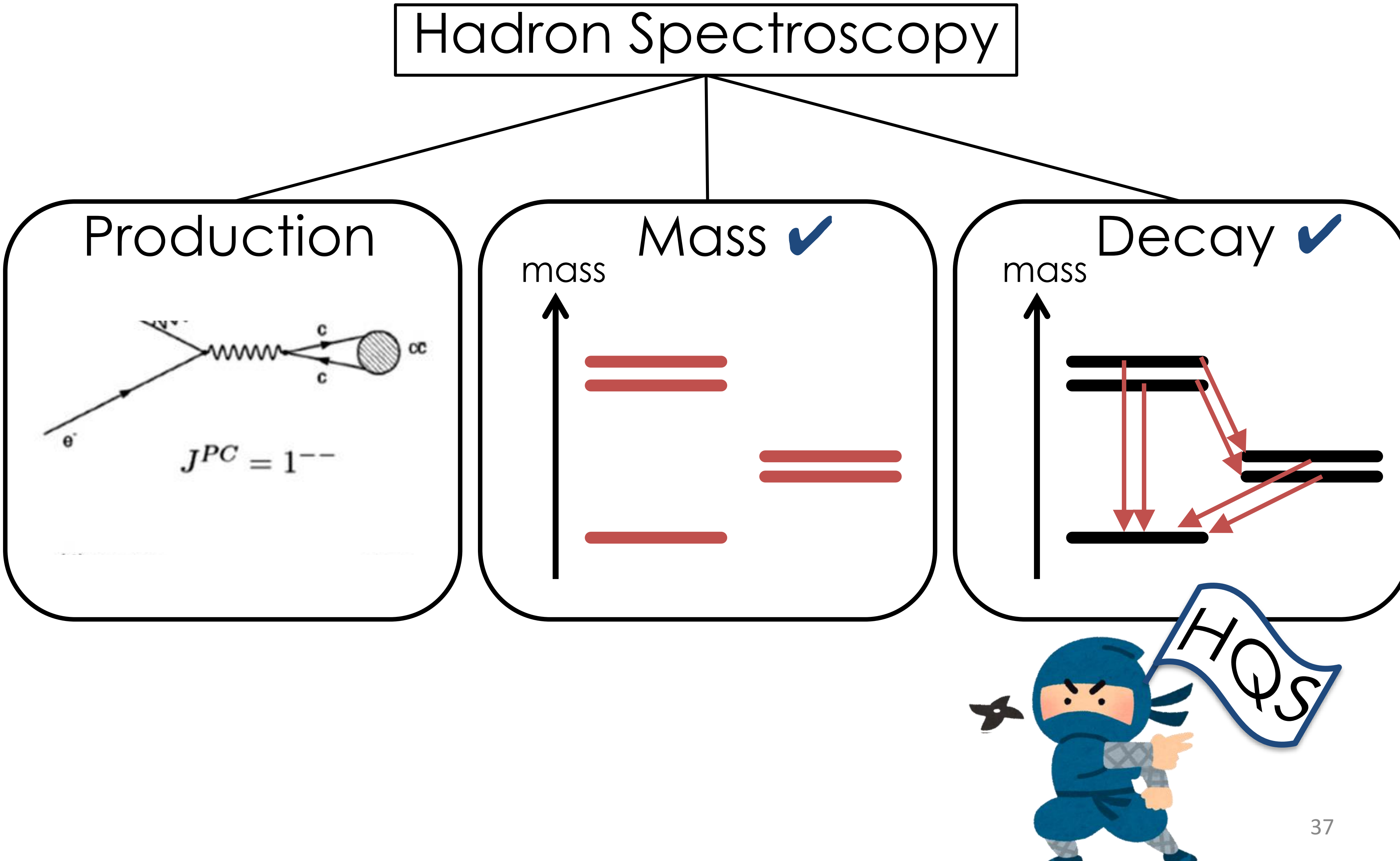
$$\mathcal{L}^{\mathrm{LO+NLO}}_{\pi H_v H_v} = \left(g + \frac{g_1}{M}\right)\mathrm{Tr}\bar{H}_v H_v\gamma_\mu\gamma_5 A^\mu + \frac{g}{2M}\left(\mathrm{Tr}v\cdot iD\bar{H}_v H_v\gamma_\mu\gamma_5 A^\mu - \mathrm{Tr}\bar{H}_v v\cdot iDH_v\gamma_\mu\gamma_5 A^\mu\right) + \frac{g}{4M}\varepsilon_{\mu\nu\rho\sigma}\left(\mathrm{Tr}iD^\nu\bar{H}_v H_v\sigma^{\rho\sigma}A^\mu - \mathrm{Tr}\bar{H}_v iD^\nu H_v\sigma^{\rho\sigma}A^\mu\right) + \frac{g_2}{M}\mathrm{Tr}\bar{H}_v\gamma_\mu\gamma_5 H_v A^\mu + \mathcal{O}(1/M^2)$$

← HQS breaking term (g_2)

NLO: unknown couplings g g_1 g_2

2. Theoretical framework for heavy hadrons

Experiment ⇄ Theory



Do you have questions?